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IIT HYDERABAD

LEPTON MASS EFFECT IN EXCLUSIVE SEMILEPTONIC B_C –MESON DECAYS

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PLAN OF PRESENTATION

- Motivation
- Feynman Diagram
- Model framework
- Meson states and meson normalization
- S- Matrix for semi-leptonic decays of B_c meson
- Formfactors, Helicity amplitude, Partial decay width
- q^2-dependence of (i)Formfactors,

  (ii)Helicity amplitude,

  (iii)Partial helicity rates and partial decay rate
- Numerical results and discussion

MOTIVATION

	$\mathcal{R}(D)$	$\mathcal{R}(D^*)$	$\mathcal{R}(J/\psi)$
SM	0.297 ± 0.017	0.252 ± 0.003	$0.25 - 0.28$
2012 ($\mathcal{BABAR}$)	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018$	
2013 ($\mathcal{BABAR}$)	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018$	
2015 (**Belle**)	$0.375 \pm 0.064 \pm 0.026$	$0.293 \pm 0.038 \pm 0.015$	
2015 (**LHCb**)		$0.336 \pm 0.027 \pm 0.030$	
2018 (**LHCb**)		$0.291 \pm 0.019 \pm 0.026$	
2018 (**LHCb**)			$0.71 \pm 0.17 \pm 0.18$
2020 (**Belle**)	$0.307 \pm 0.037 \pm 0.016$	$0.283 \pm 0.018 \pm 0.014$	

$$R(D) = \frac{B(\bar{B} \to D\tau^-\bar{\nu}_\tau)}{B(\bar{B} \to Dl^-\bar{\nu}_l)}$$

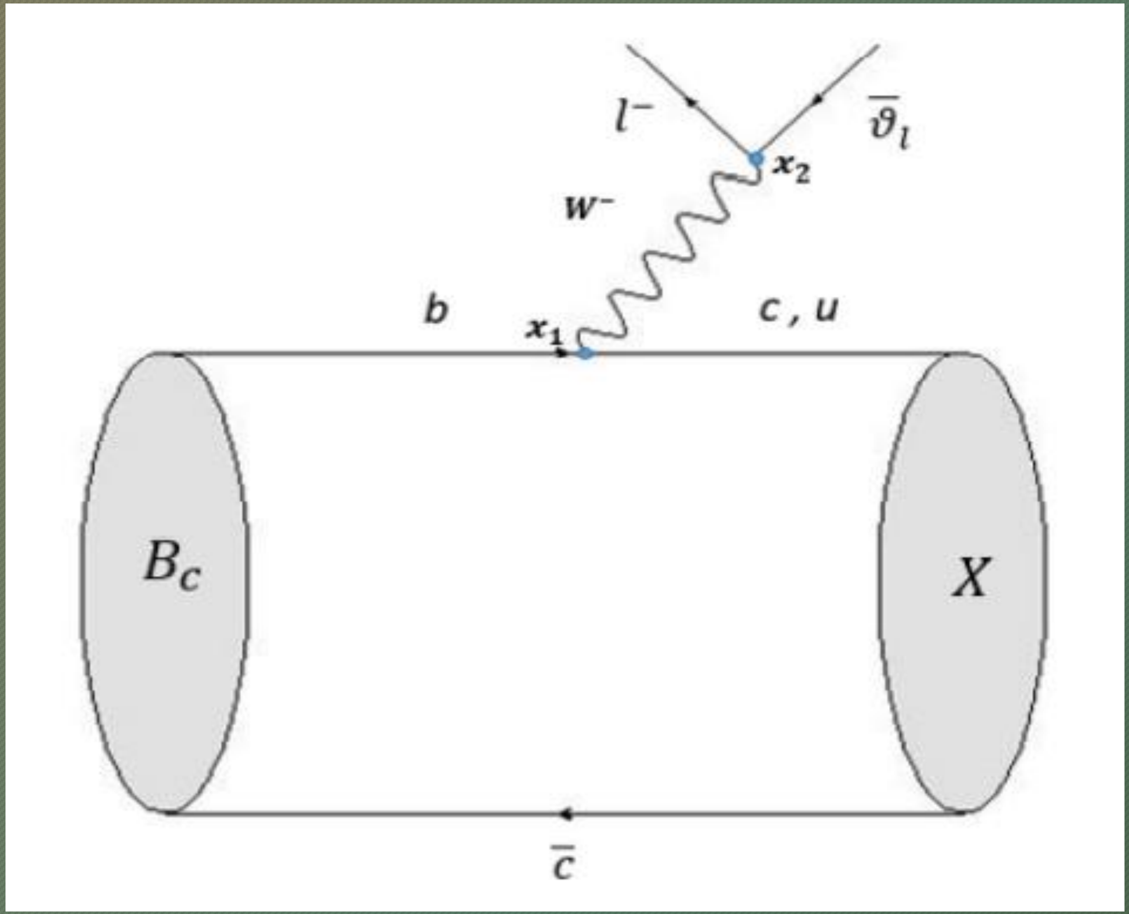
$$R(D^*) = \frac{B(\bar{B} \to D^*\tau^-\bar{\nu}_\tau)}{B(\bar{B} \to D^*l^-\bar{\nu}_l)}$$

$$R(J/\psi) = \frac{B(B_c^+ \to J/\psi\tau^+\nu_\tau)}{B(B_c^+ \to J/\psi\mu^+\nu_\mu)}$$

- **Violate the Lepton Flavor Universality**
- **Cancellation of uncertainty present in V_{cb}**
- **Higgs, leptoquarks and new vector boson**

LOWEST-ORDER FEYNMAN DIAGRAM CONTRIBUTING SEMI-LEPTONIC TRANSITION : $B_c \rightarrow X \ l, \vartheta_l$

$$S_{fi}^{B_c} = \hat{\Lambda}_{\mathrm{Bc}}\,(\overrightarrow{P}, \mathrm{S_B})\; S_{fi}^{b\bar{c}}$$



$B_c \rightarrow \mathrm{X}, l, \vartheta_l$

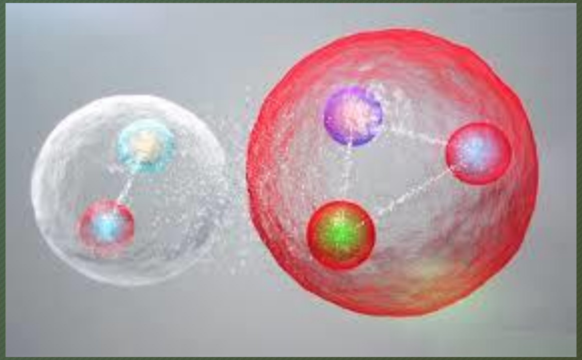
$$B_c^- \rightarrow \eta_c e^- \nu_e$$
$$B_c \rightarrow \eta_c \tau^- \nu_\tau$$
$$B_c^- \rightarrow J/\psi e^- \nu_e$$
$$B_c^- \rightarrow J/\psi \tau^- \nu_\tau$$
$$B_c^- \rightarrow D e^- \nu_e$$
$$B_c^- \rightarrow D \tau^- \nu_\tau$$
$$B_c^- \rightarrow D^* e^- \nu_e$$
$$B_c^- \rightarrow D^* \tau^- \nu_\tau$$

MODEL FRAMEWORK

RELATIVISTIC INDEPENDENT QUARK(RIQ) MODEL

In this model a meson is considered as a colour singlet assembly of constituents (quark & anti-quark) that move relativistically inside the meson bound state with an average flavour independent potential in the form :

$$U(r) = \frac{1}{2}(1+\Upsilon_0)(ar^2+V_0)$$



Where,

r = the relative distance between quark and antiquark inside meson;

a & V_0 = the potential parameters

We incorporate this interaction potential in the lagrangian density to obtain in the form :

$$£= \overline{\psi_q}[\frac{1}{2} i \Upsilon^{\mu}\partial_{\mu} - U(r) - m_q]\, \psi_q(r)$$

The Dirac Equation:

$$(\alpha.p + \beta m_q + U(r))\psi_q(r) = E\, \psi_q(r)$$

Where, $\alpha = \frac{\Upsilon_i}{\Upsilon_0}$ and $\beta = \Upsilon_0$

Solving the Dirac equations we get positive and negative energy solutions known as quark and antiquark orbitals respectively in the general form:

$$\psi^{+}_{nlj}(\vec{r}) = \begin{pmatrix} \frac{i g_{nlj}(r)}{r} \\ \overrightarrow{(\sigma.\hat{r})} \frac{f_{nlj}(r)}{r} \end{pmatrix} \varphi_{ljm_j}(\hat{r})$$

$$\psi^{-}_{nlj}(\vec{r}) = \begin{pmatrix} i\overrightarrow{\sigma}.\frac{\hat{r}\, f_{nj}(r)}{r} \\ g_{nlj}(r)/r \end{pmatrix} (-1)^{j+m_j-l}\, \varphi_{ljm_j}(\hat{r})$$

Where , $g_{nlj}(r) = N_q \left(\frac{r}{r_{nl}}\right)^{l+1} e^{-r^2/2r_{nl}^2}\, L_{n-1}^{l+1/2}\left(\frac{r^2}{r_{nl}^2}\right)$

and $f_{nlj}(r) = -N_q \frac{1}{r_{nl}\lambda_{nl}} \left(\frac{r}{r_{nl}}\right)^{l+2} e^{-r^2/2r_{nl}^2}\, [\, L_{n-2}^{l+3/2}\left(\frac{r^2}{r_{nl}^2}\right) + L_{n-1}^{l+3/2}\left(\frac{r^2}{r_{nl}^2}\right)]$

MOMENTUM PROBABILITY AMPLITUDE

- For n=1 and l=0 (ground state)
- $G_b(\overrightarrow{p_b}) = \frac{i\pi N_b}{2\alpha_b\lambda_b}\sqrt{\frac{(E_{p_b}+m_b)}{E_{p_b}}}(E_{p_b}+E_b)exp\left(-\frac{\vec{p}^2}{4\alpha_b}\right)$
- $\tilde{G}_c(\overrightarrow{p_c}) = \frac{i\pi N_c}{2\alpha_c\lambda_c}\sqrt{\frac{(E_{p_c}+m_c)}{E_{p_c}}}(E_{p_c}+E_c)exp\left(-\frac{\vec{p}^2}{4\alpha_c}\right)$

Using the momentum probability amplitudes for quarks and antiquarks we write the momentum profile function for meson as :

$$\mathcal{G}_{B_c}(\vec{p}_b, \vec{p}_{\bar{c}}) = \sqrt{G_b(\vec{p}_b)\tilde{G}_{\bar{c}}(\vec{p}_{\bar{c}})}$$

MESON STATES AND MESON NORMALIZATION

The meson state at definite momentum reflect the momentum distribution among constituent quark and antiquark

$| B_c(\vec{P}, S_{B_c}) \rangle = \hat{\Lambda}_{Bc} (\vec{P}, S_{B_c}) \; | (\vec{p_b}, \lambda_b);(\vec{p_c}, \lambda_c) \rangle$

Where $| (\vec{p_b}, \lambda_b); (\vec{p_c}, \lambda_c)\rangle = \hat{b}_b^\dagger(\vec{p_b}, \lambda_b)\tilde{b}^\dagger(\vec{p_c}, \lambda_c)|0\rangle$

$$\hat{\Lambda}_{Bc} (\vec{P}, S_B) = \frac{\sqrt{3}}{\sqrt{N_{B_c}(\vec{P})}} \sum_{\delta_b,\delta_{\bar{c}}} \varsigma_{b,\bar{c}}^{B_c} (\lambda_b, \lambda_{\bar{c}}) \int d^3\vec{p_b}\, d^3\vec{p_{\bar{c}}}\; \delta^{(3)}(\vec{p}_b + \vec{p}_{\bar{c}} - \vec{P})\, \mathcal{G}_{B_c}(\vec{p}_b, \vec{p}_{\bar{c}})$$

Imposing Normalisation condition

$$\left\langle B_c\left(\vec{P'}\right) \middle| B_c\left(\vec{P}\right)\right\rangle = \delta^3(\vec{P} - \vec{P}')$$

Where $N(\vec{P}) = \int d\vec{p_b} \; | G(\vec{p}_b , \vec{P} - \vec{p}_b)|^2$

S-Matrix and invariant transition amplitude

$$S_{fi}^{B_c} = \left\langle e^-(k_e\delta_e)\bar{\nu}(k_\nu\delta_\nu)m(\vec{K})\right| \left(\frac{-ig}{2\sqrt{2}}\right) V_{qq'} \left(\frac{-ig}{2\sqrt{2}}\right) \int d^4x_1 d^4x_2 [\bar{\psi}(x_1)\Gamma_\mu \psi(x_1)]\, D_{\mu\vartheta}(x_2 - x_1) \left(\frac{-ig}{2\sqrt{2}}\right) [\bar{\psi}(x_2)\Gamma^\vartheta \psi(x_2)] \left| M(\vec{P})\right\rangle$$

Leptonic part : $\langle e^-(k_e\delta_e)\bar{\nu}(k_\nu\delta_\nu)|\bar{\psi}(x_2)\Gamma^\vartheta \psi(x_2)|0\rangle$

Hadronic part : $\left\langle m(\vec{K})\right| \int d^4x_1 d^4x_2 [\bar{\psi}(x_1)\Gamma_\mu \psi(x_1)] \frac{G_F}{\sqrt{2}} V_{q'q''} \frac{i}{(2\pi)^4} \int d^4q e^{-i(x_2-x_1)q} \left| M(\vec{P})\right\rangle$

The hadronic amplitudes are covariantly expanded in terms of Lorentz-invariant form factors. For the transition type($0^- \to 0^-$), the expansion is

$$\langle X(k)|V_\mu(0)|B(p)\rangle = (p+k)_\mu F_+(q^2) + q_\mu F_-(q^2)$$

For ($0^- \to 1^-$)

$$\begin{aligned}\langle X(k,\epsilon^*)|V_\mu(0) - A_\mu(0)|B(p)\rangle = \ & \frac{1}{(M+m)}\epsilon^\dagger_\nu\Big\{g_{\mu\nu}(p+k)(p-k)A_0(q^2) \\ & +(p+k)_\mu(p+k)_\nu A_+(q^2) + q^\mu(p+k)^\nu A_-(q^2) \\ & +i\epsilon^{\mu\nu\alpha\beta}(p+k)_\alpha q_\beta V(q^2)\Big\}\end{aligned}$$

(FORMFACTORS FOR PSEUDOSCALAR IN FINAL MESON STATE)

$$F_{\pm} = \frac{1}{2M}\left[\sqrt{\frac{4ME_k}{N_M(0)N_m(\vec{k})}}\int d\vec{p}_b\, G_M(\vec{p}_b,-\vec{p}_b)\, G_m(\vec{k}+\vec{p}_b,-\vec{p}_b)\ \boldsymbol{Q}_{\pm}\right]$$

Where,

$$\boldsymbol{Q}_{\pm} = \left\{\frac{\left(E_{p_b}+m_b\right)\left(E_{p_c}+m_c \pm M - E_k\right)+|\vec{p}_b|^2}{\sqrt{4E_{p_b}E_{p_c}\left(E_{p_b}+m_b\right)\left(E_{p_c}+m_c\right)}}\right\}$$

(FORMFACTORS FOR VECTOR MESON IN FINAL STATE)

$$V = \frac{(M+m)}{2M}\left[\sqrt{\frac{4ME_k}{N_M(0)N_m(\vec{k})}}\int d\vec{p}_b\, G_M(\vec{p}_b,-\vec{p}_b)\, G_m(\vec{k}+\vec{p}_b,-\vec{p}_b)\left\{-\sqrt{\frac{(E_{p_b}+m_b)}{4E_{p_b}E_{p_c}(E_{p_c}+m_c)}}\right\}\right]$$

$$A_0 = \frac{1}{(M-m)}\left[\sqrt{\frac{4Mm}{N_M(0)N_m(0)}}\int d\vec{p}_b\, G_M(\vec{p}_b,-\vec{p}_b)\, G_m(\vec{p}_b,-\vec{p}_b)\left\{\frac{3(E^0_{p_c}+m_c)(E_{p_b}+m_b)-|\vec{p}_b|^2}{3\sqrt{4E^0_{p_c}E_{p_b}(E^0_{p_c}+m_c)(E_{p_b}+m_b)}}\right\}\right]$$

$$A_\pm = \frac{-E_K(M\pm m)}{2M(M+2E_k)}\left[T \mp \frac{3(M\mp E_k)}{E_k^2-m^2}\{I-A_0(M-m)\}\right]$$

where $T = J - \left(\frac{M-m}{E_k}\right)A_0$

$$J = \frac{1}{(E_{p_c}+m_c)}\left[\sqrt{\frac{4ME_k}{N_M(0)N_m(\vec{k})}}\int d\vec{p}_b\, G_M(\vec{p}_b,-\vec{p}_b)\, G_m(\vec{k}+\vec{p}_b,-\vec{p}_b)\left\{-\sqrt{\frac{(E_{p_b}+m_b)}{4E_{p_b}E_{p_c}(E_{p_c}+m_c)}}\right\}\right]$$

$$I = \sqrt{\frac{4ME_k}{N_M(0)N_m(\vec{k})}}\int d\vec{p}_b\, G_M(\vec{p}_b,-\vec{p}_b)\, G_m(\vec{p}_b,-\vec{p}_b)\left\{\frac{3(E_{p_c}+m_c)(E_{p_b}+m_b)-|\vec{p}_b|^2}{3\sqrt{4E^0_{p_c}E_{p_b}(E^0_{p_c}+m_c)(E_{p_b}+m_b)}}\right\}$$

PARTIAL DECAY WIDTH

$d\Gamma = \frac{1}{2E_M}\bar{\Sigma}\left|M_{fi}\right|^2 d\pi_3$

where $d\pi_3$=Phase space factor = $(2\pi)^4\delta^4(p-k_1-k_2-K)\frac{d^3k}{(2\pi)^3 2E_k}\frac{d^3k_1}{(2\pi)^3 2E_{k_1}}\frac{d^3k_2}{(2\pi)^3 2E_{k_2}}$

Differential decay rates considering angular distribution with the momentum transfer squared q^2

$$\frac{d\Gamma}{dq^2 d(cos\theta)} = \frac{1}{(2\pi)^3}\sum\frac{G_F^2}{12M_p^2}|V_{bc}|^2\frac{(q^2-m_e^2)}{q^2}\left|\vec{K}\right| L^{\mu\vartheta}H_{\mu\vartheta}$$

$$L^{\mu\vartheta}H_{\mu\vartheta} = \frac{2}{3}(q^2-m_e^2)\left[\frac{8}{3}(1+cos^2\theta)\hat{H}_U + \frac{3}{4}sin^2\theta\hat{H}_L - \frac{3}{4}cos\theta\hat{H}_p + \frac{m_e^2}{2q^2}\left(\frac{3}{4}sin^2\theta\hat{H}_U + \frac{3}{2}cos^2\theta\hat{H}_L + 3cos\theta\hat{H}_{SL} + \frac{1}{2}\hat{H}_S\right)\right]$$

Where $H_U = |H_+|^2 + |H_-|^2$

$H_L = |H_0|^2$

$H_P = |H_+|^2 - |H_-|^2$

$H_S = 3|H_t|^2$

$H_{SL} = Re(H_0 H_t)$

Then substituting the $L^{\mu\vartheta}H_{\mu\vartheta}$ value and integrating the above equation w.r.to $cos\theta$

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_U}{dq^2} + \frac{d\Gamma_L}{dq^2} + \frac{d\tilde{\Gamma}_U}{dq^2} + \frac{d\tilde{\Gamma}_L}{dq^2} + \frac{d\tilde{\Gamma}_S}{dq^2}$$

Where , $\frac{d\Gamma_i}{dq^2} = \frac{1}{(2\pi)^3} \sum \frac{G_F^2}{12M_p^2} |V_{bc}|^2 \frac{(q^2 - m_l^2)}{q^2} |\vec{K}| H_i$

and $\frac{d\tilde{\Gamma}_i}{dq^2} = \frac{m_l^2}{2q^2} \frac{d\Gamma_i}{dq^2}$, $i = U, L, P, S$

Integrating over q^2 we get

$$\Gamma = \Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

For Pseudoscalar Meson In Final state

$$\Gamma = \Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

For Vector meson In Final State

$$\Gamma = \Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

INPUT PARAMETERS

For ground state we take the quark masses, corresponding binding energies and potential parameters:

$$(a, V_0) \equiv (0.017166 GeV^3, -0.1375 GeV)$$

$$(m_u, m_b, m_c) \equiv (0.07875, 4.77659, 1.49276) GeV$$

$$(E_u, E_b, E_c) \equiv (0.47125, 4.76633, 1.57951) GeV$$

q^2-dependence of Formfactors

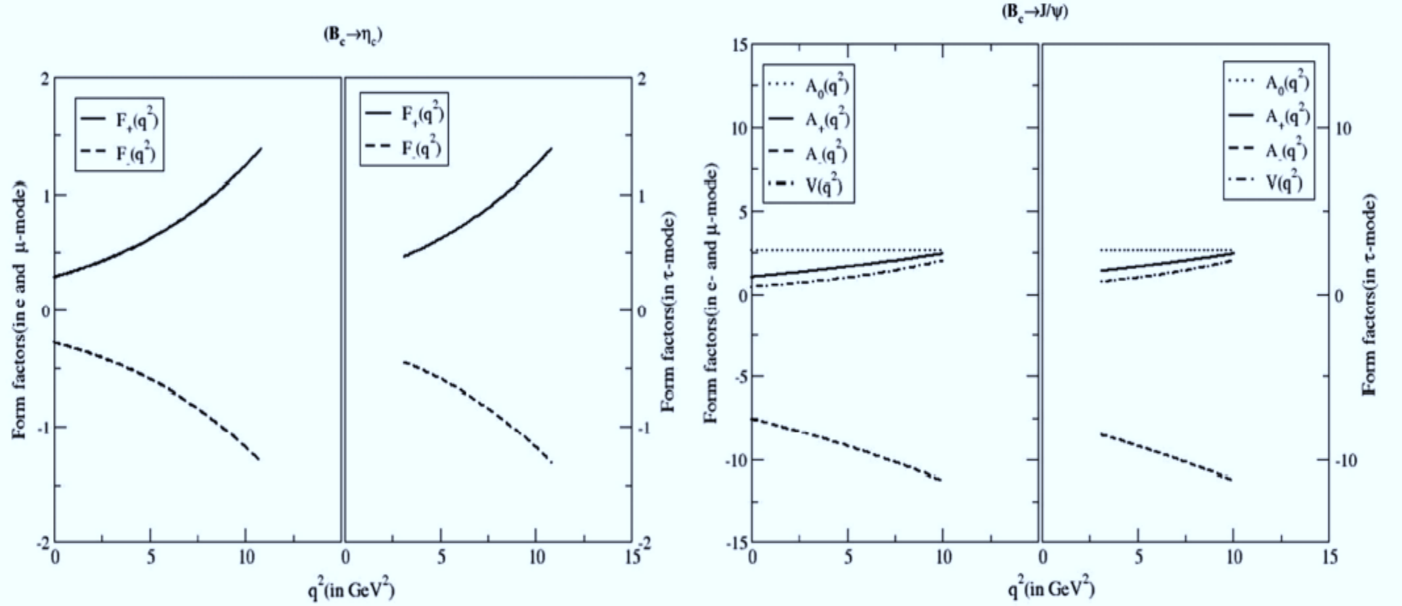


FIG. 2. The q^2-dependence of invariant form factors for semileptonic $B_c \to \eta_c(J/\psi)$ decays.

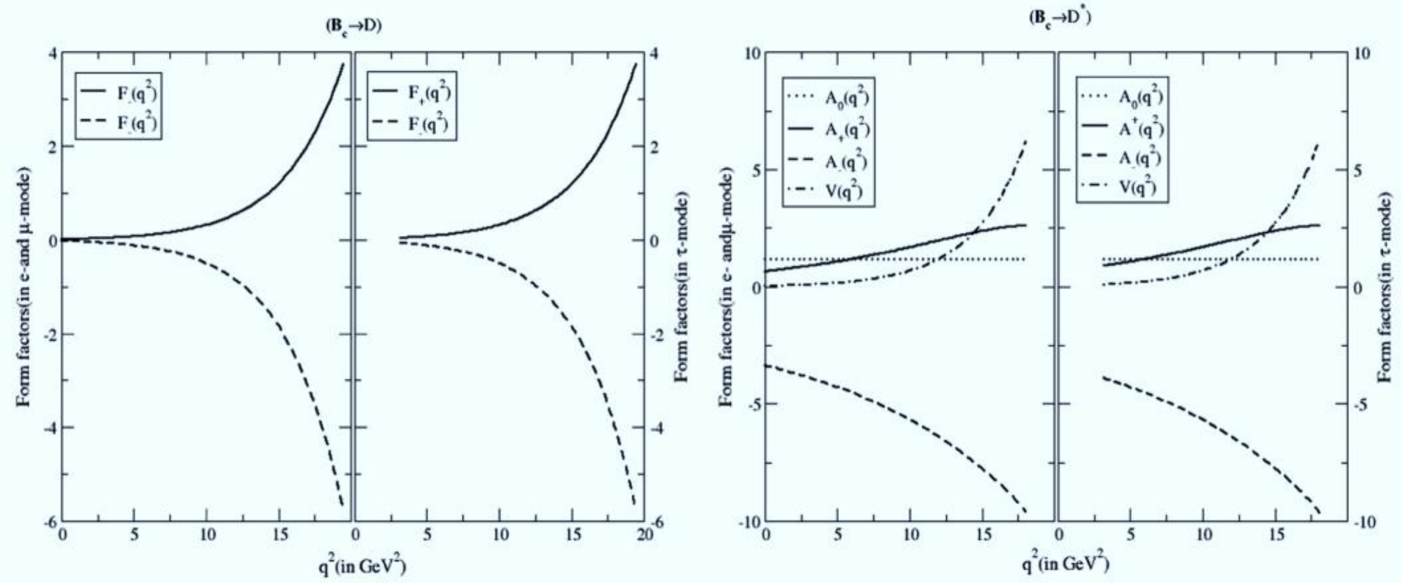


FIG. 3. The q^2-dependence of invariant form factors for semileptonic $B_c \to D(D^*)$ decays.

q^2-dependence of helicity amplitude

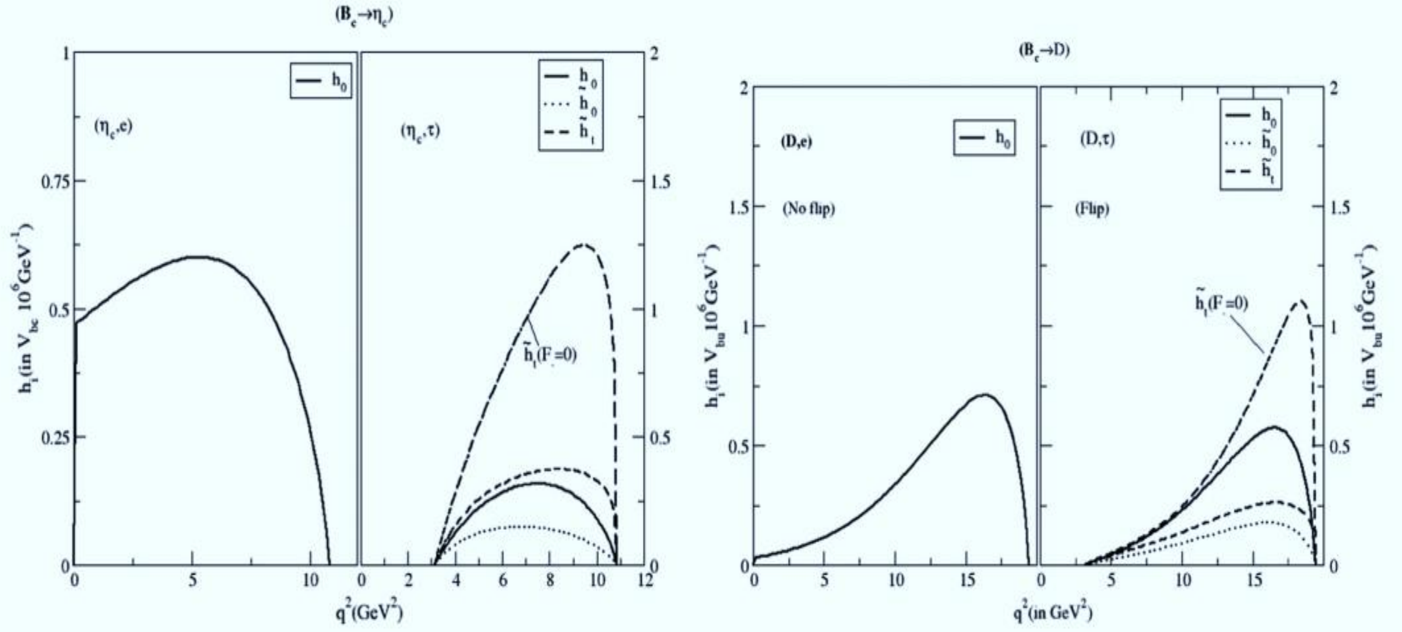


FIG. 4. Reduced helicity amplitudes h_i and $\tilde{h}_i(i = t, 0)$ as functions of q^2 for semileptonic $B_c \to \eta_c$ and $B_c \to D$ decays.

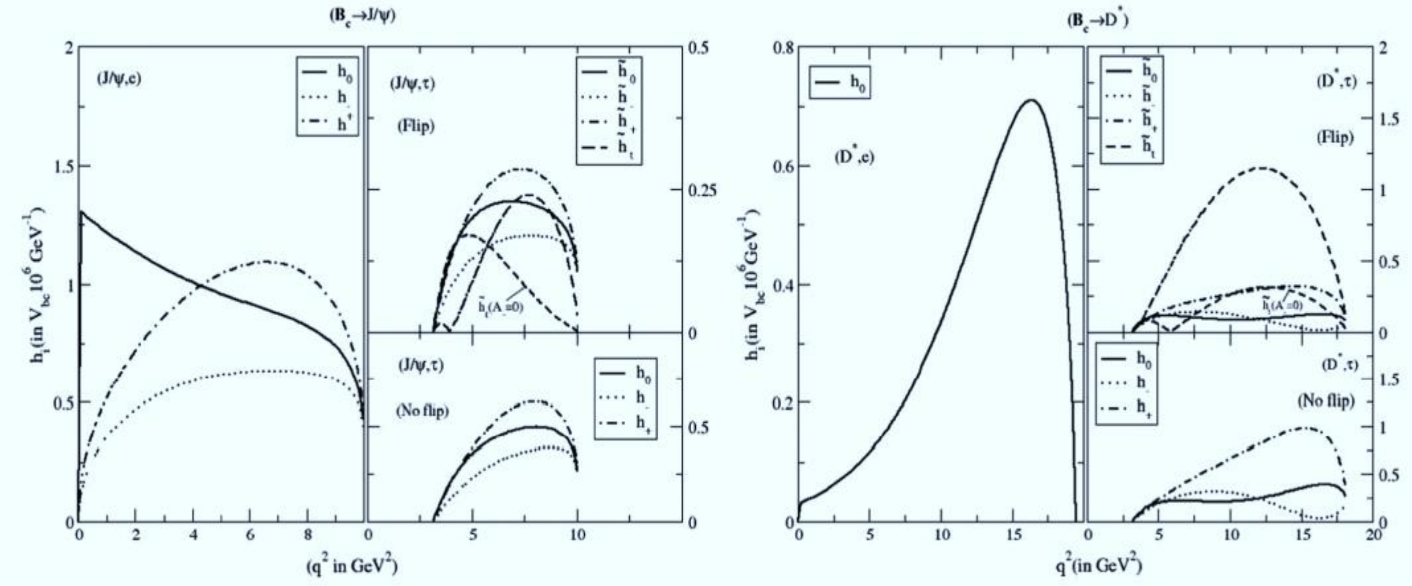


FIG. 5. Reduced helicity amplitudes h_i and $\tilde{h}_i(i = t, +, -, 0)$ as functions of q^2 for semileptonic $B_c \to J/\psi$ and $B_c \to D^*$ decays.

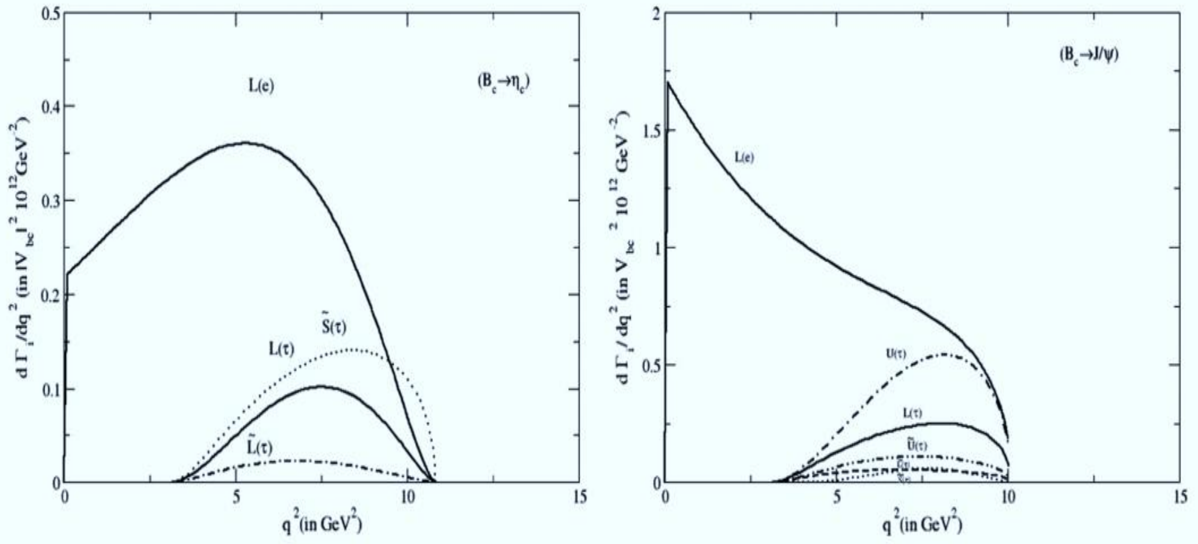


FIG. 6. Partial helicity rates $\frac{d\Gamma_i}{dq^2}$ and $\frac{d\tilde{\Gamma}_i}{dq^2}$ as functions of q^2 for semileptonic $B_c \to \eta_c$ and $B_c \to D$ decays.

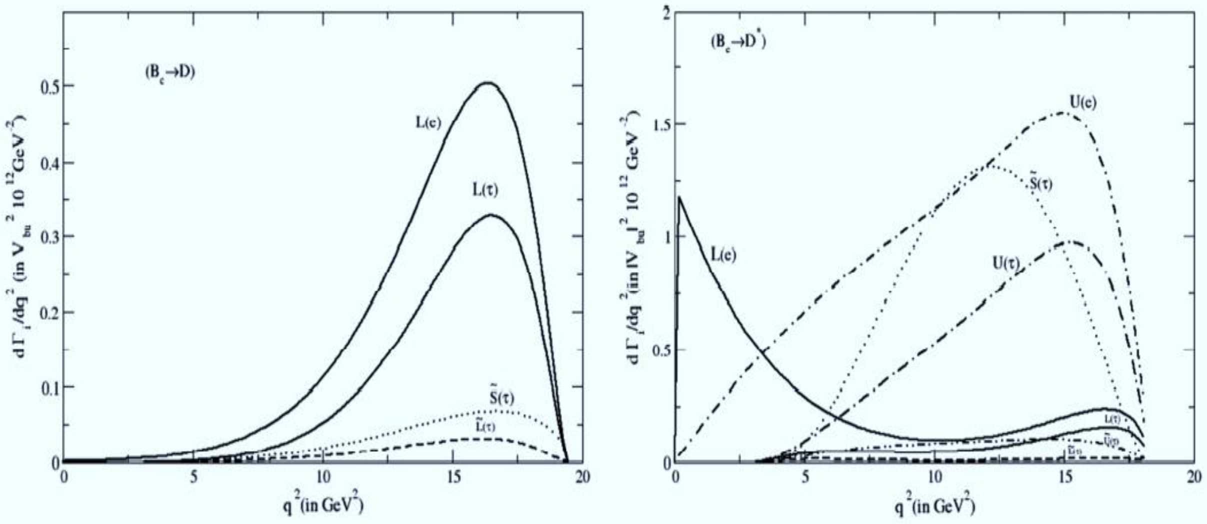


FIG. 7. Partial helicity rates $\frac{d\Gamma_i}{dq^2}$ and $\frac{d\tilde{\Gamma}_i}{dq^2}$ as functions of q^2 for semileptonic $B_c \to J/\psi$ and $B_c \to D^*$ decays.

q^2-dependence of partial helicity rates

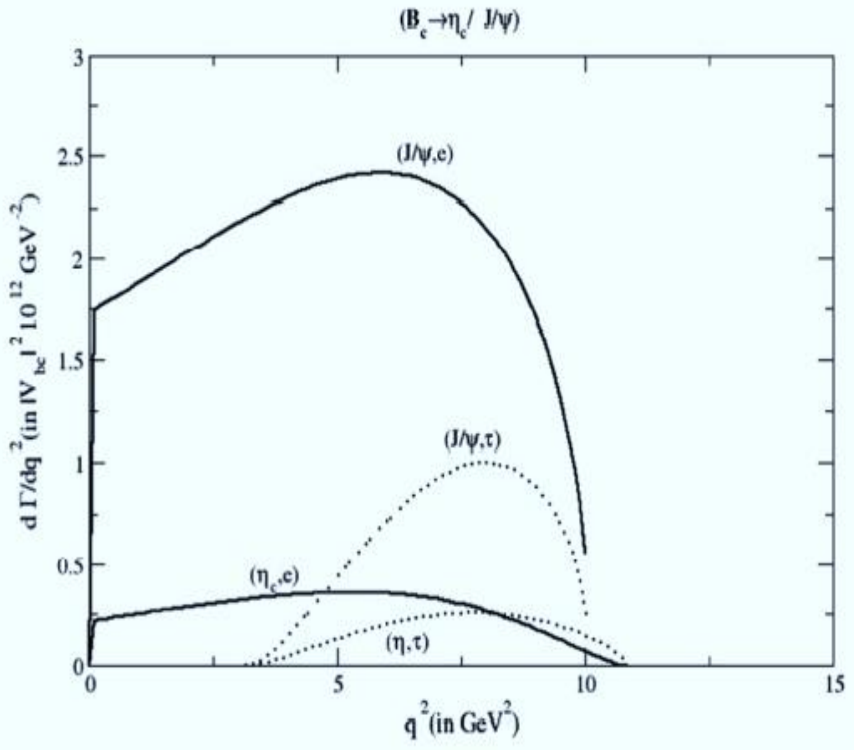


FIG. 8. q^2-spectrum of s.l. decay rates for $B_c \to \eta_c$ and $B_c \to J/\psi$ decays.

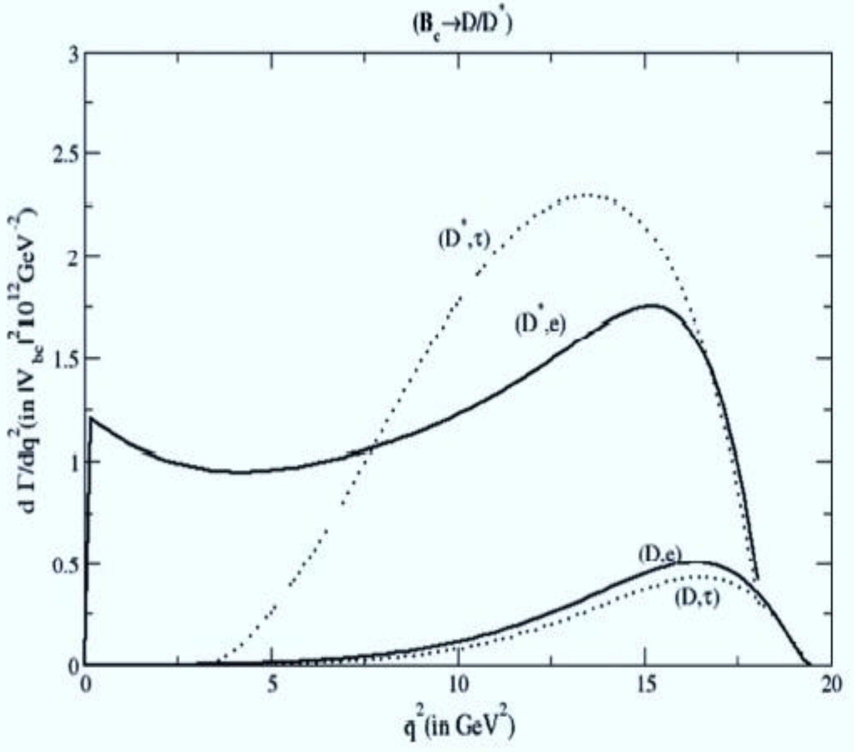


FIG. 9. q^2– spectrum of s.l. decay rates for $B_c \to D$ and $B_c \to D^*$ decays.

q^2-dependence of partial decay rate

NUMERICAL RESULTS AND DISCUSSION

DECAY WIDTH

TABLE I. Helicity rates(in 10^{-15} GeV) of semileptonic B_c-meson decays into charmonium and charm-meson state:

Decay mode	U	$\tilde{U}$	L	$\tilde{L}$	P	S	$\tilde{S}$	$\widetilde{SL}$	Γ
$B_c^- \to \eta_c e^- \nu_e$			4.844	4.432×10^{-7}			15.397×10^{-7}	4.712×10^{-7}	4.844
$B_c \to \eta_c \tau^- \nu_\tau$			0.756	0.172			1.194	0.253	2.122
$B_c^- \to J/\psi e^- \nu_e$	18.634	6.052×10^{-7}	16.283	27.813×10^{-7}	8.368	1.188	66.653×10^{-7}	22.856×10^{-7}	34.918
$B_c^- \to J/\psi \tau^- \nu_\tau$	3.823	0.846	1.922	0.437	1.704	0.614	0.307	0.197	7.336
$B_c^- \to D e^- \nu_e$			0.047	4.611×10^{-10}			1.072×10^{-9}	4.038×10^{-10}	0.047
$B_c^- \to D \tau^- \nu_\tau$			0.028	0.003			0.007	0.0027	0.038
$B_c^- \to D^* e^- \nu_e$	0.2439	4×10^{-9}	0.078	7.760×10^{-9}	0.169	0.081	4.092×10^{-8}	3.648×10^{-9}	0.322
$B_c^- \to D^* \tau^- \nu_\tau$	0.113	0.015	0.0156	0.0021	0.092	0.046	0.151	0.0094	0.297

BRANCHING FRACTION

TABLE II. Branching ratios(in%) of semileptonic B_c decays into ground state charmonium and charm meson state:

Decay mode	This work	[24]	[46]	[23]	[55,56]	[10]	[44]	[49]	[25]	[11,12]	[57]	[58]
$B_c \to \eta_c e\nu$	0.37	0.83	0.81	0.98	0.75	0.97	0.59	0.44	0.95	0.86	0.162	0.45
$B_c \to \eta_c \tau\nu$	0.16	0.27	0.22	0.27	0.23	⋯	0.20	0.14	0.24	⋯	⋯	⋯
$B_c \to J/\psi e\nu$	2.68	2.19	2.07	2.30	1.9	2.35	1.20	1.01	1.67	2.33	1.67	1.37
$B_c \to J/\psi \tau\nu$	0.56	0.61	0.49	0.59	0.48	⋯	0.34	0.29	0.40	⋯	⋯	⋯
$B_c \to D e\nu$	0.0037	⋯	0.0035	0.018	⋯	0.004	0.004	0.0032	0.0033	⋯	⋯	⋯
$B_c \to D \tau\nu$	0.0029	⋯	0.0021	0.0094	0.002	⋯	⋯	0.0022	0.0021	⋯	⋯	⋯
$B_c \to D^* e\nu$	0.0251	⋯	0.0038	0.034	⋯	0.018	0.018	0.011	0.006	⋯	⋯	⋯
$B_c \to D^* \tau\nu$	0.0230	⋯	0.0022	0.019	0.008	⋯	⋯	0.006	0.0034	⋯	⋯	⋯

RATIOS OF BRANCHING FRACTION

Ratio of Branching Fractions(R)	This work	[25]	[46]	[49]
$R_{\eta_c} = \frac{\mathcal{B}(B_c\to\eta_c l\nu)}{\mathcal{B}(B_c\to\eta_c \tau\nu)}$	2.312	3.96	3.68	3.2
$R_{J}/\psi = \frac{\mathcal{B}(B_c\to J/\psi l\nu)}{\mathcal{B}(B_c\to J\psi \tau\nu)}$	4.785	4.18	4.22	3.4
$R_D = \frac{\mathcal{B}(B_c\to D l\nu)}{\mathcal{B}(B_c\to D\tau\nu)}$	1.275	1.57	1.67	1.42
$R_{D^*} = \frac{\mathcal{B}(B_c\to D^* l\nu)}{\mathcal{B}(B_c\to D^* \tau\nu)}$	1.091	1.76	1.72	1.66

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