Hadronic corrections to the muon $g - 2$

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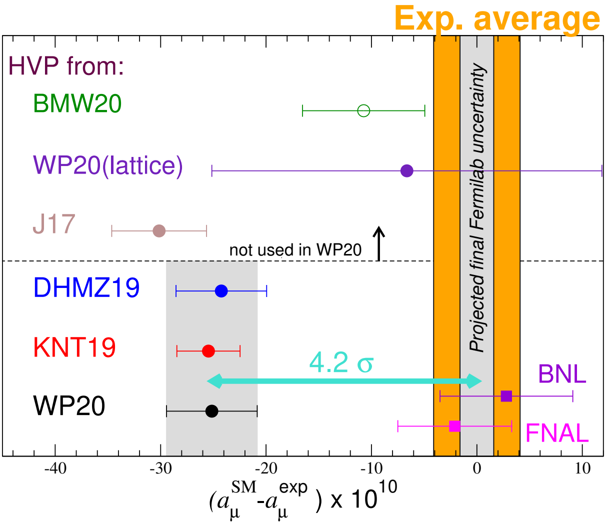
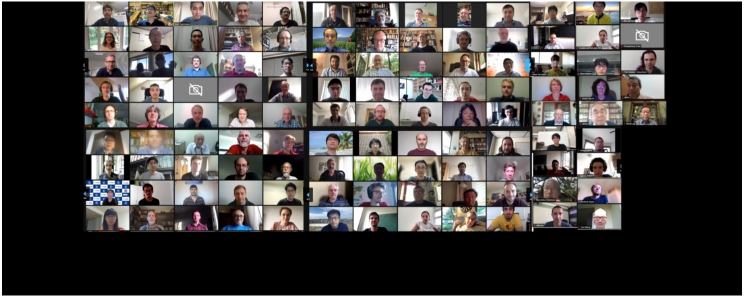
Anomalies 2021

The situation after the Fermilab announcement



The Muon $g-2$ Theory Initiative



- Formed in 2017, series of workshops since (last plenary one virtually at KEK in June 2021) https://www-conf.kek.jp/muong-2theory/
- Map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results
- White paper 2006.04822: **The anomalous magnetic moment of the muon in the Standard Model** https://muon-gm2-theory.illinois.edu/

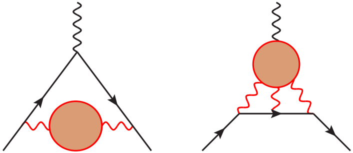
The anomalous magnetic moment of the muon in the Standard Model

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

see talks by M. Passera (SM theory overview), L. Roberts (experiment), C. Lehner (lattice)

The SM prediction for $(g-2)_\mu$: hadronic effects



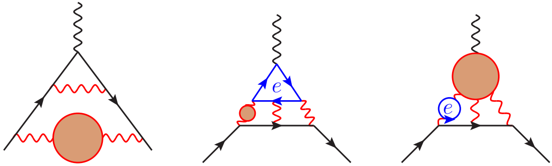
- **Hadronic vacuum polarization**: need hadronic two-point function

$$\Pi_{\mu\nu} = \langle 0|T\{j_\mu j_\nu\}|0\rangle$$

- **Hadronic light-by-light scattering**: need hadronic four-point function

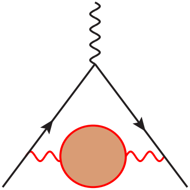
$$\Pi_{\mu\nu\lambda\sigma} = \langle 0|T\{j_\mu j_\nu j_\lambda j_\sigma\}|0\rangle$$

- Main challenge to match experimental precision: hadronic contributions

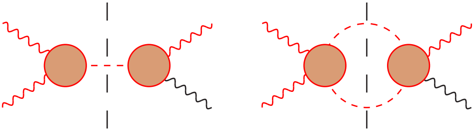


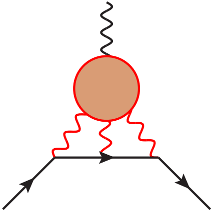
- Once $\Pi_{\mu\nu}$ and $\Pi_{\mu\nu\lambda\sigma}$ known, higher-order iterations determined
- Standard for NLO HVP Calmet et al. 1976
- NNLO HVP found to be relevant recently Kurz et al. 2014
- NLO HLbL already further suppressed Colangelo et al. 2014

Hadronic light-by-light scattering



- In the past: hadronic models, inspired by various QCD limits, but error estimates difficult
- Dispersive approach: use again **analyticity**, **unitarity**, **crossing**, and **gauge invariance** for data-driven approach Colangelo, MH, Procura, Stoffer 2014, ...
- For simplest intermediate states: relation to $\pi^0 \to \gamma^*\gamma^*$ **transition form factor** and $\gamma^*\gamma^* \to \pi\pi$ **partial waves**





HLbL scattering: white paper strategy

- Reference points:

$$a_\mu^{\text{HLbL}}\Big|_{\text{“Glasgow consensus” 2009}} = 105(26) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}}\Big|_{\text{Jegerlehner, Nyffeler 2009}} = 116(39) \times 10^{-11}$$

- Strategy in the white paper
  - Take well-controlled results for the low-energy contributions
  - Combine errors in quadrature
  - Take best guesses for medium-range and short-distance matching
  - Add these errors linearly, since errors hard to disentangle at the moment
- **Recommended value**

$$a_\mu^{\text{HLbL}}\,(\text{phenomenology}) = 92(19) \times 10^{-11}$$

- **Lattice QCD**: first complete calculation RBC/UKQCD 2019 (Mainz 2021 after WP, see below)

$$a_\mu^{\text{HLbL}}\,(\text{lattice, } uds) = 79(35) \times 10^{-11}$$

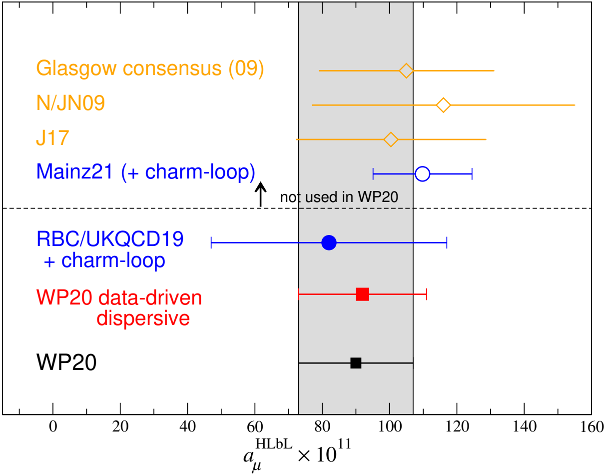
$\hookrightarrow$ can combine with phenomenological value

HLbL scattering: white paper details

Contribution	PdRV(09)	N/JN(09)	J(17)	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	$-19(19)$	$-19(13)$	$-20(5)$	$-16.4(2)$
S-wave $\pi\pi$ rescattering	$-7(7)$	$-7(2)$	$-5.98(1.20)$	$-8(1)$
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	$\}$ $-1(3)$
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

All to be compared to projected **final E989 precision**: $\Delta a_\mu^{\text{E989}} = 16 \times 10^{-11}$

Status of HLbL scattering



Hadronic vacuum polarization from e^+e^- data

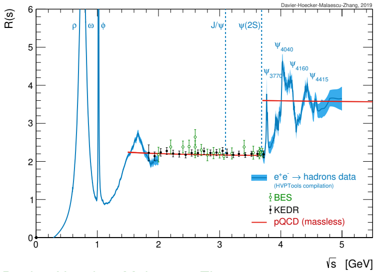
HVP from e^+e^- data

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} \text{d}s \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s) \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons}(+\gamma))(s)$$

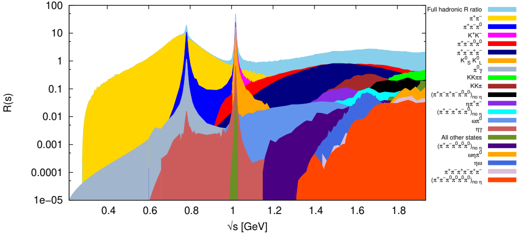
$$= 6931(40) \times 10^{-11}$$

- The "theory" prediction a_μ^{SM} is actually **based on experiments** (ISR, direct scan)
  $\hookrightarrow$ propagation of experimental uncertainties
- Uncertainty estimate includes:
  - different methodologies for the combination of data sets Davier et al. 2019, Keshavarzi et al. 2020
  - conservative estimate of systematic errors from tensions in the data
  - cross checks from analyticity/unitarity constraints Colangelo et al. 2018, Ananthanarayan et al. 2018, Davier et al. 2019, MH et al. 2019
  - full NLO radiative corrections Campanario et al. 2019

Hadronic vacuum polarization from e^+e^- data



Davier, Hoecker, Malaescu, Zhang 2019



Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
  - up to about 2 GeV: sum of exclusive channels
  - above: inclusive data + narrow resonances + pQCD
- **Tensions in the data**: most notably between KLOE and BaBar 2π data

Breakdown of the HVP error

HVP from e^+e^- data

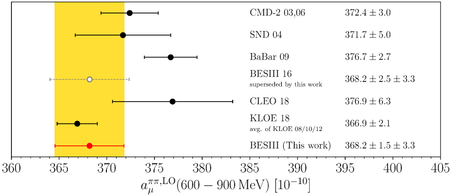
$$a_\mu^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error

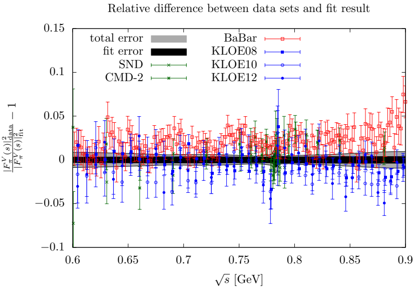
  $\hookrightarrow$ would get $4.2\sigma \to 4.8\sigma$ when ignoring additional systematic error
- There was broad consensus to adopt **conservative error estimates**

  $\hookrightarrow$ **merging procedure** in WP20 covers tensions in the data and different methodologies for the combination of data sets
- Systematic effect dominated by [fit w/o KLOE - fit w/o BaBar]/2

Cross checks from analyticity and unitarity



BESIII 2009.05011



Colangelo, MH, Stoffer 2018

- For “simple” channels $e^+e^- \to 2\pi, 3\pi$ can derive form of the cross section from **general principles of QCD** (analyticity, unitarity, crossing symmetry)
  $\hookrightarrow$ strong cross check on the data sets (covering about 80% of HVP)
- Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests

Cross checks from analyticity and unitarity

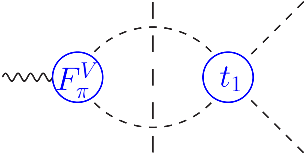
- In direct integration: local combination of data

  $\hookrightarrow$ **local scale factor** in case tensions arise

- $e^+e^- \to 2\pi$ determined by pion vector form factor F_π^V

- **Unitarity** for **pion vector form factor**

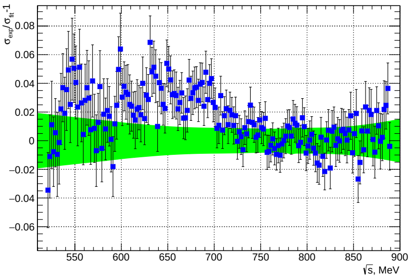
$$\text{Im}\, F_\pi^V(s) = \theta\big(s - 4M_\pi^2\big) F_\pi^V(s) e^{-i\delta_1(s)} \sin \delta_1(s)$$

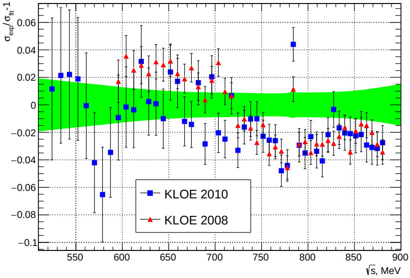


  $\hookrightarrow$ **final-state theorem**: phase of F_π^V equals $\pi\pi$ P-wave phase δ_1 Watson 1954

- Can derive a **global fit function** that depends on
  - Two values of δ_1 (elastic 2π intermediate states)
  - ω mass, width, and residue (3π intermediate states)
  - Conformal polynomial (4π intermediate states)

New data since WP20

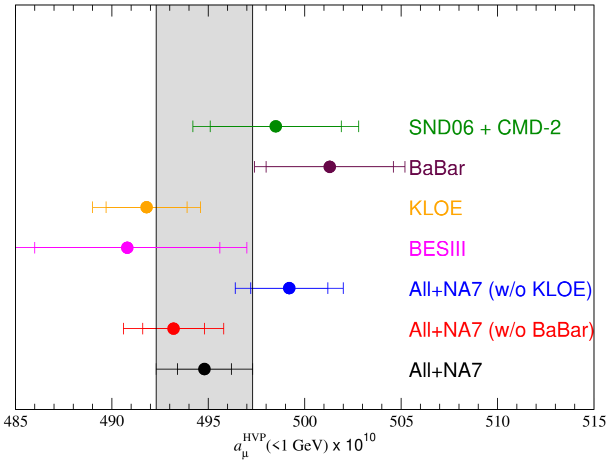




BaBar vs. SND 20 2004.00263 KLOE vs. SND 20

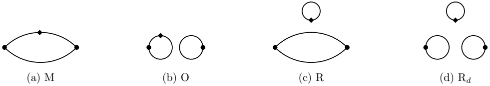
- New data from SND experiment not yet included in WP20 number
  $\hookrightarrow$ lie between BaBar and KLOE
- **More $\pi\pi$ data to come** from: CMD3, BESIII, BaBar, Belle II
- New data on 3π: BESIII, BaBar
- **MUonE project**: extract space-like HVP from μe scattering

$\pi\pi$ contribution below 1 GeV

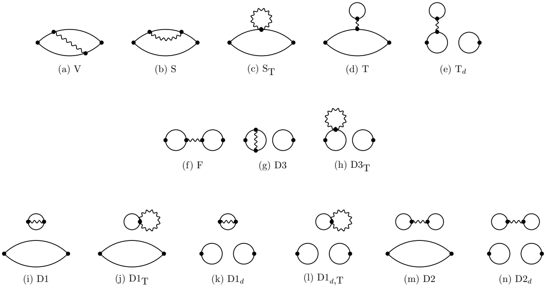


Isospin breaking on the lattice

- **Strong isospin breaking** $\propto m_u - m_d$



- **QED effects** $\propto \alpha$

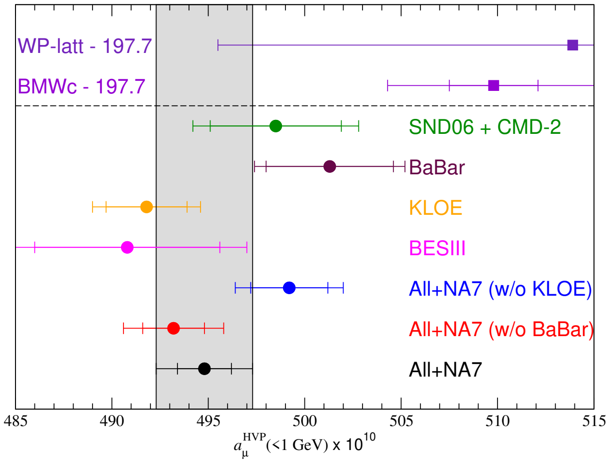


plots from Gülpers et al. 2018

- **Matches data-driven convention for leading-order HVP**

  $\hookrightarrow$ diagram (f) F without additional gluons is subtracted

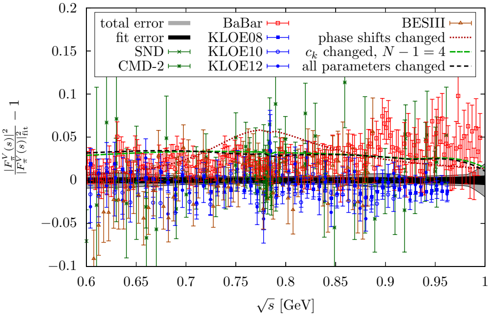
$\pi\pi$ contribution below 1 GeV

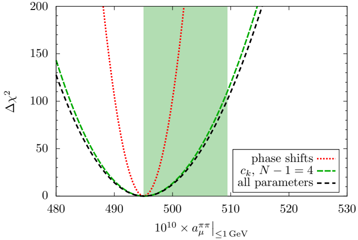


Assumption: suppose all changes occur in $\pi\pi$ channel below 1 GeV

$\hookrightarrow a_\mu^{\text{total}}[\text{WP20}] - a_\mu^{2\pi,<1\text{ GeV}}[\text{WP20}] = 197.7 \times 10^{-10}$

Changing the $\pi\pi$ cross section below 1 GeV

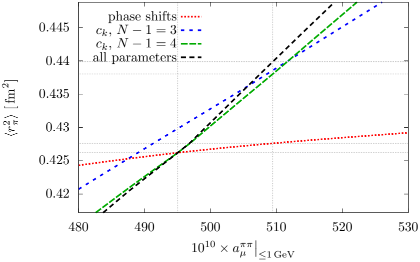


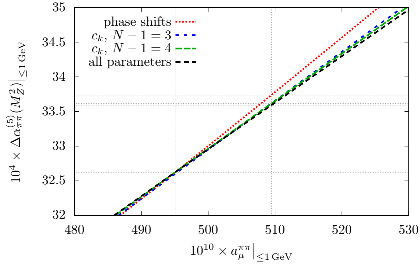


Colangelo, MH, Stoffer 2020

- Changes in 2π cross section **cannot be arbitrary** due to analyticity/unitarity constraints, but increase is actually possible
- Three scenarios:
  1. "Low-energy" scenario: **$\pi\pi$ phase shifts**
  2. "High-energy" scenario: **conformal polynomial**
  3. Combined scenario

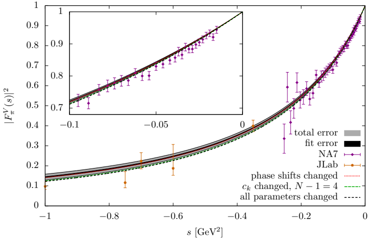
$\hookrightarrow$ 2. and 3. lead to uniform shift, 1. concentrated in ρ region

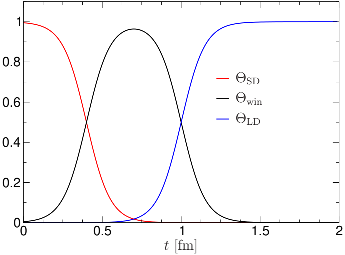


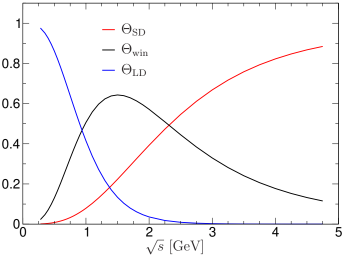


Correlations with other observables:

- **Pion charge radius $\langle r_\pi^2 \rangle$**
  - $\hookrightarrow$ significant change in scenarios 2. and 3.
  - $\hookrightarrow$ can be tested in lattice QCD
- **Hadronic running of α**
- **Space-like pion form factor**







- **Weight functions in Euclidean time** proposed by RBC/UKQCD 2018

  $\hookrightarrow$ long-distance, intermediate, and short-distance window

- For intermediate window $a_\mu^{\text{int}}[\text{RBC/UKQCD}] = 231.9(1.5) \times 10^{-10}$ and $a_\mu^{\text{int}}[\text{BMWc}] = 236.7(1.4) \times 10^{-10}$ **differ by 2.3σ**

- Difference between BMWc and e^+e^- in intermediate window is **3.7σ**, but $\pi\pi$ channel below 1 GeV split 69 : 28 : 3, relevant changes above 1 GeV?

- Detailed study of windows key tool for **comparison among lattice and with e^+e^-**

Conclusions

- **Hadronic light-by-light scattering**
  - Use dispersion relations to remove model dependence as far as possible
  - Evaluation of subleading terms and comparison to lattice-QCD calculations in progress
- **WP20 value for HVP**
  - based on decades of $e^+e^- \to$ hadrons measurements
  - consensus accounting for different methodologies in combining data sets
  - includes cross checks from analyticity/unitarity
  - systematic errors for tensions in the data assigned beyond a simple χ^2 inflation
- If HVP is changed, **new tensions** arise
  - in electroweak fit
  - with e^+e^- data
- To make progress in HVP
  - new data for $e^+e^- \to \pi^+\pi^-$ channel: CMD3, BaBar, Belle II, BESIII
  - detailed comparisons between different lattice calculations (global and windows)
  - detailed comparison between lattice and e^+e^- data
  - MUonE

Relation to global electroweak fit

Hadronic running of α

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ enters as input in **global electroweak fit**
  $\hookrightarrow$ integral weighted more strongly towards high energy
- Changes in $R_{\text{had}}(s)$ have to occur at low energies, $\lesssim 2\,\text{GeV}$ Crivellin et al. 2020, Keshavarzi et al. 2020, Malaescu et al. 2020
- This seems to happen for BMWc calculation (translated from the space-like), with only moderate increase of tensions in the electroweak fit ($\sim 1.8\sigma \to 2.4\sigma$)
  $\hookrightarrow$ need **large changes in low-energy cross section**

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)\times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

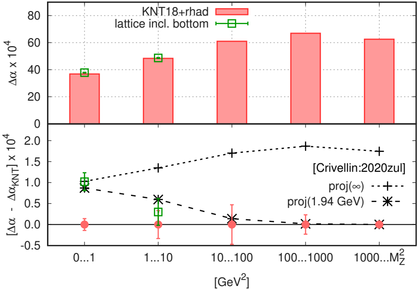
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha M_Z^2}{3\pi}P\int_{s_{\text{thr}}}^{\infty}ds\frac{R_{\text{had}}(s)}{s(M_Z^2-s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2)+\frac{\alpha}{\pi}\big(\hat{\Pi}(M_Z^2)-\hat{\Pi}(-M_Z^2)\big)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020