Gravitational waves in RS theories

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Based on works done in collaboration with:
T. Konstandin, E. Megías, G. Nardini, A. Wulzer (2006-2021)

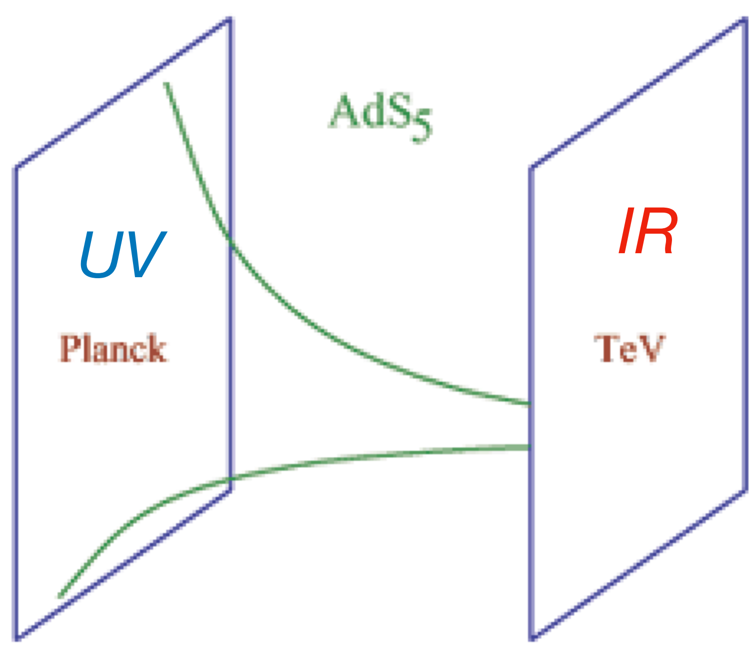
Warped extra dimensions as solution to the hierarchy problem

- Proposed in 1999 by L. Randall and R. Sundrum (RS) *9905221*

- It was based on AdS_5 space with line element $ds^2 = e^{-2A}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2,\ A = ky$
  RS

$$k \sim M_{Pl}\,, \quad \rho \sim TeV = e^{-A(y_1)}k, \quad A(y_1) \sim 35$$

- With two branes

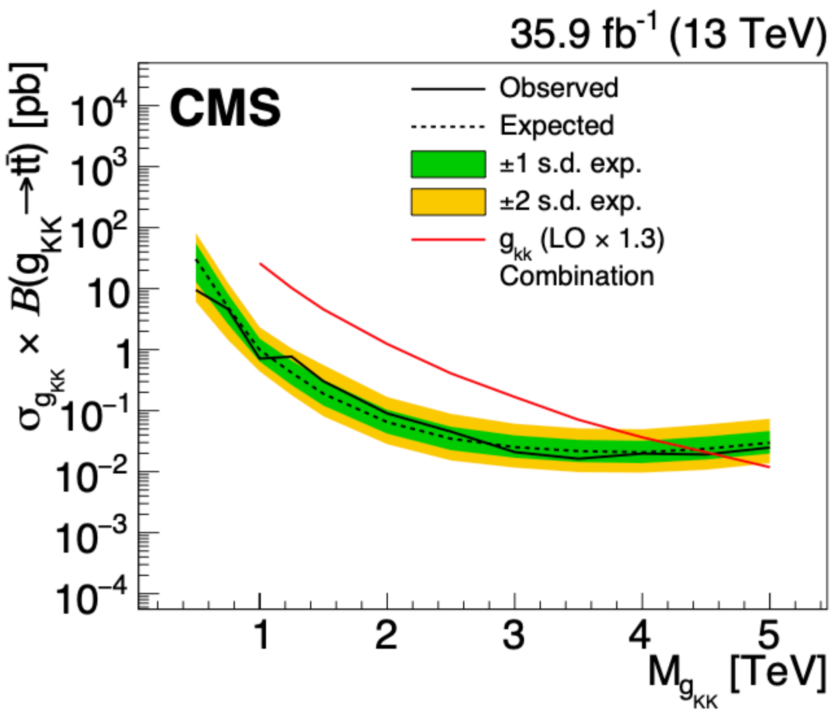


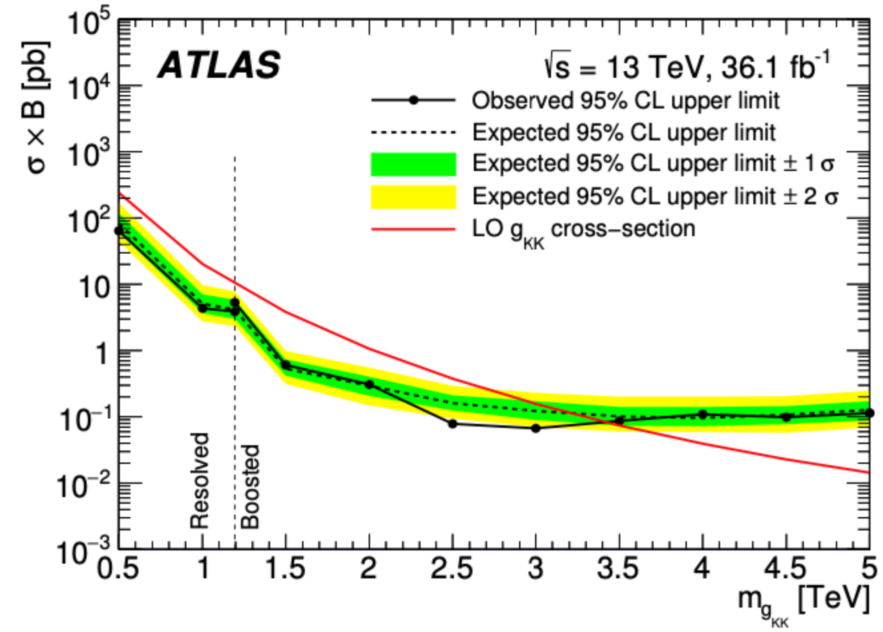
$$AdS \Leftrightarrow CFT \quad correspondence$$

- The Higgs is localized toward the IR brane (composite):
- Heavy (light) fermions are localized toward the IR (UV) brane: composite (elementary)
- The theory predicts TeV KK resonances, localized toward the IR brane (composite)

Collider challenges

- The LHC data are putting severe bounds on the mass of the lightest KK resonances, e.g. for KK gluons:





- These limits point toward the possibility that nature might have chosen values of $\rho \gg TeV$

- The warped factor still explaining the relation $\rho \Leftrightarrow M_{Pl}$

- But a little hierarchy problem of course would remain for $\rho \gg 1\, TeV$

- Heavy KK resonances would escape LHC detection $\Rightarrow$ More energetic colliders…

or by GW's detection…? (this talk)

Phases in the RS theory

- The theory with a warped extra dimension and two branes requires stabilization of the brane distance
- This is achieved by a bulk scalar field ϕ with brane potentials, creating an effective potential in terms of the radion field

Goldberger-Wise, 9907447

- At low temperature the Higgs is confined: confinement phase
- At high temperature the Higgs melts and there is another phase: deconfinement phase
- The phase transition from the deconfined to the confined phase is first order and can give rise to a stochastic GW background (SGWB). *P. Creminelli et al., 0107141*

The confined phase

The 5-dimensional action of the model reads as

$$S = \int d^5x \sqrt{|\det g_{MN}|}\left[-\frac{1}{2\kappa^2}R + \frac{1}{2}g^{MN}(\partial_M\phi)(\partial_N\phi) - V(\phi)\right]$$

$$-\sum_a \int_{B_a} d^4x \sqrt{|\det \bar{g}_{\mu\nu}|}\,\Lambda_a(\phi) + S_{\rm GHY}$$

With brane potentials $\Lambda_a(\phi) = \Lambda_a + \frac{1}{2}\gamma_a(\phi - v_a)^2$

EoM can be expressed in terms of the superpotential $W(\phi)$

$$\phi'(r) = \frac{1}{2}W'(\phi)\,, \quad A'(r) = \frac{\kappa^2}{6}W(\phi) \qquad V(\phi) = \frac{1}{8}\left[W'(\phi)\right]^2 - \frac{\kappa^2}{6}W^2(\phi)$$

s integration constant

The superpotential $W(\phi)$ is expressed in terms of $W = \sum_n s^n W_n$ with

$$W_0(\phi) = \frac{6}{\ell\kappa^2} + \frac{u}{\ell}\phi^2 \qquad W_1(\phi) = \frac{1}{\ell\kappa^2}\left(\frac{\phi}{v_0}\right)^{4/u} e^{\kappa^2(\phi^2 - v_0^2)/3}$$

leads to solutions for the metric and field ϕ

$$\bar{\phi}_0(r) = \bar{v}_0 e^{u\bar{r}} \qquad \bar{\phi}_1(r) = \frac{1}{2u\bar{v}_0} e^{u\bar{r}}\left[e^{(4-2u)\bar{r}} e^{\bar{v}_0^2/3\left(e^{2u\bar{r}}-1\right)} - 1\right]$$

$$A_0(r) = \bar{r} + \frac{\bar{v}_0^2}{12}\left(e^{2u\bar{r}} - 1\right) \qquad A_1(r) = \frac{1}{12}\left[e^{4A_0(\bar{r})} - 1\right] + \frac{2+u}{24u}\left(1 - \frac{\bar{\phi}_0^2}{\bar{v}_0^2}\right)$$

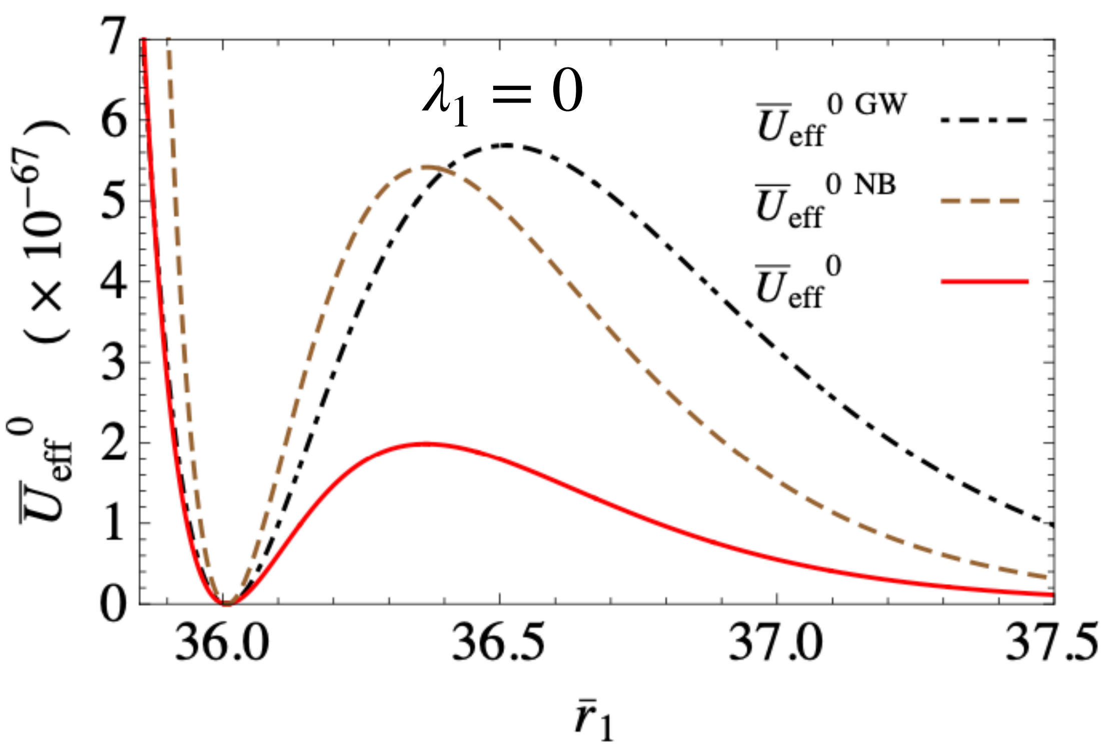
$$s(\bar{r}_1) = \frac{2u\bar{v}_0^2 e^{-u\bar{r}_1}\left(v_1/v_0 - e^{u\bar{r}_1}\right)}{e^{(4-2u)\bar{r}_1} e^{\bar{v}_0^2/3\left(e^{2u\bar{r}_1}-1\right)} - 1}$$

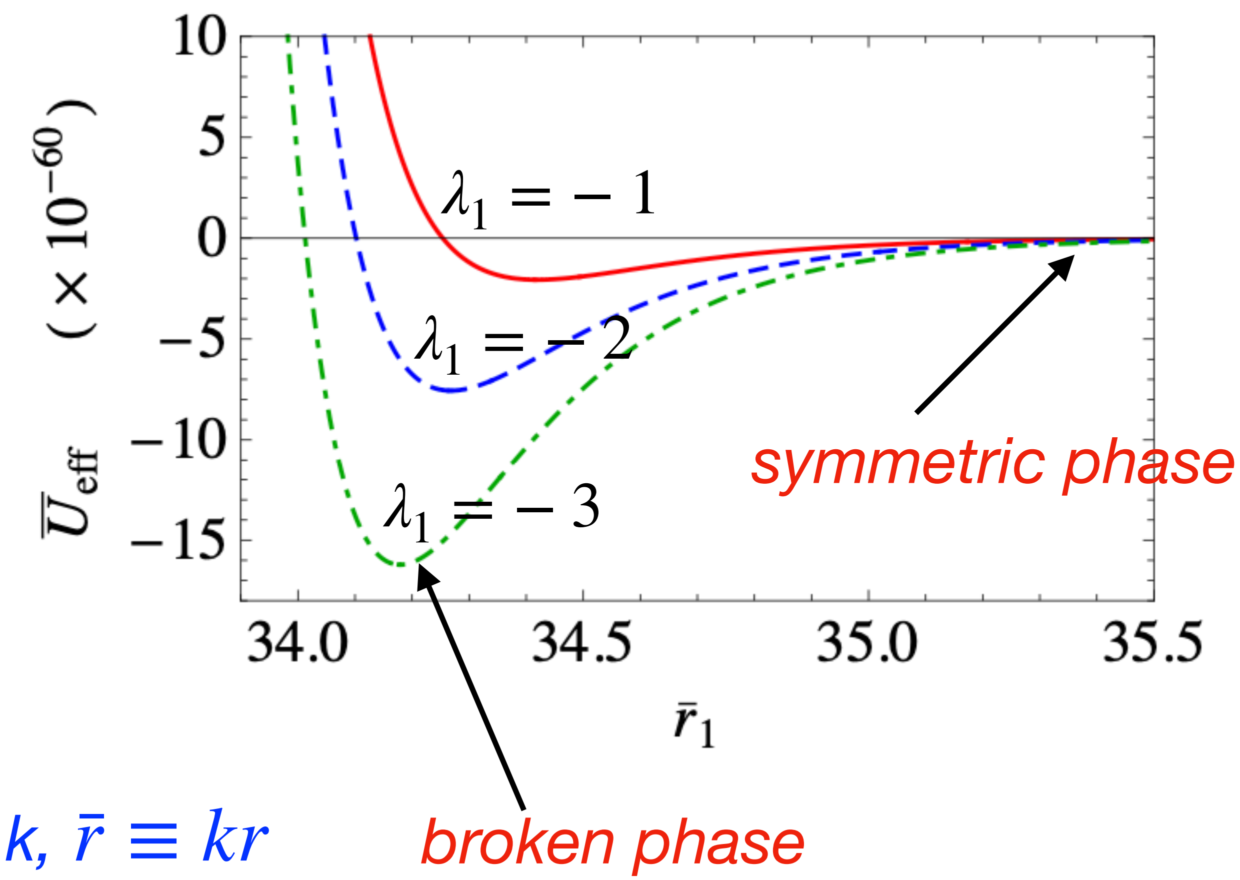
The effective potential as a function of r_1

$$U_{\text{eff}}(r_1) = [\Lambda_1 + W_0(v_1)]e^{-4A_0(r_1)}[1 - 4A_1(r_1)s(r_1)]$$
$$+ s(r_1)\left[e^{-4A_0(r_1)} W_1(v_1) - W_1(v_0)\right]$$

- The effective potential is then a function of the brane distance r_1 and depends on the IR tension λ_1 defined as $\Lambda_1 + W_0(\mathrm{v}_1) \equiv 12\, k M_5^3\, \lambda_1$

E. Megias et al., 2005.04127





Units of k, $\bar{r} \equiv kr$

Potential normalized as $\lim_{r_1 \to \infty} \overline{U}_{\rm eff}(r_1) = 0$

The potential is controlled by λ_1 and $\rho \equiv k e^{-A_0(r_1)}$

The deconfined phase

- At finite temperature the system allows for an additional 5D gravitational solution with a black hole (BH) singularity located in the bulk

$$ds^2_{BH} = -\frac{1}{h(y)}dy^2 + e^{-2A(y)}(h(y)dt^2 - d\vec{x}^{\,2})$$

$blackening\ factor\ h(y_h) = 0$

- In the AdS/CFT correspondence this BH metric describes the high temperature phase of the system where the radion is sent to its symmetric phase, zero VEV

- The phase transition starts when the free energy of the BH deconfined phase equals the free energy of the confined phase

all fields except IR ones

$$F_d(T) = E_0 + F_{min} - \frac{\pi^2}{90} g_d^{eff} T^4$$

all fields

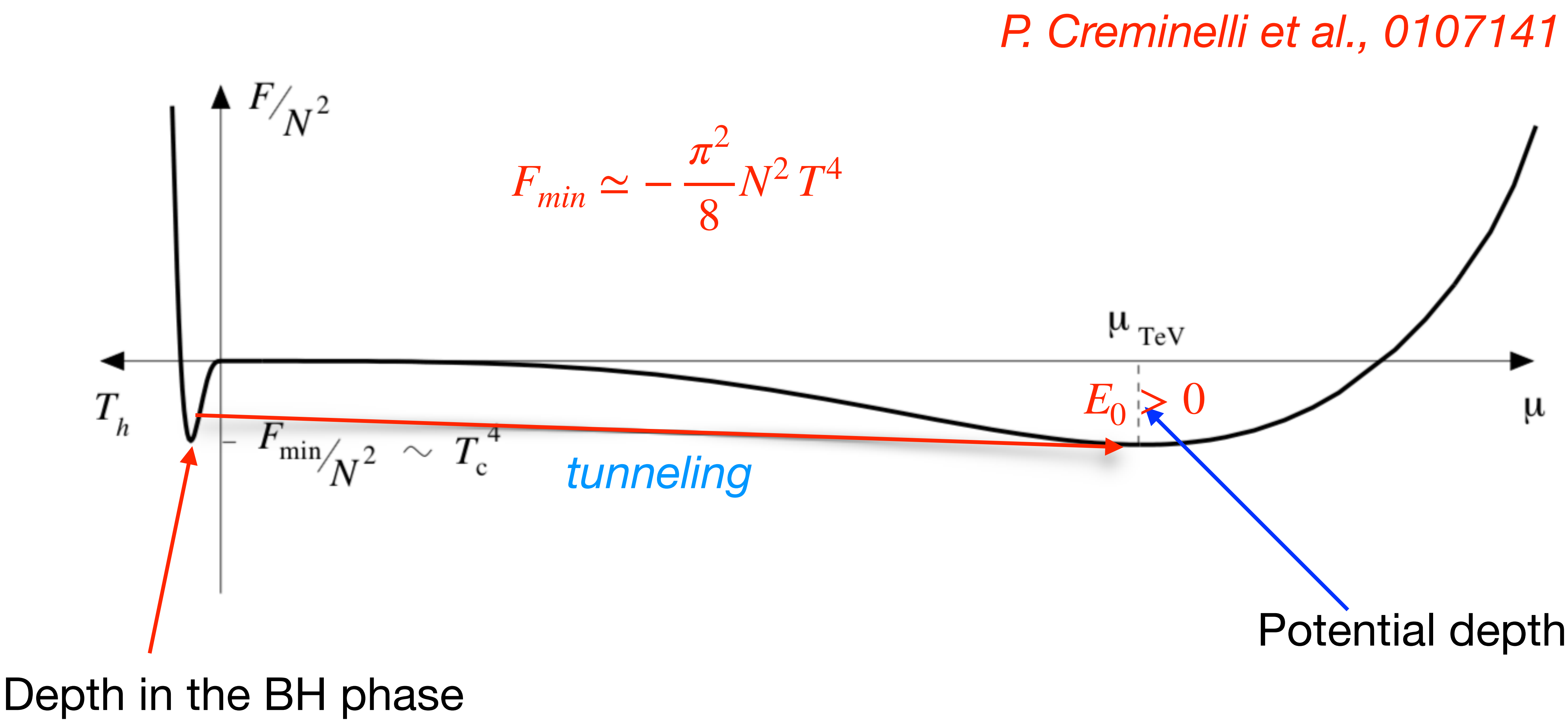
$$F_c(T) = -\frac{\pi^2}{90} g_c^{eff} T^4$$

Potential depth $E_0 > 0$

Depth in the BH phase

Deconfinement/confinement phase transition

Cartoon



The phase transition

- Phase transition takes place when the bubble nucleation rate equals the universe expansion rate at T_n

- This happens when the euclidean action S_n, with O_n symmetry [n=3 (n=4) high temperature (low temperature)], becomes $\sim 10^2$

- In the thick wall approximation:

$$S_3(T) \simeq \frac{\sqrt{3} N^2 \rho^3}{\pi T \sqrt{E_0 + F_{min}}}, \quad S_4(T) \simeq \frac{9 N^2 \rho^4}{4(E_0 + F_{min})}$$

Controlled by λ_1

- The value of nucleation temperature T_n depends on euclidean actions which in turn depend on: (λ_1, N)

- N is the # of degrees of freedom in the holographic theory

- By means of the AdS/CFT duality, N is connected to k through the 5D square gravitational coupling constant $G_5^2 \equiv (k/M_5)^3$ via
$$\frac{N^2}{16\pi^2} = G_5^{-2}$$

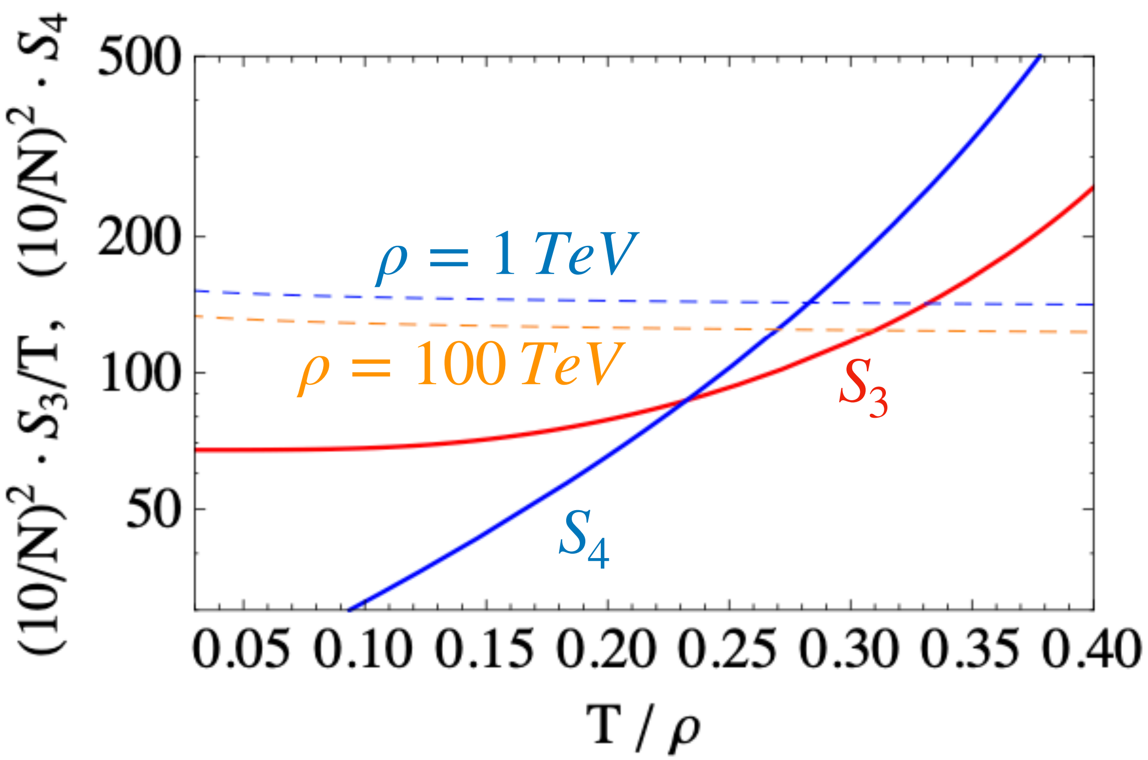
- The 5D gravitational theory is weakly coupled in the limit $N \gg 1$

- However in the limit $N \to \infty$ there is no phase transition as $S_n \to \infty$!!!

- The idea is to keep N large, but not too much to not forbid the phase transition

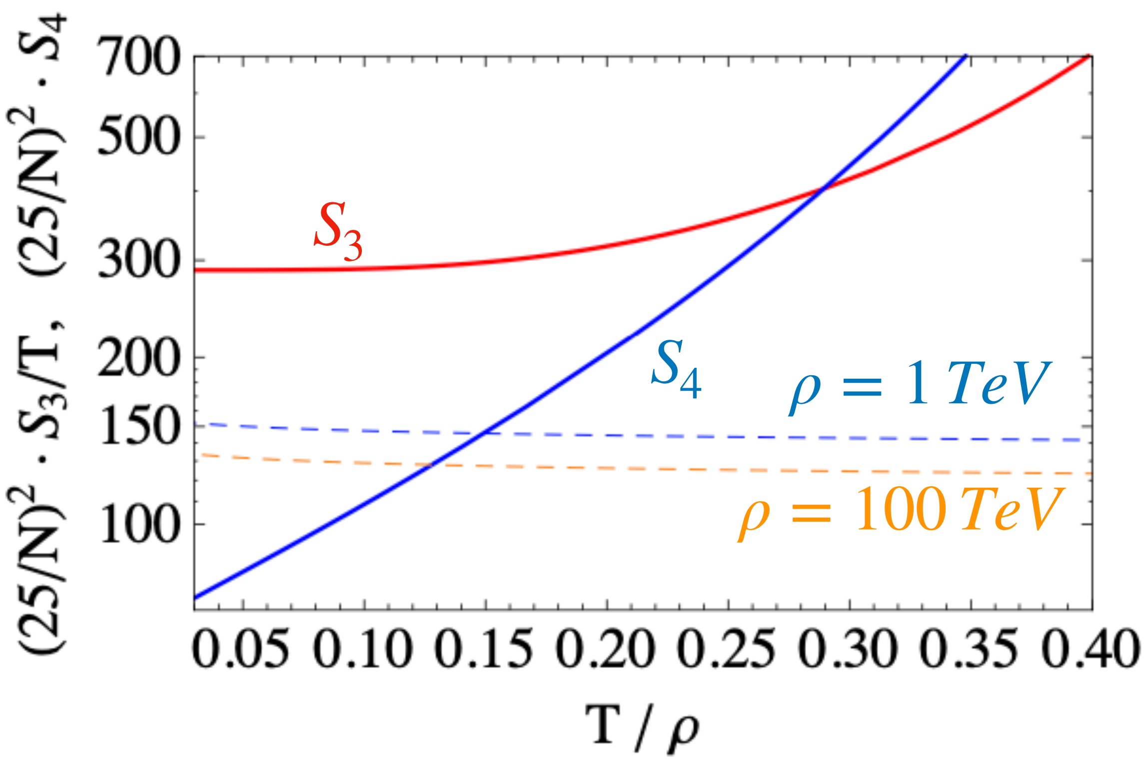
- T_n decreases with increasing N and decreasing $|\lambda_1|$

$N = 10, \quad \lambda_1 = -3$



At high temperature

$N = 25, \quad \lambda_1 = -5$



At low temperature

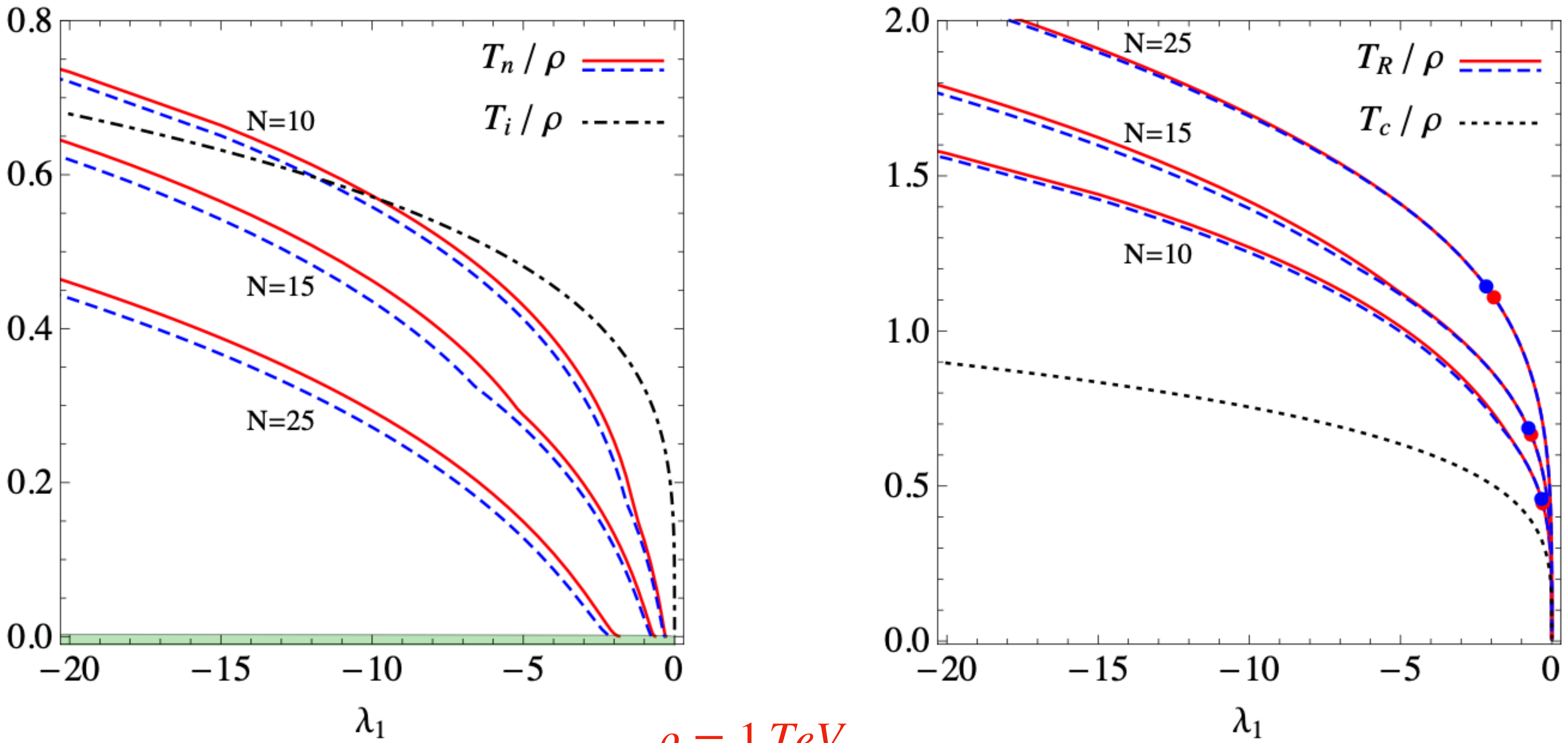
- When the radion $\chi\,[\langle\chi\rangle = \rho]$ phase transition happens the nucleation temperature is smaller than the VEV: $\rho/T_n \gg 1$ and the phase transition is very strong first order

- The cooling can trigger a brief period of cosmological inflation with few e-folds of inflation

- The universe ends up in the confined phase at the reheat temperature $T_R > T_n$

- In most cases (but not always) the reheat temperature is around the ρ scale

The behavior of the different temperatures as functions of N and λ_1



$\rho = 1\,TeV$

$\rho = 100\,TeV$

Gravitational waves

See talks by Hitoshi Murayama and David Weir at this conference

- A cosmological first order phase transition generates a stochastic gravitational wave background (SGWB)

- The power spectrum depends on phase transition quantities

inverse duration of phase transition

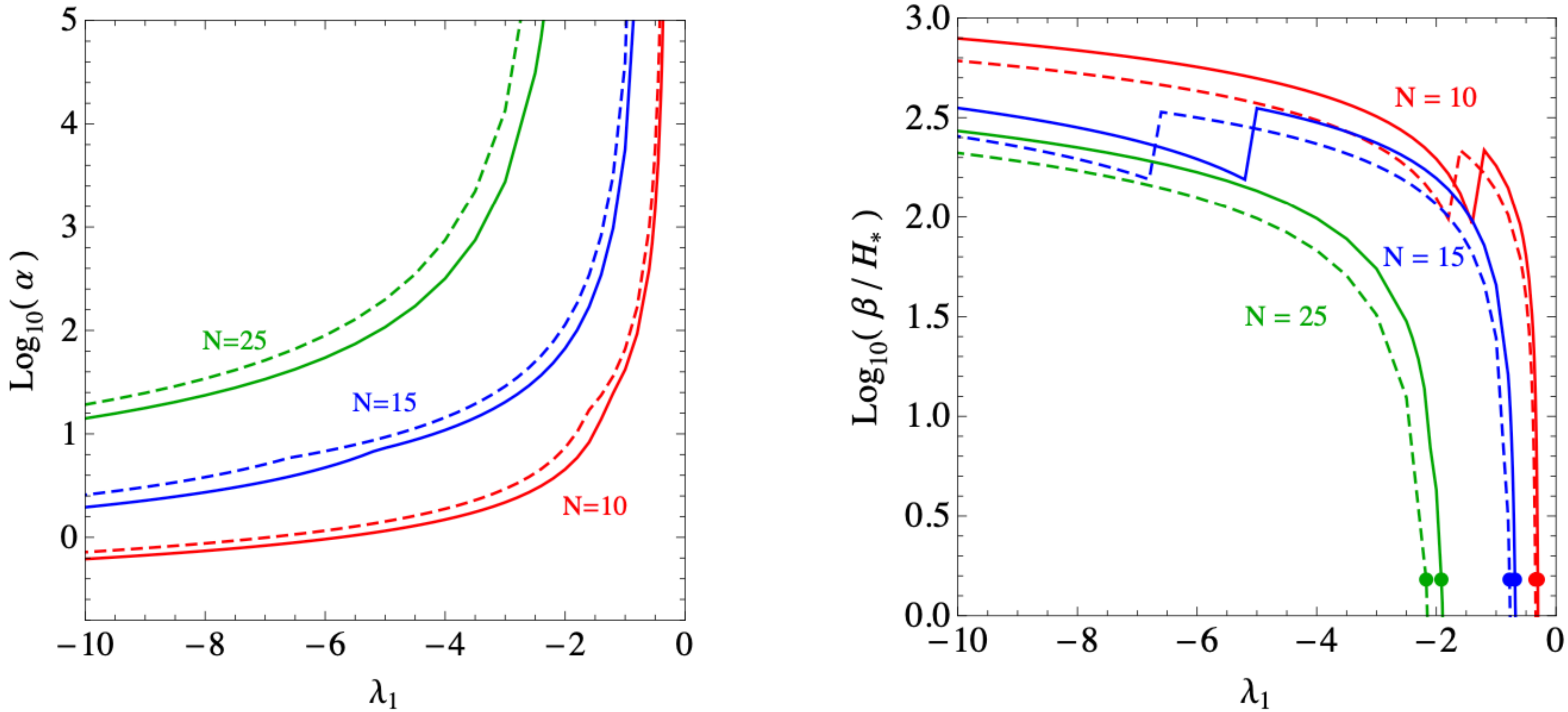
$$\alpha \simeq \frac{F_d(T_n) - F_c(T_n)}{\rho_d(T_n) - E_0}$$

radiation energy density

$$\frac{\beta}{H_\star} \simeq T_n \frac{dS_E}{dT}\bigg|_{T=T_n}$$

- In the next two decades several GW observatories will have the potential to observe, or constrain, the SGWB produced in the confinement/deconfinement phase transition

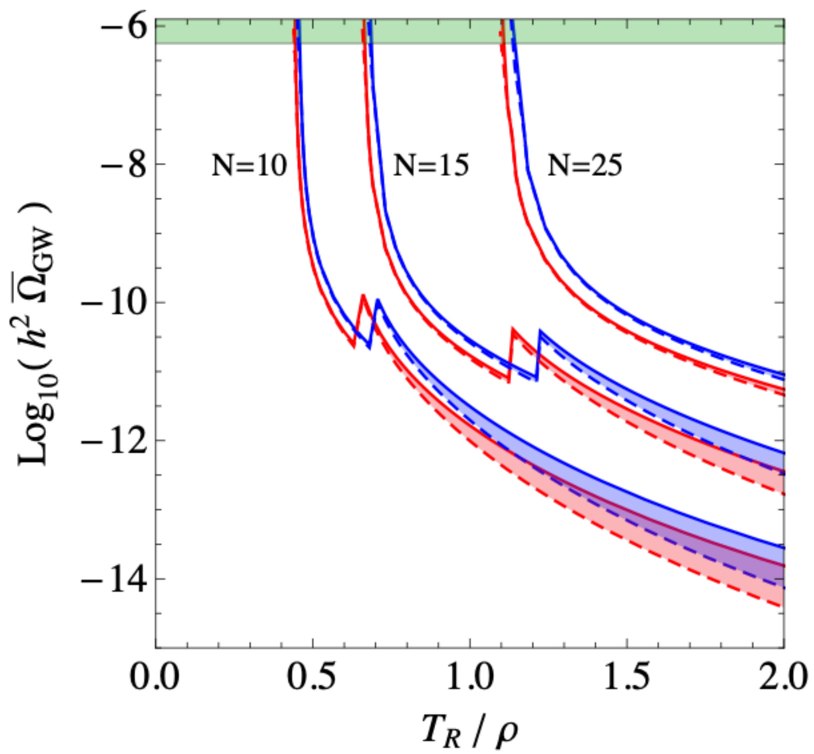
The behavior of the GW parameters in RS as functions of N and λ_1

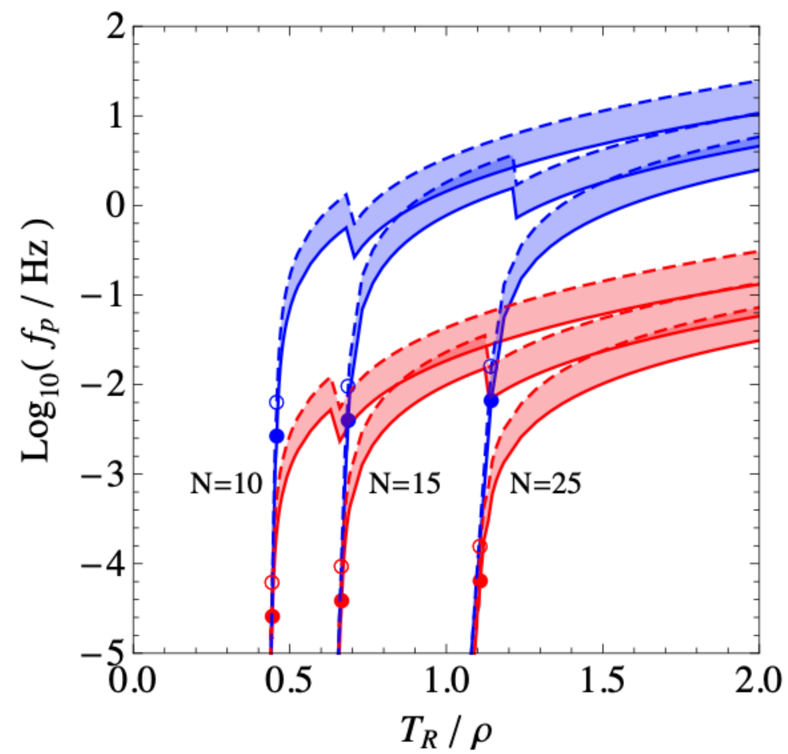


$\rho = 1\, TeV$

$\rho = 100\, TeV$

The amplitude and peak frequency in RS

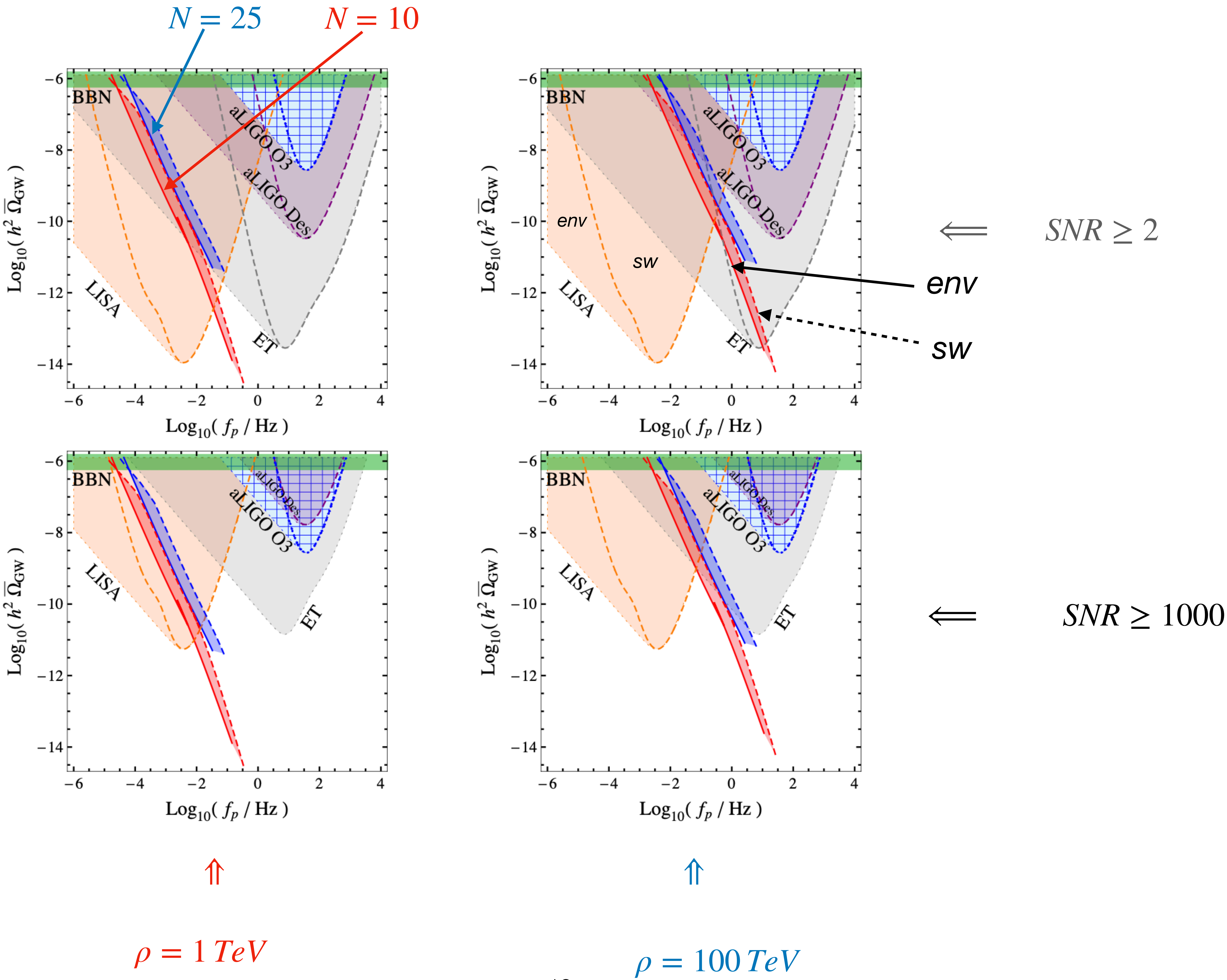




Strips are between envelop (solid) and sound-wave (dashed) approximations

Red strips are for $\rho = 1\,TeV$

Blue strips are for $\rho = 100\,TeV$



N = 25
N = 10
BBN
aLIGO O3
aLIGO Des
LISA
ET
env
sw
Log10(h² Ω̄GW)
Log10(fp / Hz)
⟸ SNR ≥ 2
⟸ SNR ≥ 1000
⇑
ρ = 1 TeV
⇑
ρ = 100 TeV

Conclusions

- Warped extra dimension is an interesting alternative to SUSY to solve the hierarchy problem (dual to CFT,…)
- It triggers a confinement/deconfinement first order phase transition
- Gravitational waves are useful tools to detect the existence of the confinement/deconfinement phase transition

- Future interferometers will thus probe heavy resonances

$$m_{KK} \lesssim 10^5\, TeV\ (LISA)$$

$$10^2\, TeV \lesssim m_{KK} \lesssim 10^8\, TeV\ \ (aLIGO\, Design)$$

$$m_{KK} \lesssim 10^9\, TeV\ (ET)$$

E. Megías, G. Nardini, M.Q., 2005.04127, 2103.02705