Dark Baryon solution to the Neutron Lifetime Puzzle

Benjamín Grinstein

with

Bart Fornal

Anomalies 2021
November 11

“Missing Neutrons May Lead a Secret Life as Dark Matter”,
C. Moskowitz, Scientific American
(January 29, 2018)

BARRY UNIVERSITY · MIAMI · 1940 · FLORIDA

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

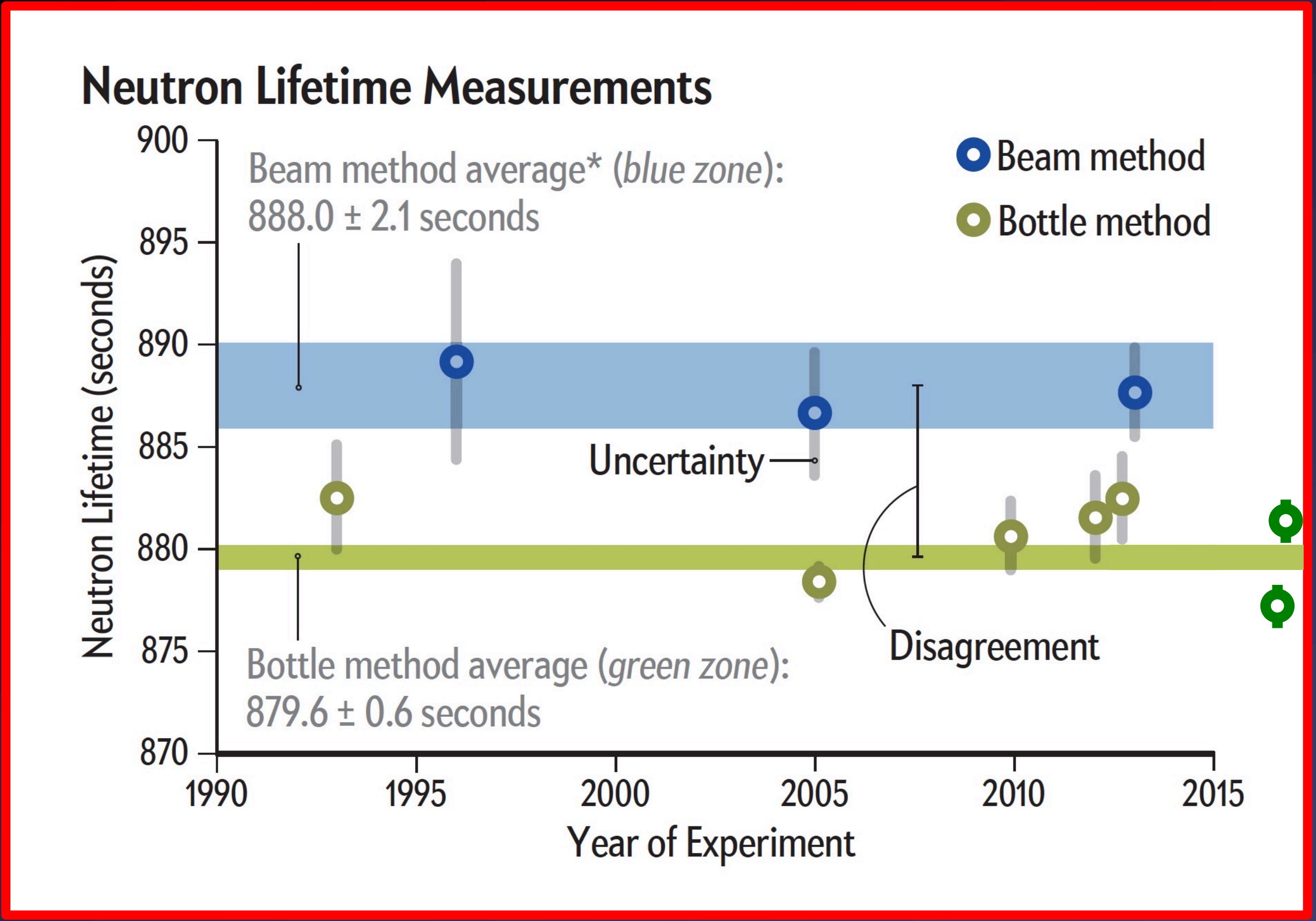
UNIVERSITY OF CALIFORNIA · SAN DIEGO ·
LET THERE BE LIGHT

Why this, in this conference?

- Effect at > 4σ
- Many independent experiments

- How is this different from $g-2$?
  - Many independent experiments
  - No trivial/obvious explanation
  - Not theory driven

Neutron lifetime measurements



Source: https://www.scientificamerican.com, modified

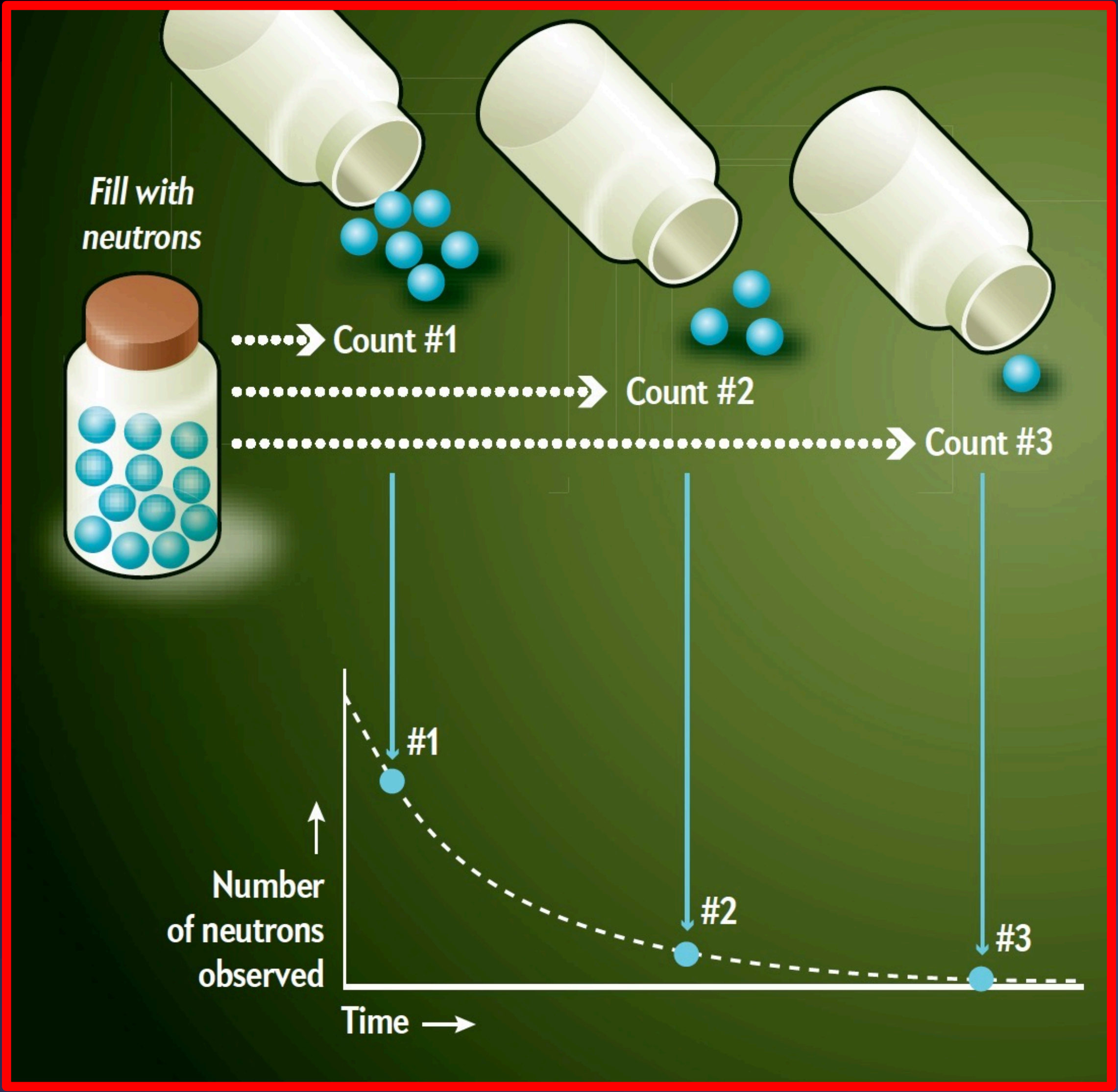
$$\tau_n^{\text{beam}} = 888.0 \pm 2.0\ \text{s}$$

$$\tau_n^{\text{bottle}} = 879.6 \pm 0.6\ \text{s}$$

Discrepancy

$$\frac{\Delta\tau_n}{\tau_n} \approx 1\%$$

Bottle experiments



Source: https://www.scientificamerican.com

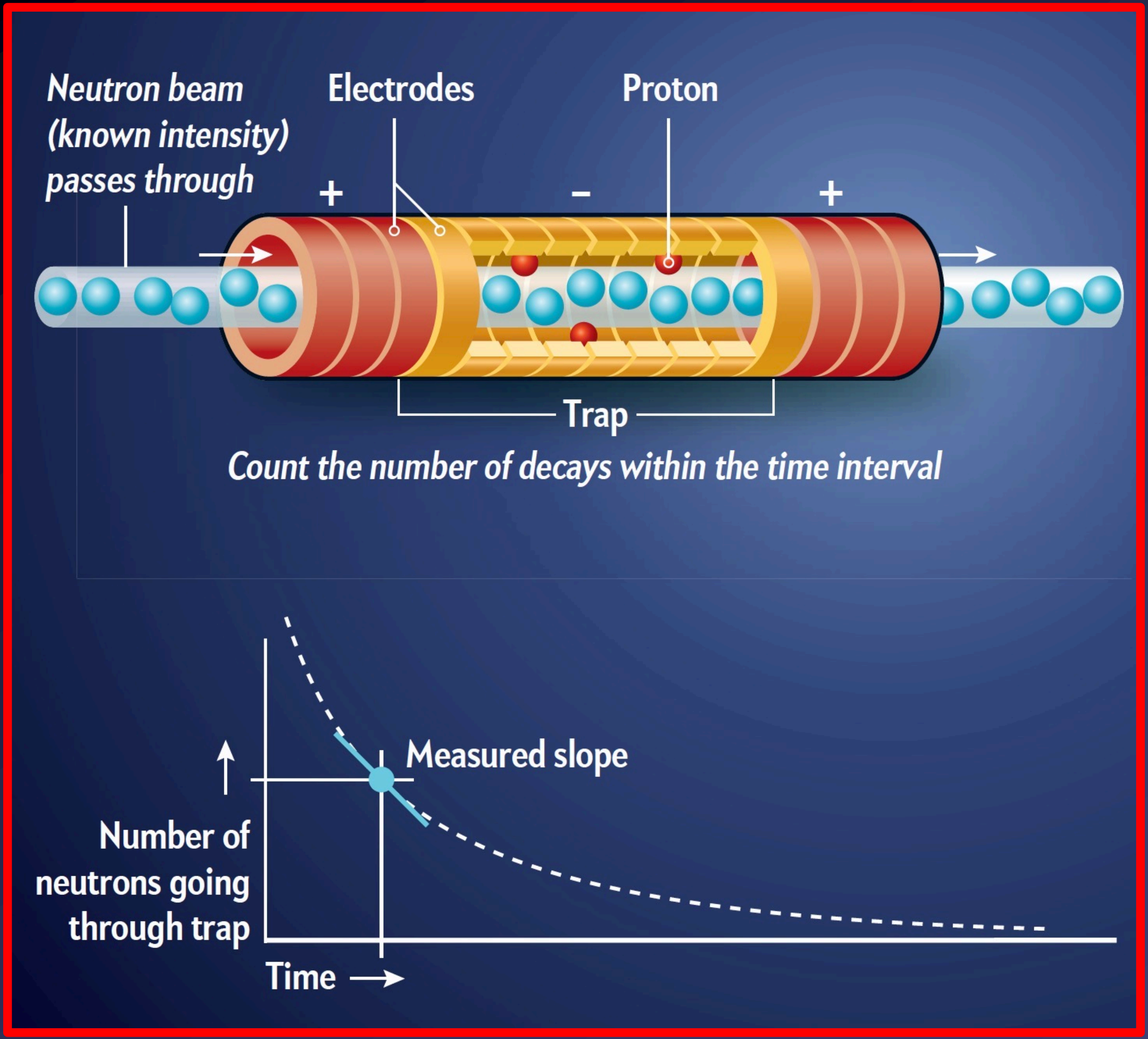
Data points fit to an exponential decay

$$N = N_0\, e^{-\lambda t}$$

Lifetime

$$\tau = \frac{1}{\lambda}$$

Beam experiments



Source: https://www.scientificamerican.com

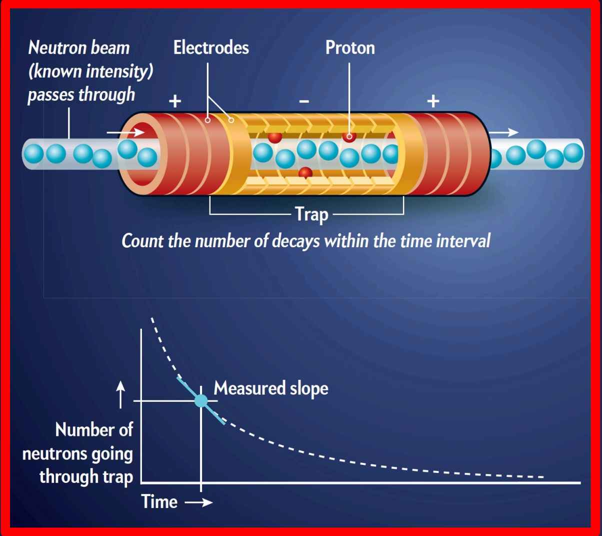
Only the decay rate is measured

$$\frac{dN}{dt} = -\lambda N$$

Since neutron decays only via beta decay

$$n \to p + e^- + \bar{\nu}_e$$

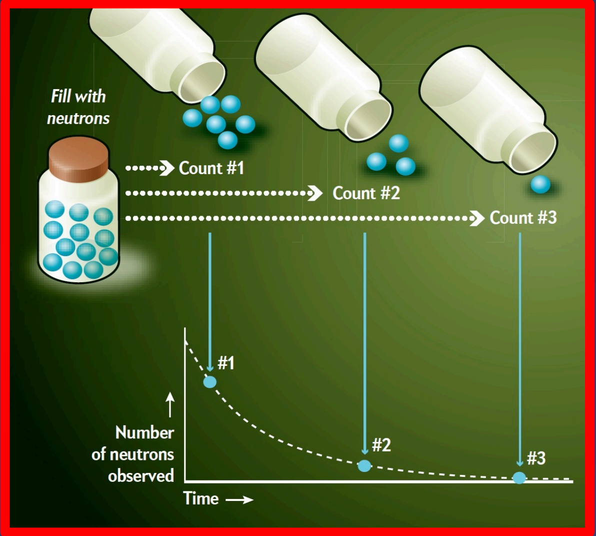
Beam experiments



equality should hold:

$$\tau_n^{\text{beam}} = \tau_n^{\text{bottle}}$$

Bottle experiments



but

$$\tau_n^{\text{beam}} = 888.0(2.1)\,\text{s} \;>\; \tau_n^{\text{bottle}} = 879.3(0.8)\,\text{s}$$

Neutron lifetime in the Standard Model

Theoretical prediction

$$\boxed{\tau_n = \frac{4908.6(1.9)\ \mathrm{s}}{|V_{ud}|^2(1+3g_A^2)}}$$

Czarnecki, Marciano & Sirlin,
PRL 120, 202002 (2018)

$$\boxed{\mathcal{M} = \tfrac{1}{\sqrt{2}}\, G_F V_{ud}\, g_V \left[\bar{p}\,\gamma_\mu n - g_A \bar{p}\,\gamma_5\gamma_\mu n\right] \left[\bar{e}\,\gamma^\mu(1-\gamma_5)\nu\right]}$$

Using the PDG average for g_A

$$\boxed{880.5\,\mathrm{s} < \tau_n < 886.0\,\mathrm{s}}$$

Lattice result

$$\boxed{870\,\mathrm{s} < \tau_n < 900\,\mathrm{s}}$$

$$\boxed{g_A = 1.271 \pm 0.013}$$

Chang et al.,
Nature 558, 91 (2018)

Neutron lifetime in the Standard Model

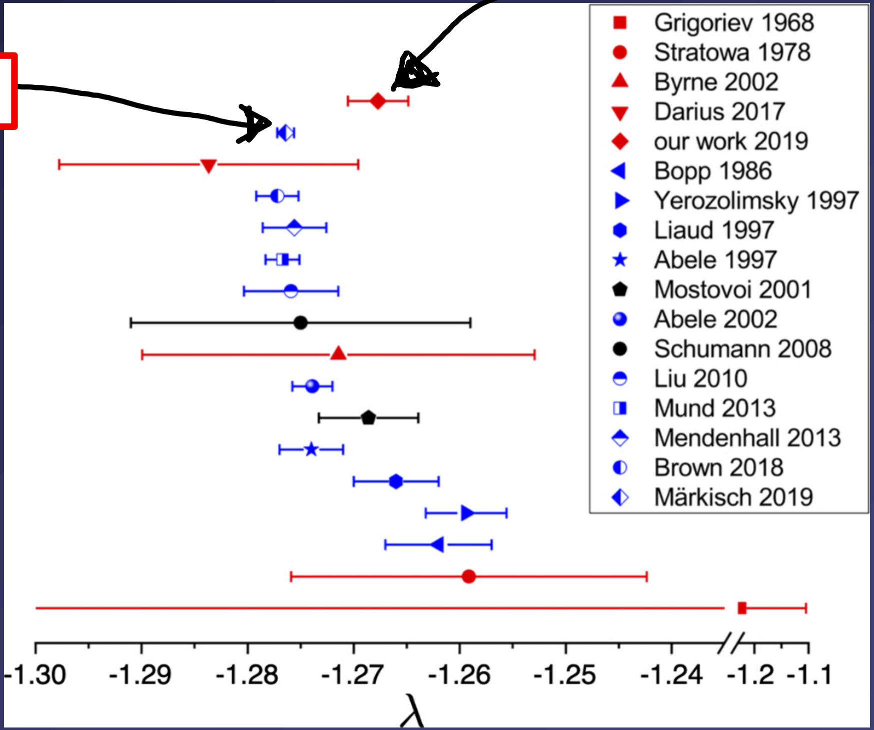
Theoretical prediction

$$\boxed{\tau_n = \frac{4908.6(1.9)\ \mathrm{s}}{|V_{ud}|^2(1+3g_A^2)}}$$

Czarnecki, Marciano & Sirlin,
PRL 120, 202002 (2018)

gives bottle τ

gives beam τ



PERKEO : β spectrum of polarized neutrons

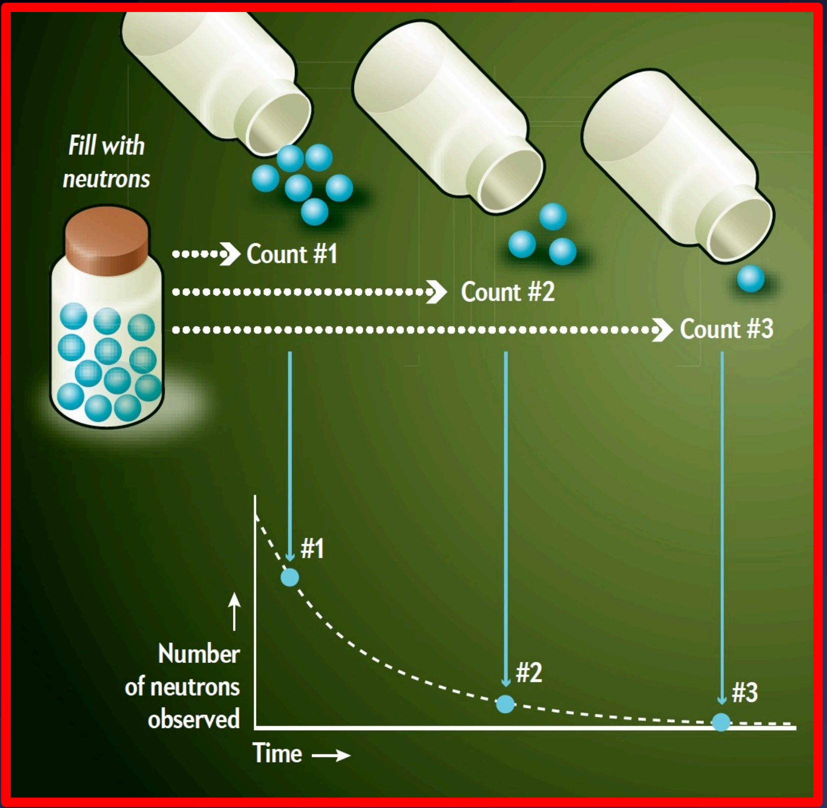
aSPECT: $\beta - \bar{\nu}$ angular correlations

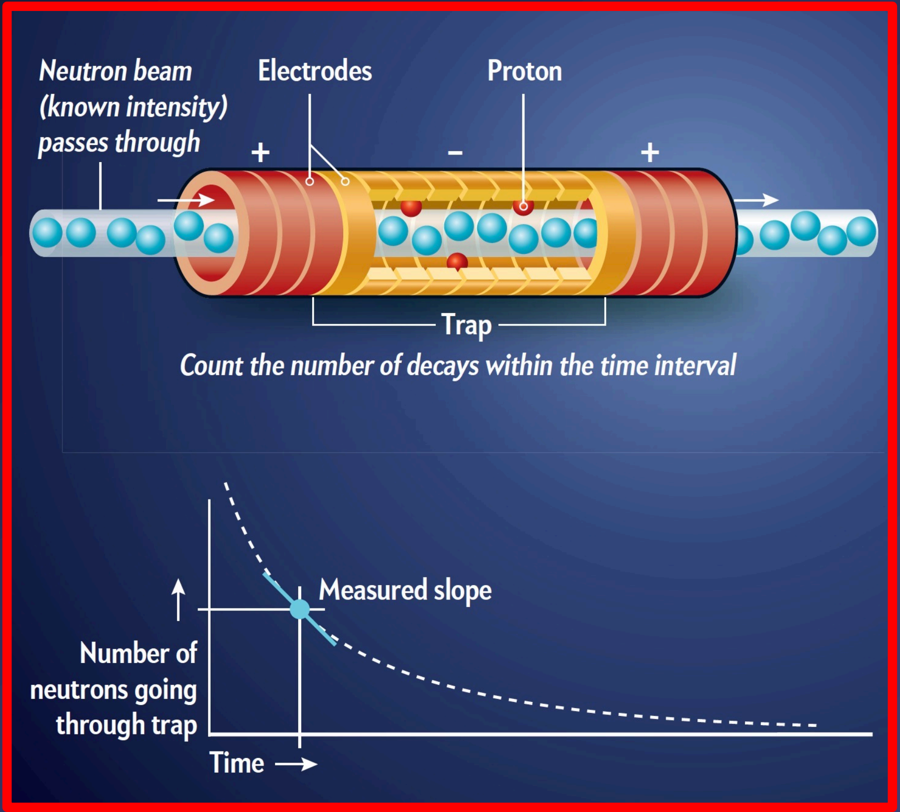
aSPECT vs PERKEO
2.8σ tension

$$\boxed{\lambda = g_A/g_V}$$

Beck et al,
Phys.Rev.C 101 (2020) 5, 055506

Neutron lifetime discrepancy





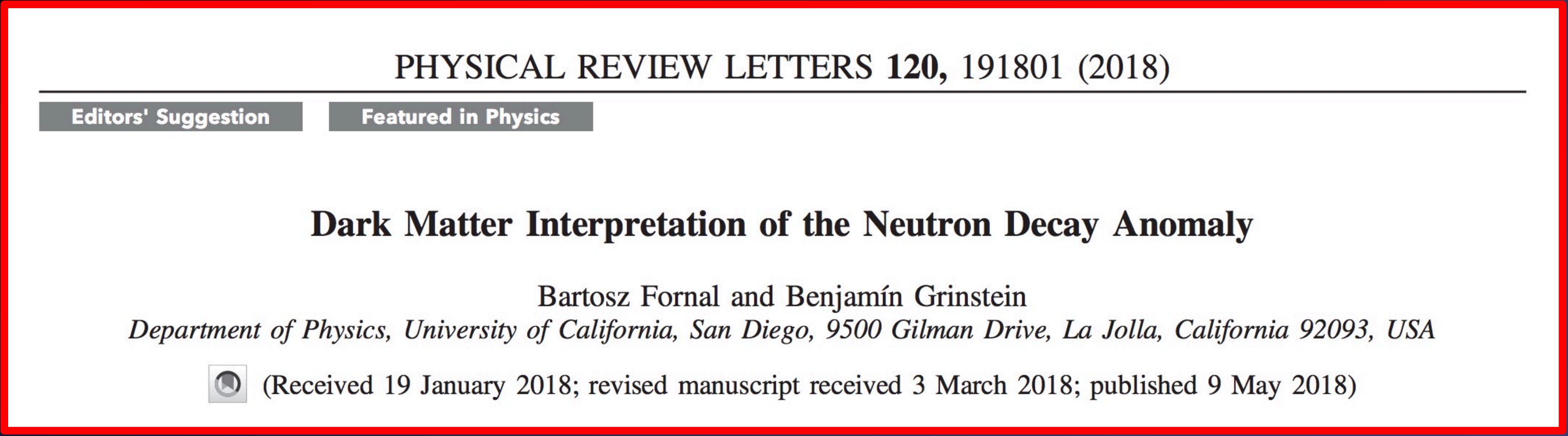
Source: https://www.scientificamerican.com

Beam measures protons only! And only slope (decay rate).

$$\frac{1}{N(n)}\frac{dN(p)}{dt} = -\lambda \mathrm{Br}(n \to p)$$

$$\tau_n^{\mathrm{beam}} = \frac{\tau_n}{\mathrm{Br}(n \to p + \mathrm{anything})}$$

Neutron dark decay



PHYSICAL REVIEW LETTERS **120,** 191801 (2018)

Editors' Suggestion **Featured in Physics**

Dark Matter Interpretation of the Neutron Decay Anomaly

Bartosz Fornal and Benjamín Grinstein
Department of Physics, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093, USA

(Received 19 January 2018; revised manuscript received 3 March 2018; published 9 May 2018)

$$\mathrm{Br}(n \to p + \text{anything}) \approx 99\%$$

Remaining 1% :

- $n \to \text{ SM particles (other than } p)$ ✗
- $n \to \text{ dark particle(s) } + \text{ SM particle(s)}$ ✓
- $n \to \text{ dark particles}$ ✓

$$\tau_{n \to \text{dark}} \approx 1 \text{ day}$$

Hypothesis is model independent

This simple hypothesis can be tested

Ongoing beam and bottle experiments

beam

https://www.ncnr.nist.gov/

NIST Center for Neutron Research

beam

https://nat-web.com

J–PARC, Japan

Obvious test 1:

➡ **Measure lifetime by exponential decay along experimental axis.**

Ongoing beam and bottle experiments

beam

https://www.ncnr.nist.gov/

NIST Center for Neutron Research

beam

https://nat-web.com

J–PARC, Japan

bottle

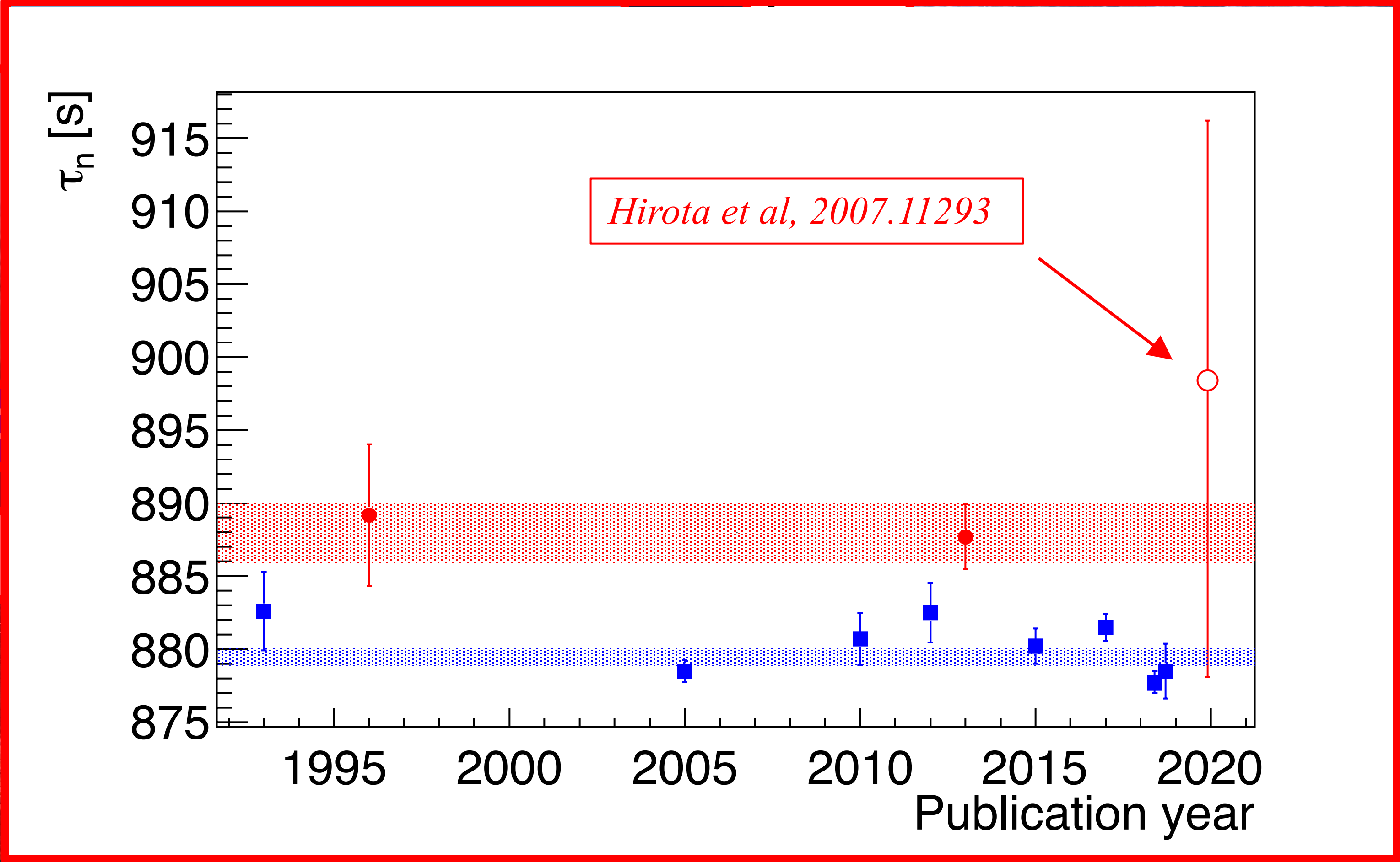
http://www.lanl.gov/

Los Alamos Neutron Science Center

Ongoing beam and bottle experiments

beam

NIST C



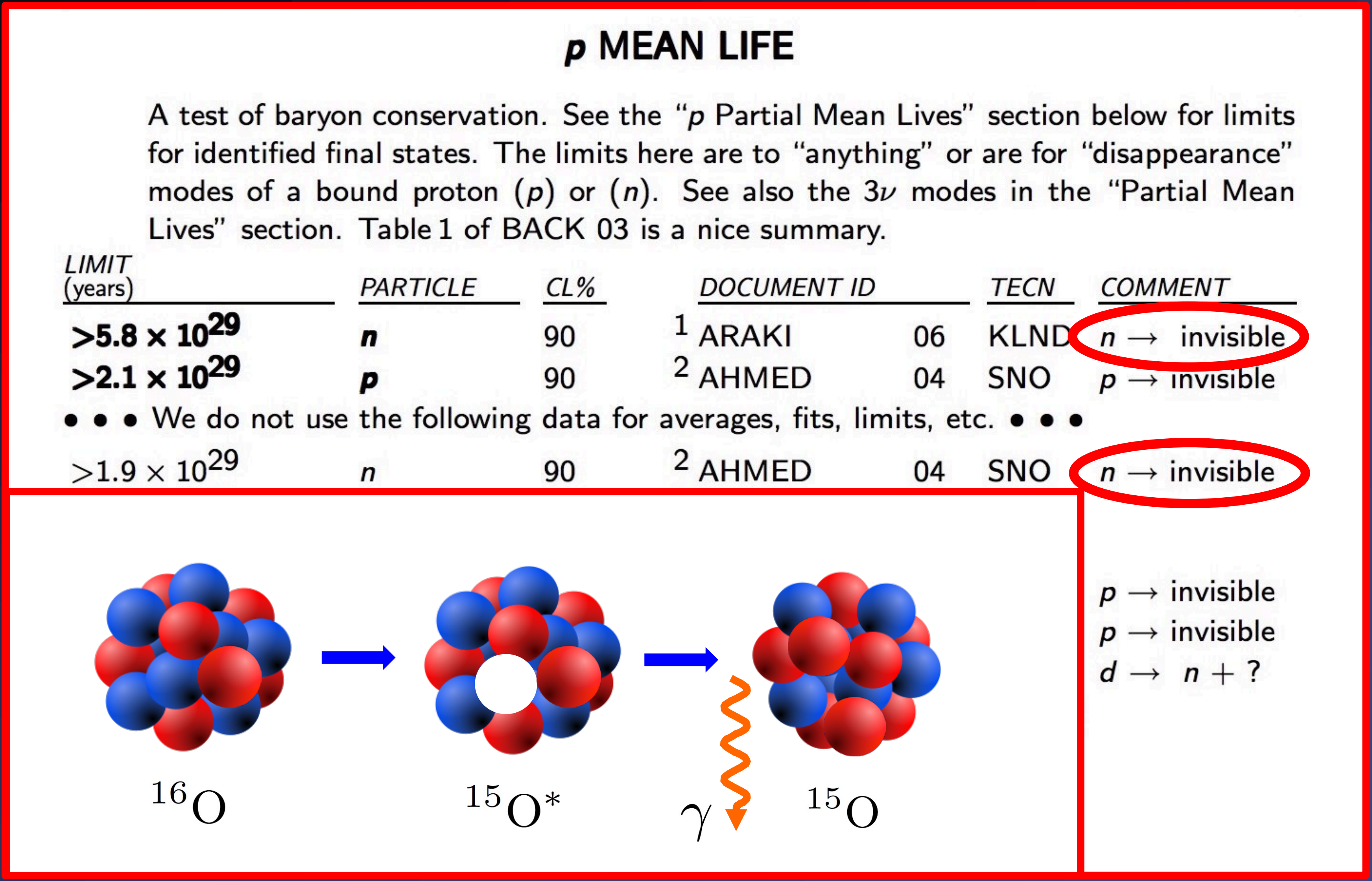
bottle

http://www.la

Los Alamos Neutron Science Center

in bottle experiments

Nuclear physics bounds

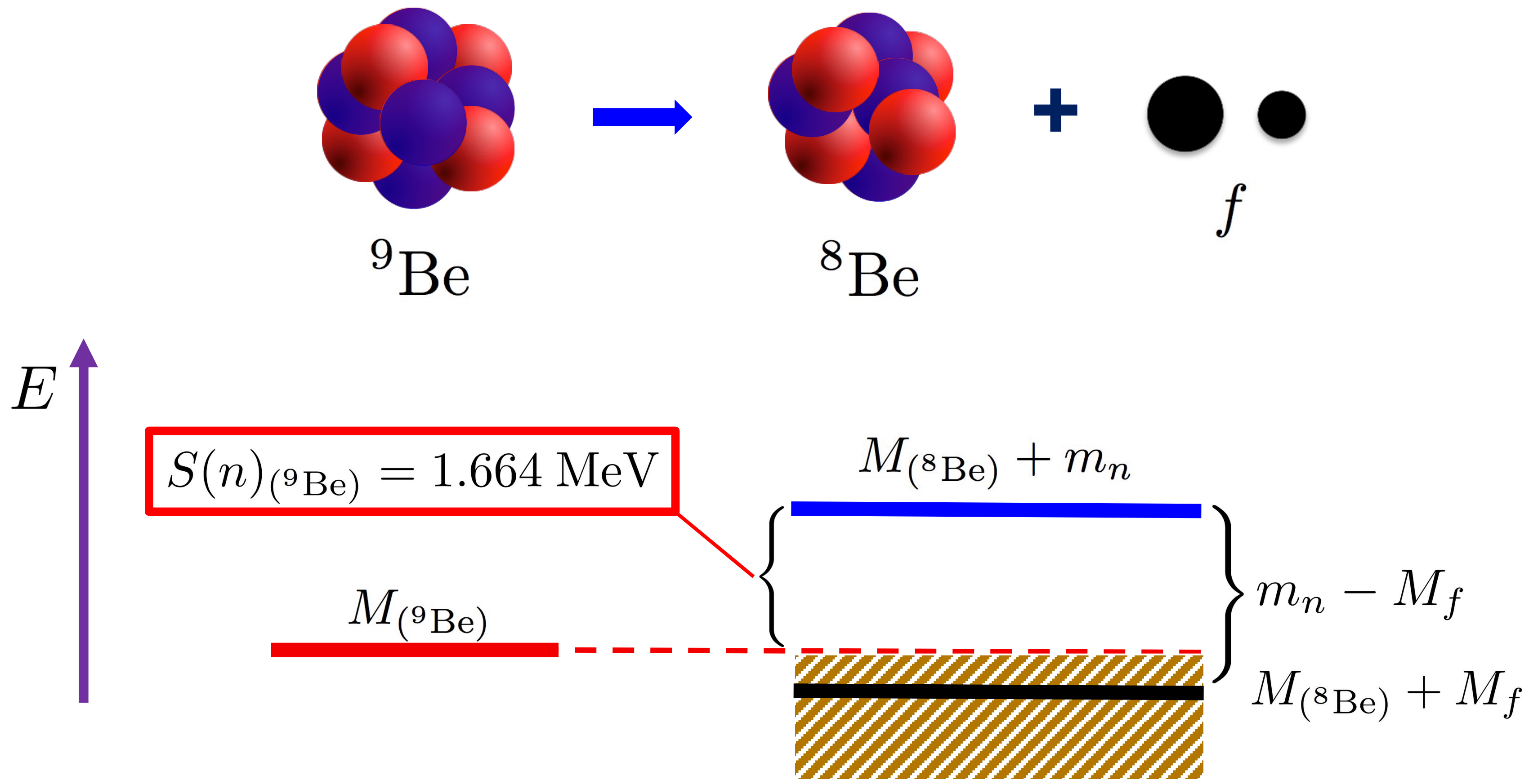


p MEAN LIFE

A test of baryon conservation. See the "*p* Partial Mean Lives" section below for limits for identified final states. The limits here are to "anything" or are for "disappearance" modes of a bound proton (*p*) or (*n*). See also the 3ν modes in the "Partial Mean Lives" section. Table 1 of BACK 03 is a nice summary.

LIMIT (years)	*PARTICLE*	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
$>5.8 \times 10^{29}$	***n***	90	[1] ARAKI	06	KLND	$n \to$ invisible
$>2.1 \times 10^{29}$	***p***	90	[2] AHMED	04	SNO	$p \to$ invisible
• • • We do not use the following data for averages, fits, limits, etc. • • •						
$>1.9 \times 10^{29}$	*n*	90	[2] AHMED	04	SNO	$n \to$ invisible
						$p \to$ invisible
						$p \to$ invisible
						$d \to n + ?$

Nuclear physics bounds – ^{9}Be



^{9}Be → ^{8}Be + f decay forbidden if:

^{9}Be → 2α + f decay forbidden if:

Pfutzner & Riisager 1803.01334

$$\boxed{937.900\ \mathrm{MeV} < M_f < 939.565\ \mathrm{MeV}}$$

$$\boxed{937.992\ \mathrm{MeV} < M_f < 939.565\ \mathrm{MeV}}$$

Dark Matter scenario

Dark particle stability

$$\boxed{\chi \cancel{\to} p + e^{-} + \bar{\nu}_e}$$

requires

$$\boxed{m_\chi < m_p + m_e = 938.783 \text{ MeV}}$$

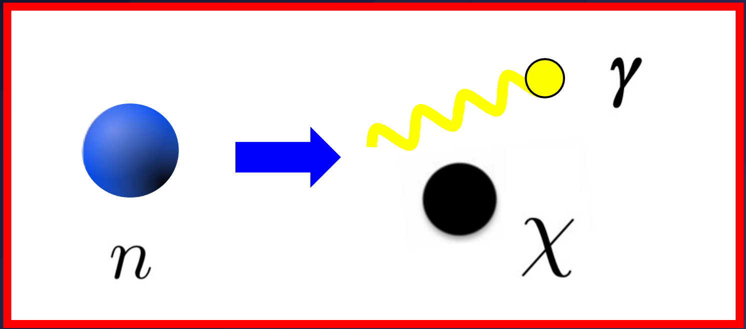
The (possible) dark matter mass is in the range

$$\boxed{937.992 \text{ MeV} < m_\chi < 938.783 \text{ MeV}}$$

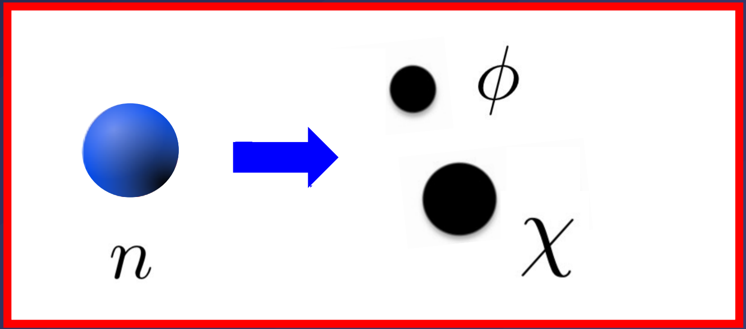
New neutron decay channels

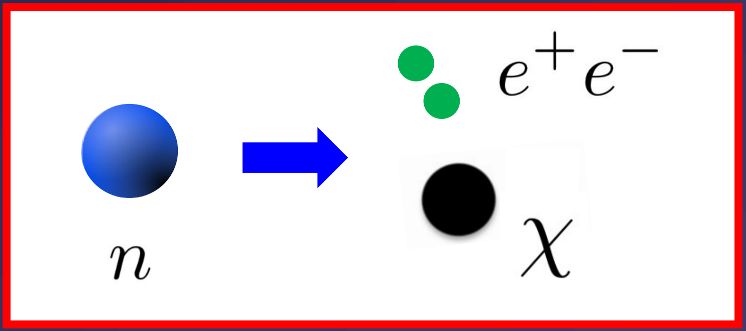
$$937.993\,\mathrm{MeV} < M_f < 939.565\,\mathrm{MeV}$$

Scenario I



Scenario II

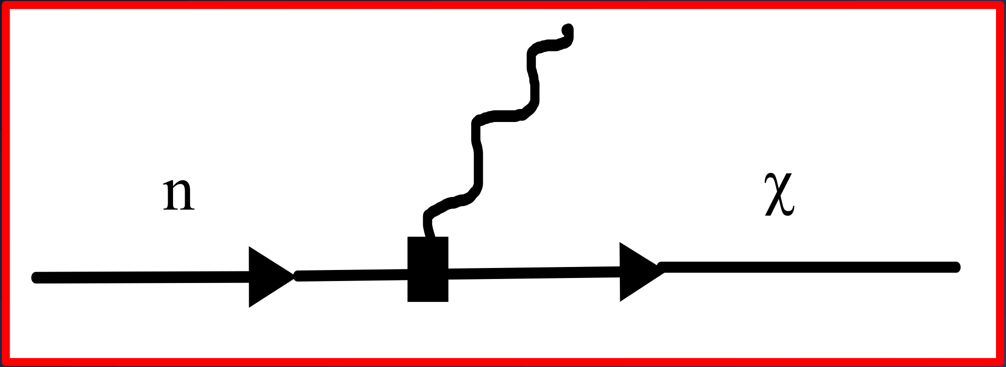




Scenario III

• • •

Case (1): Neutron ⟶ dark particle + photon



Dark particle mass $937.900\ \mathrm{MeV} < m_\chi < 939.565\ \mathrm{MeV}$

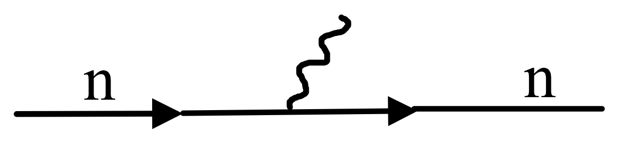
Dark decay photon energy $0 < E_\gamma < 1.664\ \mathrm{MeV}$

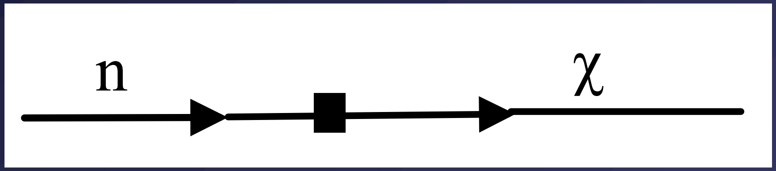
Dark matter case $0.782\ \mathrm{MeV} < E_\gamma < 1.664\ \mathrm{MeV}$

Note: did not update lower bound from ^{9}Be → 2α

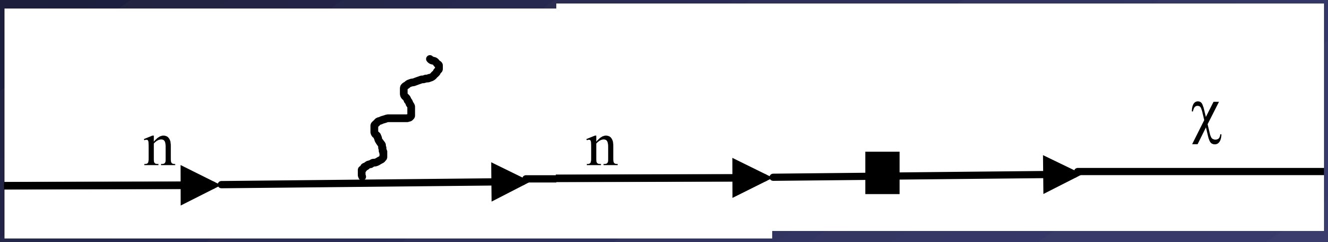
Case (1): Neutron ⟶ dark particle + photon

Example: $\chi n \gamma$ interaction from mixing

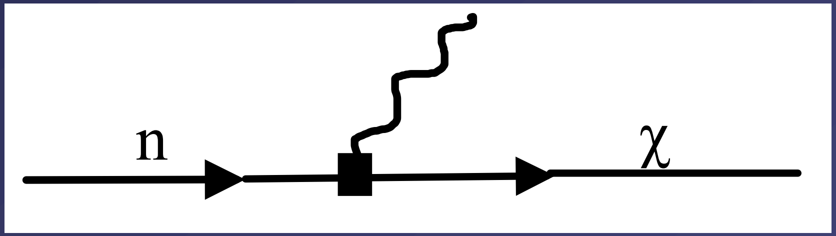




gives

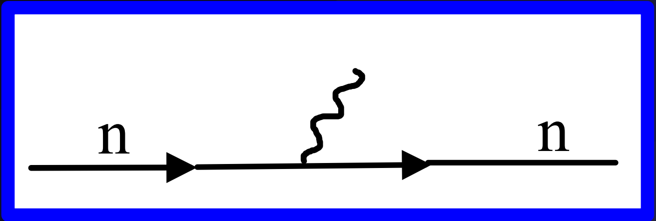


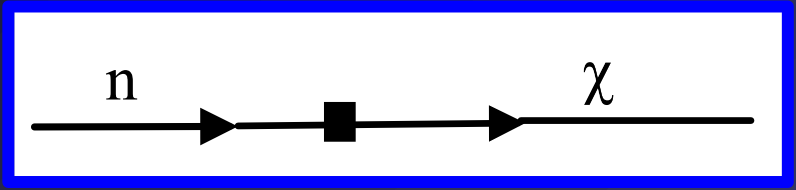
In terms of mass eigenstates, neutron dark decay



Case (1): Quantitative description

Example of a theory with *χn* interaction





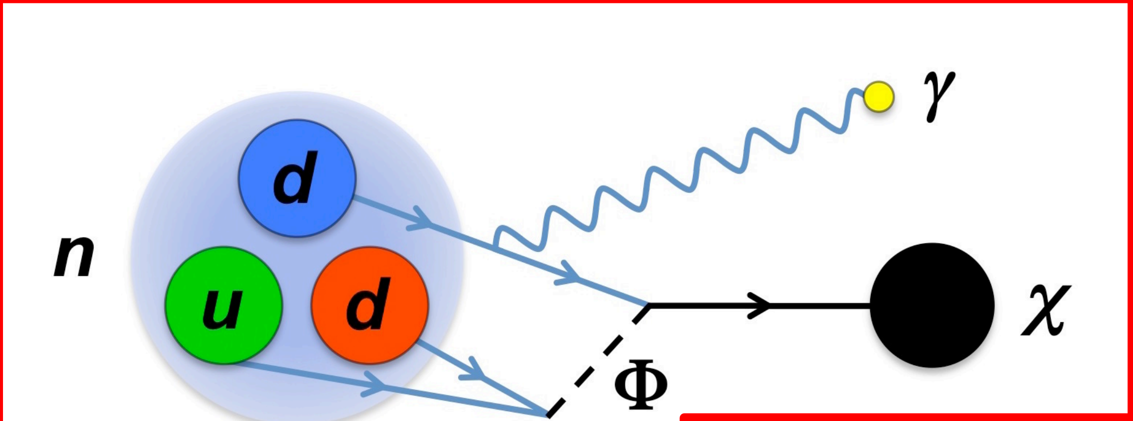
$$\mathcal{L}^{\text{eff}} = \bar{n}\left(i\not{\partial} - m_n + \frac{g_n e}{2m_n}\sigma^{\mu\nu}F_{\mu\nu}\right)n + \bar{\chi}\left(i\not{\partial} - m_\chi\right)\chi + \varepsilon\left(\bar{n}\chi + \bar{\chi}n\right)$$

In terms of mass eigenstates, neutron dark decay

$$\mathcal{L}^{\text{eff}}_{n\to\chi\gamma} = \frac{g_n e}{2m_n}\frac{\varepsilon}{(m_n - m_\chi)}\,\bar{\chi}\,\sigma^{\mu\nu}F_{\mu\nu}\,n$$

$$\epsilon \approx 10^{-13}\ \text{GeV}$$

Microscopic Model 1 (minimal)



$0 < E_\gamma < 1.664\ \text{MeV}$

Lagrangian

$$\mathcal{L}_1 = \left(\lambda_q\, \epsilon^{ijk}\, \overline{u^c_{L}}_i\, d_{Rj} \Phi_k + \lambda_\chi \Phi^{*i} \bar{\chi}\, d_{Ri} + \text{h.c.}\right) - M_\Phi^2 |\Phi|^2 - m_\chi\, \bar{\chi}\, \chi$$

Dark decay rate

$$\Delta\Gamma_{n\to\chi\gamma} = \frac{g_n^2 e^2}{8\pi}\left(1 - \frac{m_\chi^2}{m_n^2}\right)^3 \frac{m_n\, \varepsilon^2}{(m_n - m_\chi)^2}$$

$$\varepsilon = \frac{\beta\, \lambda_q \lambda_\chi}{M_\Phi^2}$$

$$\langle 0 | \epsilon^{ijk}\, \overline{u^c_{L}}_i d_{Rj} d_{Rk} | n \rangle = \beta\, P_R\, u_n$$

Aoki et al, PRD 96 (2017) 014506, [1705.01338].

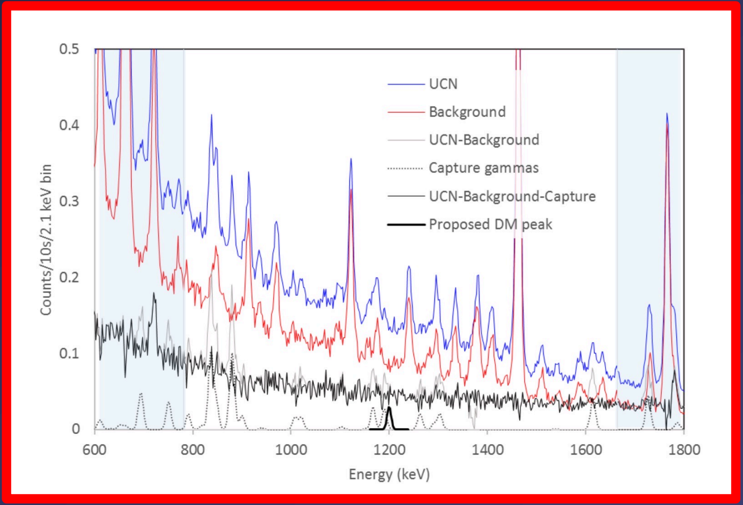
To explain the neutron lifetime discrepancy

$$\frac{\Delta\Gamma_{n\to\chi\gamma}}{\Gamma_n} \approx 1\% \quad \longrightarrow \quad \frac{M_\Phi}{\sqrt{|\lambda_q \lambda_\chi|}} \approx 400\ \text{TeV}$$

Neutron dark decay – experimental search

monochromatic photons

$$0.782\ \mathrm{MeV} < E_\gamma < 1.664\ \mathrm{MeV}$$



Tang *et al.*, *Search for the neutron decay $n \to X + \gamma$ where X is a dark matter particle*, PRL 121, 022505 (2018)

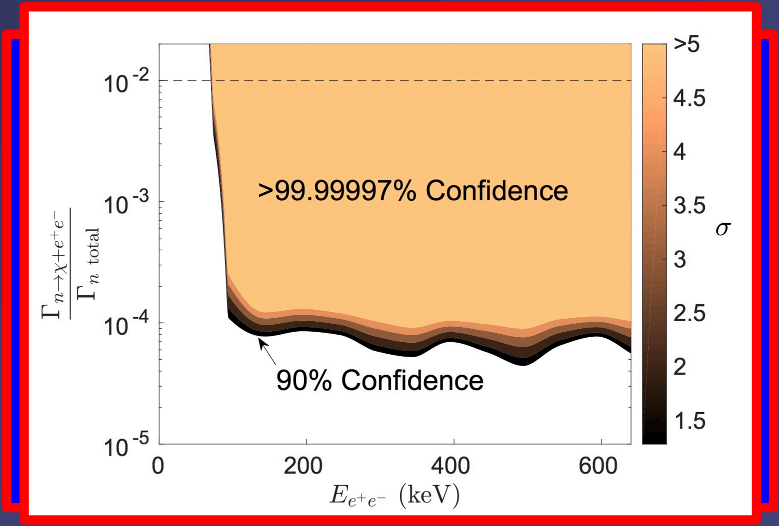
2.2σ exclusion

electron-positron pairs

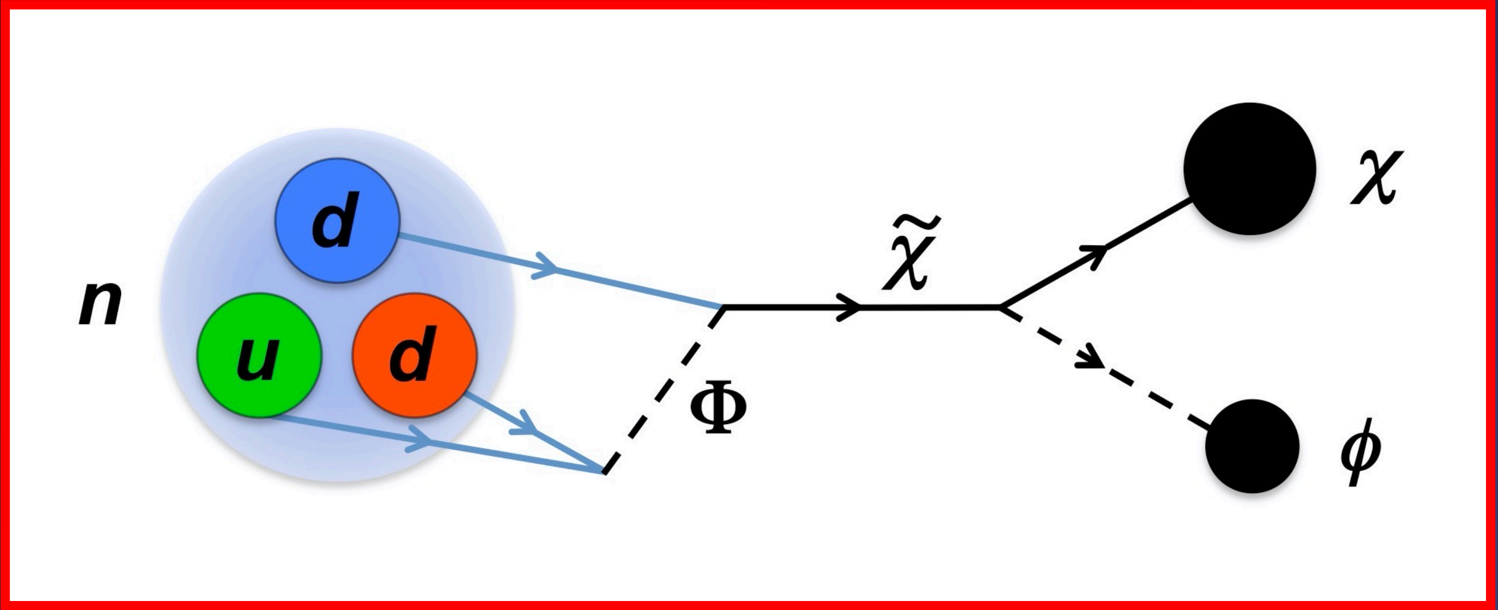
$$2\,m_e + 100\ \mathrm{keV} \le E_{e^+e^-} < 1.665\ \mathrm{MeV}$$

Sun et al., *Search for dark matter decay of the free neutron from the UCNA experiment $n \to \chi + e^+e^-$*, PRC 97, 052501 (2018) --Los Alamos

Klopf et al., *Constraints on the Dark Matter Interpretation of the Neutron Decay Anomaly with the PERKEO II experiment*, PRL 122, 222503 (2019) --ILL Grenoble



Case (2): Neutron ⟶ two dark particles



Constraints on masses

$$937.900\ \mathrm{MeV} < m_{\tilde{\chi}}$$

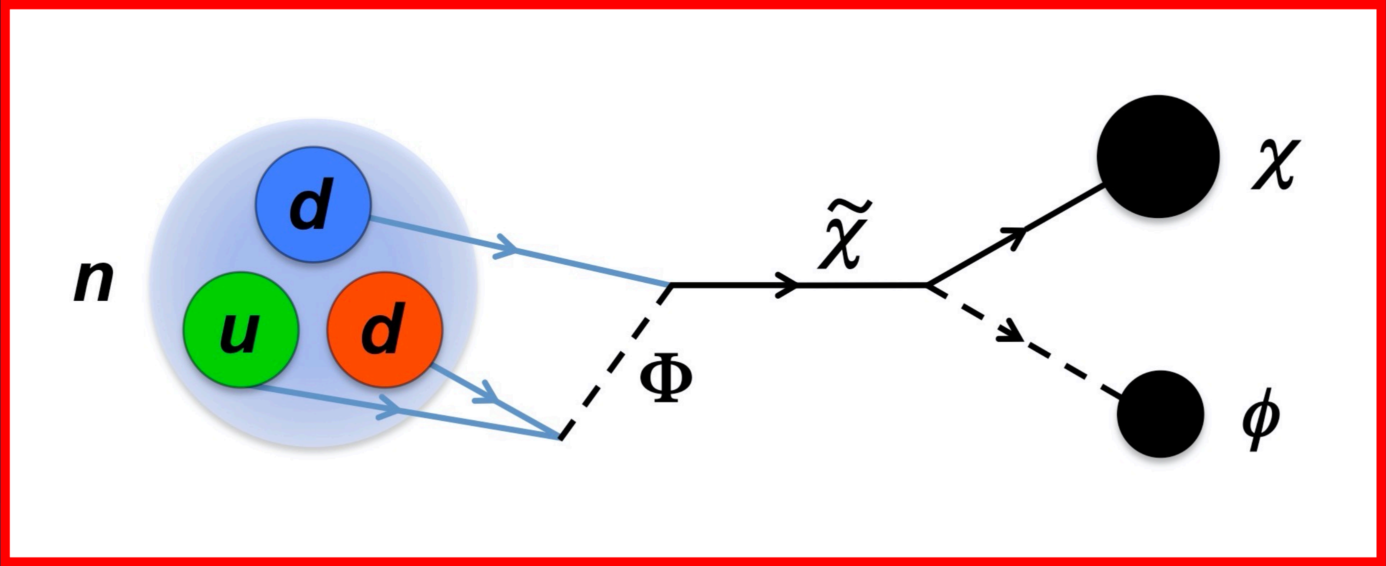
$$937.900\ \mathrm{MeV} < m_\chi + m_\phi < 939.565\ \mathrm{MeV}$$

Dark matter case

$$|m_\chi - m_\phi| < 938.783\ \mathrm{MeV}$$

Note: did not update lower bound from ^{9}Be → 2α

Model 2

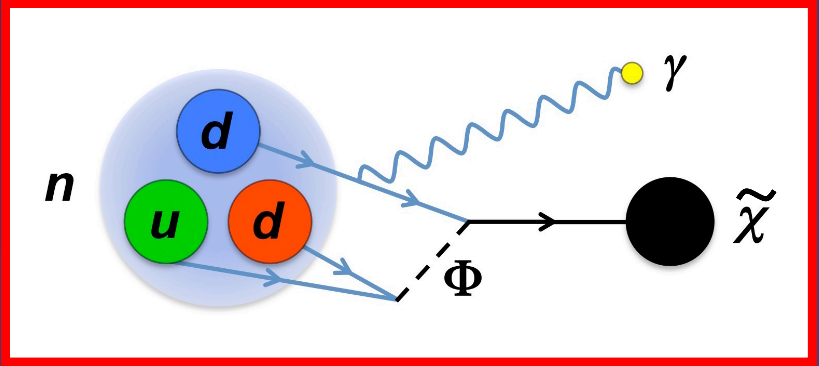


For $m_{\tilde{\chi}} > m_n$

$$\frac{\Delta\Gamma_{n\to\chi\phi}}{\Gamma_n} \approx 1\% \xrightarrow{\lambda_\phi \approx 0.04} \frac{M_\Phi}{\sqrt{|\lambda_q \lambda_\chi|}} \approx 1600\ \mathrm{TeV}$$

For 937.9 MeV < $m_{\tilde{\chi}} < m_n$

$$\frac{\Delta\Gamma_{n\to\tilde{\chi}\gamma} + \Delta\Gamma_{n\to\chi\phi}}{\Gamma_n} \approx 1\%$$



Active Field: Other topics

Unstable Nucleus Decay*

Mesogenesis (baryogenesis)

Hyperon Decays

Dark Matter

Neutron Stars

Complete Models

Hydrogen decay

Other solutions

*Must cover

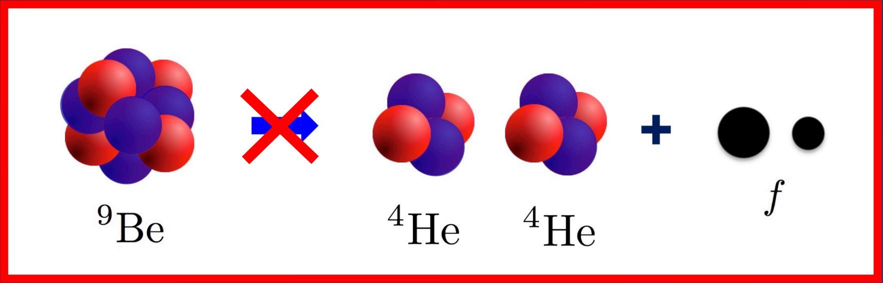
Nuclear dark decays

Stable nuclei remain stable if

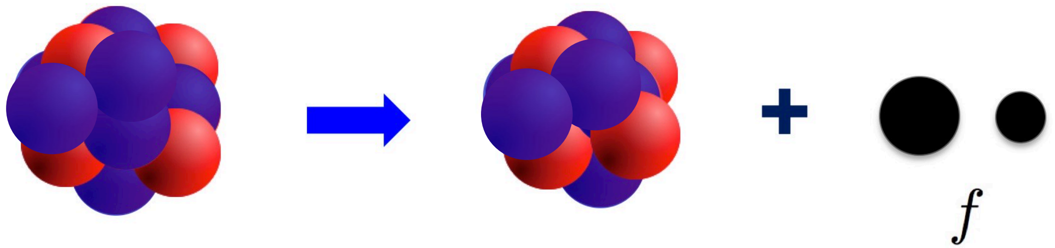
$$m_n - 1.572\,\mathrm{MeV} < M_f < m_n$$

i.e.,

$$937.993\,\mathrm{MeV} < M_f < 939.565\,\mathrm{MeV}$$



Dark decays possible in unstable nuclei with ***S(n)* < 1.572 MeV**



$$M + m_n - S(n) \quad > \quad M + M_f$$

$$937.993\,\mathrm{MeV} < M_f < m_n - S(n)$$

^{11}Be decay

An example of an unstable nucleus with $S(n) < 1.572$ MeV is ^{11}Be with $\boxed{S(n)_{(^{11}\mathrm{Be})} = 0.502\ \mathrm{MeV}}$ that could decay via

Pfutzner & Riisager, PRC 97, 042501(R) (2018)

$$\boxed{^{11}\mathrm{Be} \to {}^{10}\mathrm{Be} + \tilde{\chi}^{*} \to {}^{10}\mathrm{Be} + \chi + \phi}$$

^{11}Be primer:

$$\boxed{\mathrm{Br}\left({}^{11}\mathrm{Be} \xrightarrow{\beta^{-}} {}^{11}\mathrm{B}\right) = 97.1\%}$$

(Note: $^{11}\mathrm{B}$ is stable)

$$\boxed{\mathrm{Br}\left({}^{11}\mathrm{Be} \xrightarrow{\beta^{-},\,\alpha} {}^{7}\mathrm{Li} + {}^{4}\mathrm{He}\right) = 2.9\%}$$

Theoretical estimate for β-delayed proton emission:

$$\boxed{\mathrm{Br}({}^{11}\mathrm{Be} \xrightarrow{\beta} {}^{11}\mathrm{B} \to {}^{10}\mathrm{Be} + p) \approx 2 \times 10^{-8}}$$

Hint from ^{11}Be decays?

Pfutzner & Riisager, *Examining the possibility to observe neutron dark decay in nuclei*, PRC 97, 042501(R) (2018)

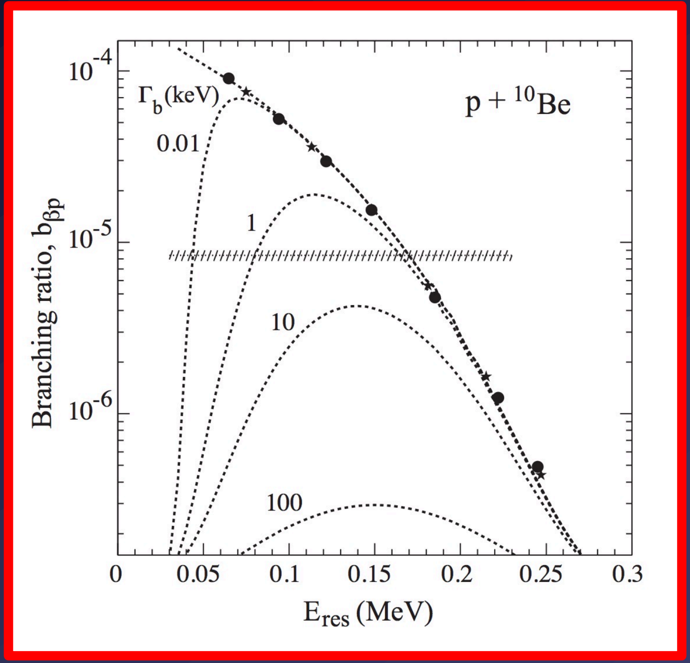
^{11}Be: one-neutron halo nucleus, can calculate!

Beta-delayed, proton emission $^{11}\mathrm{Be} \to {}^{11}\mathrm{B}\ e^- \to ({}^{10}\mathrm{Be}\ p)e^-$

experiment $\mathrm{Br}\left({}^{11}\mathrm{Be} \to {}^{10}\mathrm{Be} + ?\right) \approx 8 \times 10^{-6}$

400 x halo nucleus prediction

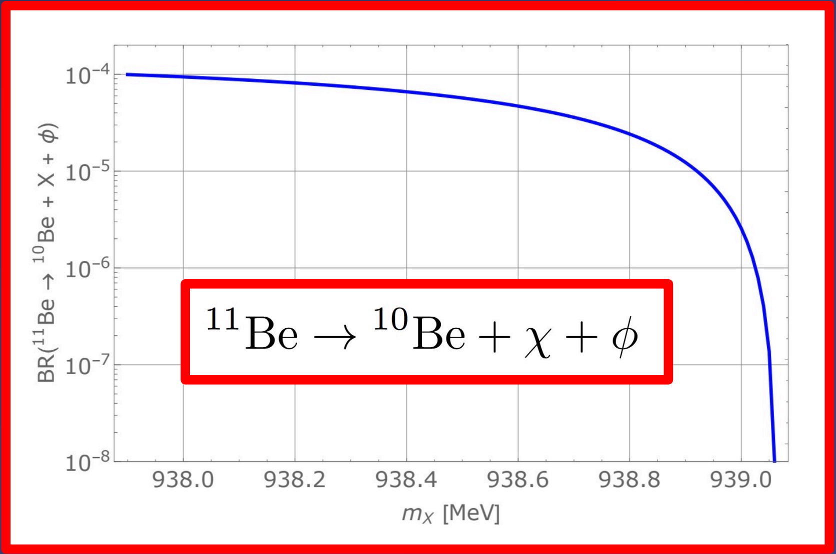
Riisager *et al.*, *^{11}Be(βp), a quasi-free neutron decay?*, PLB 732, 305 (2014)



?

Resonance
or
dark decay

?



$^{11}\mathrm{Be} \to {}^{10}\mathrm{Be} + \chi + \phi$

$S_n({}^{11}\mathrm{Be}) = 0.5\ \mathrm{MeV}$

^{11}Be decay experiments

Are there protons in the final state of ^{11}Be decays?

This would test ALL neutron dark decay channels with:

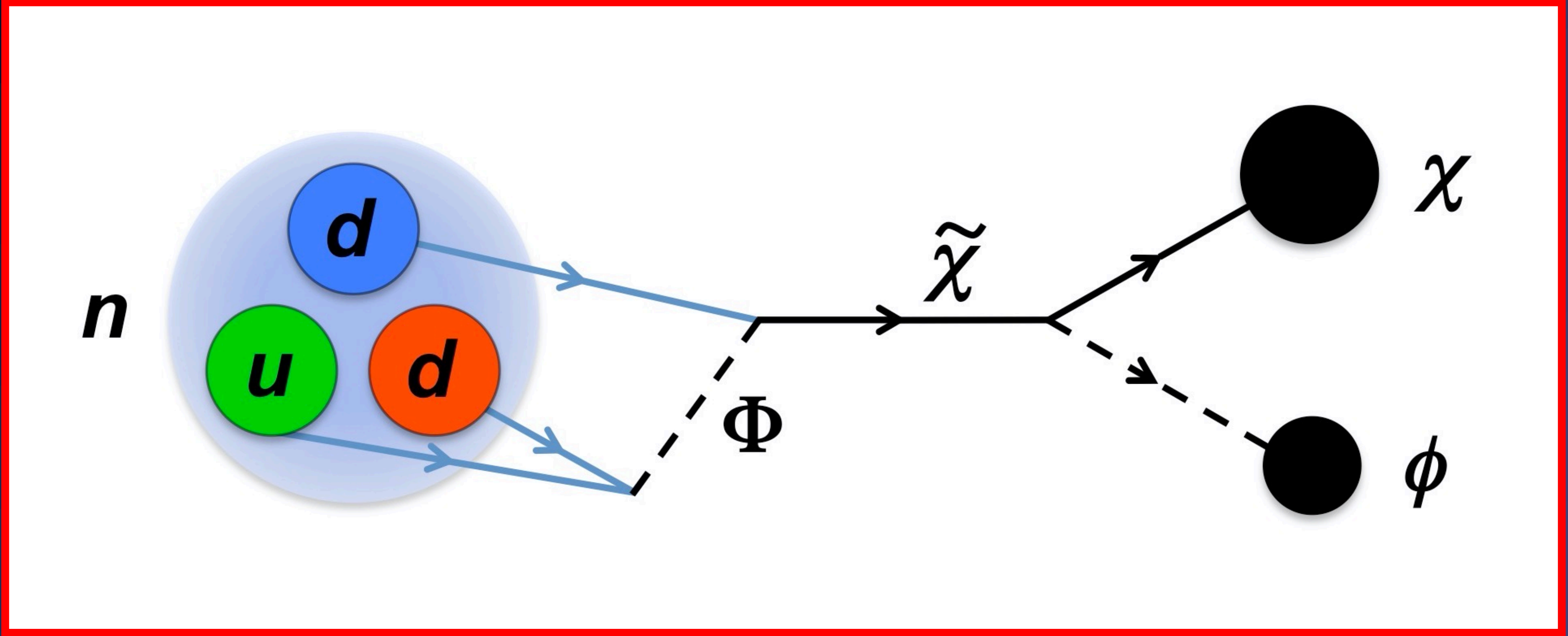
$$\boxed{937.993 \text{ MeV} < M_f < (939.565 - 0.501) \text{ MeV}}$$

$$\Downarrow$$

$$\boxed{937.993 \text{ MeV} < M_f < 939.064 \text{ MeV}}$$

Results inconclusive – stay tuned!

It would be truly amazing if the good old neutron turned out to be the particle enabling us to probe the dark matter sector of the universe

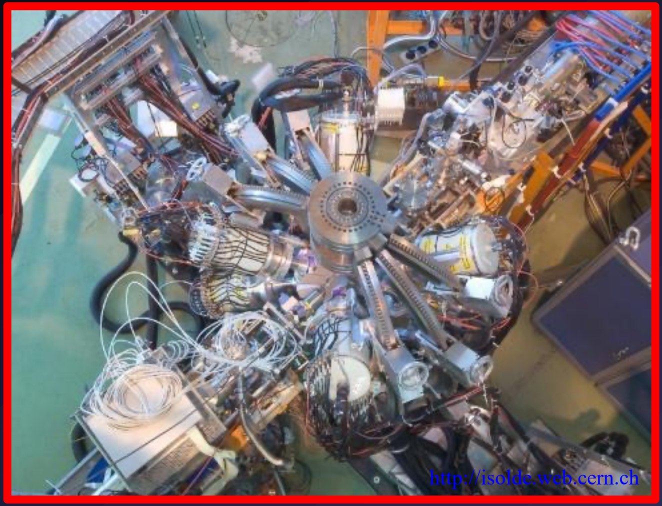


Thank you!

Back up slides

^{11}Be decay experiments

CERN – ISOLDE



http://isolde.web.cern.ch

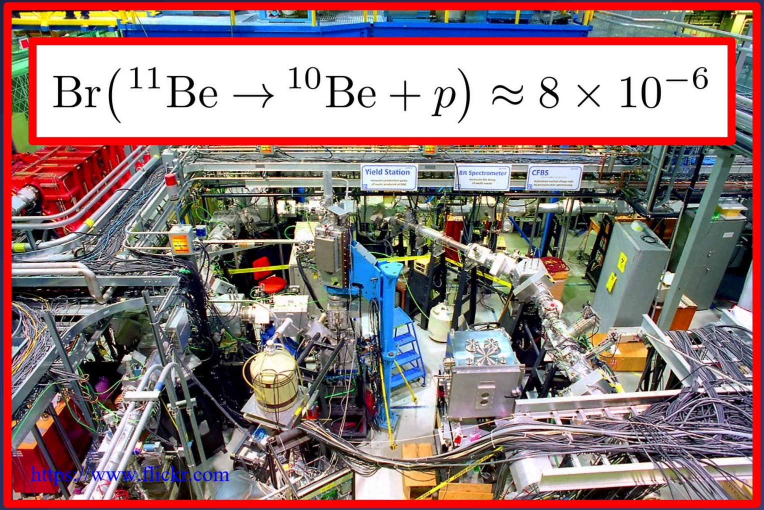
Not the last word:
$Br(^{11}\text{Be} \rightarrow {}^{10}\text{Be} + X) < 2 \times 10^{-6}$

Search for beta-delayed proton emission from ^{11}Be, Riisager et al Eur.Phys.J.A 56 (2020) 3, 100

Possible α-particle contamination?:

Clarification of large-strengt
decay of ^{11}Be, J.Refsgaard et al, Phys.Rev.C 99 (2019) 4, 044316

TRIUMF & MSU



$$\text{Br}\left({}^{11}\text{Be} \rightarrow {}^{10}\text{Be} + p\right) \approx 8 \times 10^{-6}$$

https://www.flickr.com

Evidence of proton resonance:

E_x=11.425(20)MeV, Γ = 12(5)keV

and 2× previous rate:
$Br(^{11}\text{Be} \rightarrow {}^{10}\text{Be} + p + X) = 1.3(3) \times 10^{-5}$

n emission in
^{11}Be: Y. Ayyad *et al, PRL 123 (2019) 082501*

Are there protons in the final state of ^{11}Be decays?

This would test ALL neutron dark decay channels with:

$$937.993 \text{ MeV} < M_f < 939.064 \text{ MeV}$$

$S_p(^{11}\text{B})$ = 11.228 MeV
$E_{res} \sim$ 0.1 MeV
$\Gamma \leq$ 1keV

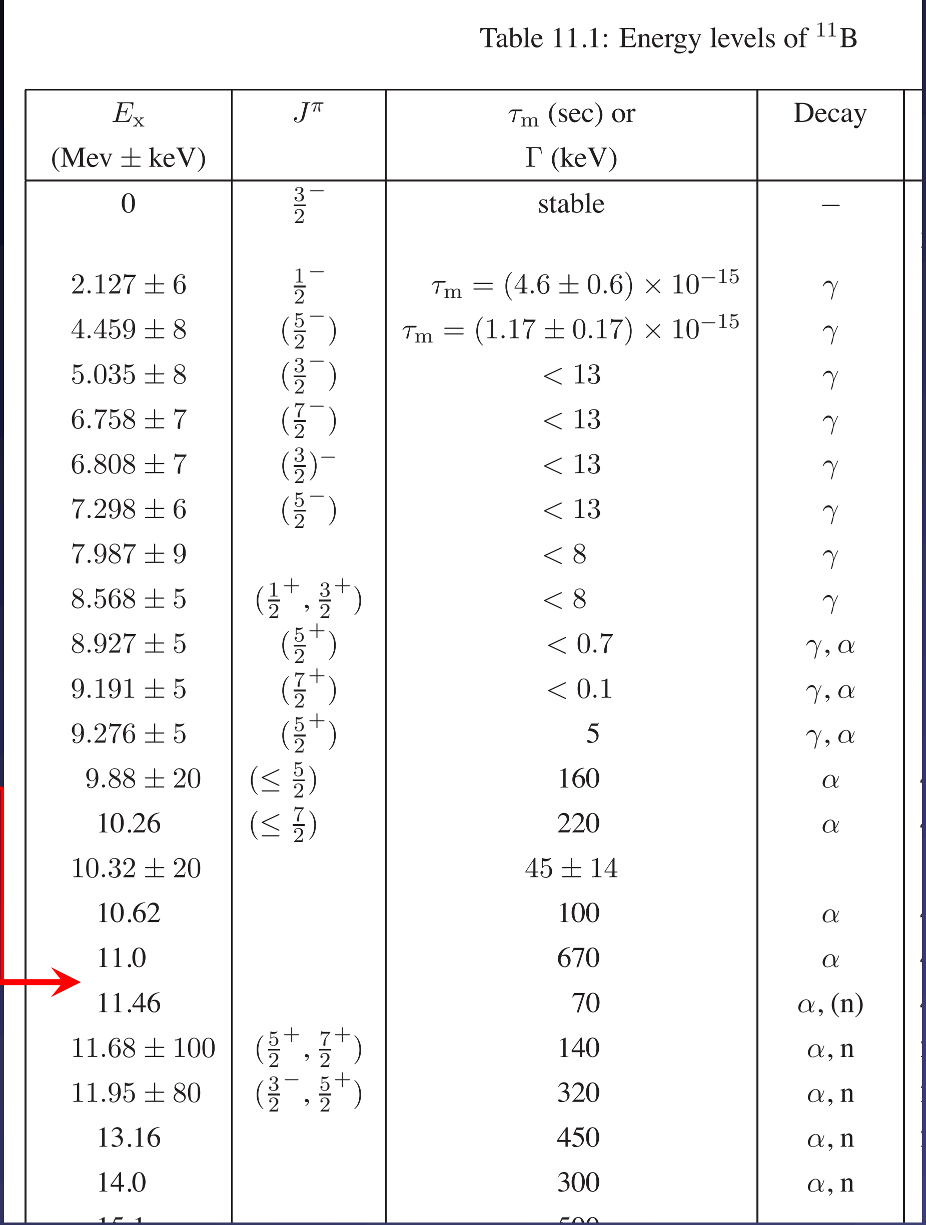


Table 11.1: Energy levels of ^{11}B

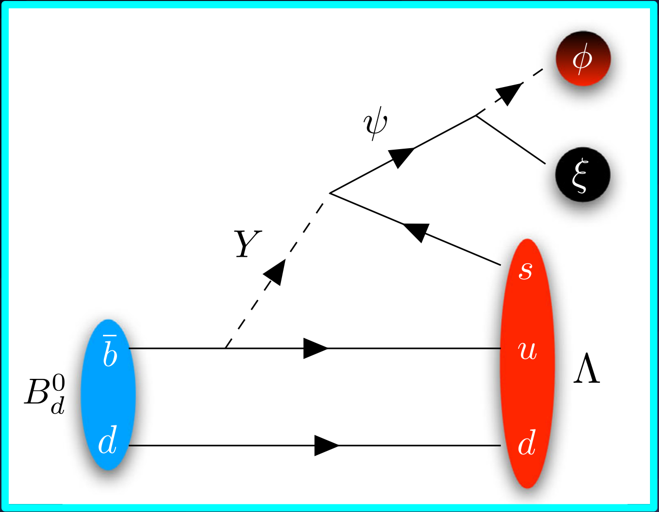
E_x (Mev ± keV)	J^π	τ_m (sec) or Γ (keV)	Decay
0	$\frac{3}{2}^-$	stable	–
2.127 ± 6	$\frac{1}{2}^-$	$\tau_\text{m} = (4.6 \pm 0.6) \times 10^{-15}$	γ
4.459 ± 8	$(\frac{5}{2}^-)$	$\tau_\text{m} = (1.17 \pm 0.17) \times 10^{-15}$	γ
5.035 ± 8	$(\frac{3}{2}^-)$	< 13	γ
6.758 ± 7	$(\frac{7}{2}^-)$	< 13	γ
6.808 ± 7	$(\frac{3}{2})^-$	< 13	γ
7.298 ± 6	$(\frac{5}{2}^-)$	< 13	γ
7.987 ± 9		< 8	γ
8.568 ± 5	$(\frac{1}{2}^+, \frac{3}{2}^+)$	< 8	γ
8.927 ± 5	$(\frac{5}{2}^+)$	< 0.7	γ, α
9.191 ± 5	$(\frac{7}{2}^+)$	< 0.1	γ, α
9.276 ± 5	$(\frac{5}{2}^+)$	5	γ, α
9.88 ± 20	$(\leq \frac{5}{2})$	160	α
10.26	$(\leq \frac{7}{2})$	220	α
10.32 ± 20		45 ± 14	
10.62		100	α
11.0		670	α
11.46		70	α, (n)
11.68 ± 100	$(\frac{5}{2}^+, \frac{7}{2}^+)$	140	α, n
11.95 ± 80	$(\frac{3}{2}^-, \frac{5}{2}^+)$	320	α, n
13.16		450	α, n
14.0		300	α, n

New narrow, near-threshold resonance in ^{11}B suggested also by a numerical calculation (*a posteriori*)

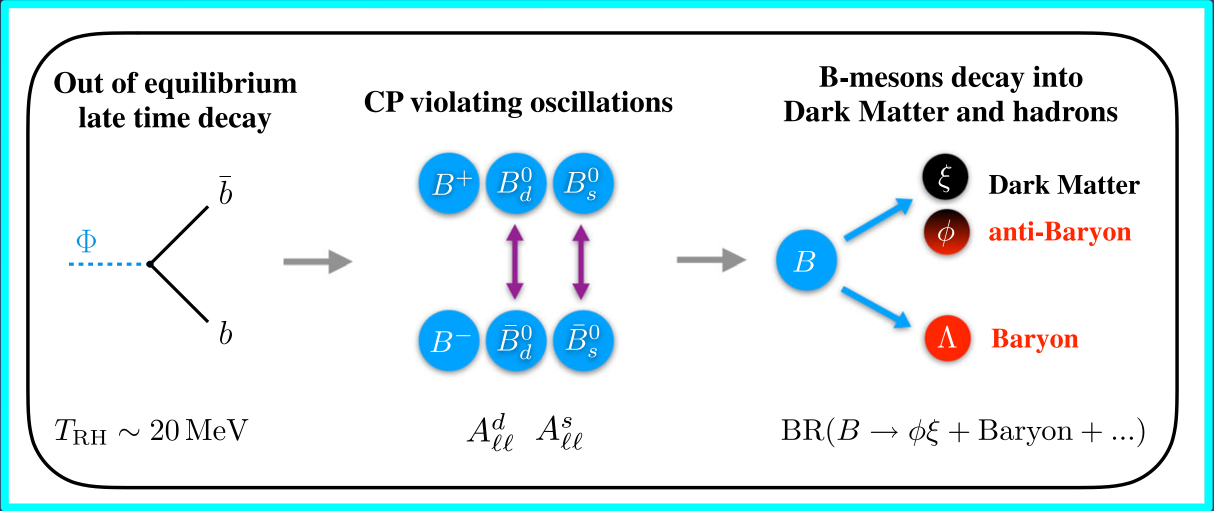
Convenient Location of a Near-Threshold Proton-Emitting Resonance in ^{11}B, J.Okołowicz, et al, PRL 124 (2020) 4, 042502

Mesogenesis

Elor et al, PRD 99, 035031 (2019)



- If *d* quark couples to diquark, so may *s* and *b* quarks.
- New neutral scalar decays late, out of equilibrium, into $B\overline{B}$ pairs
- Before decaying, $B\overline{B}$ oscillations
- B mesons CP violating decay into baryon plus dark



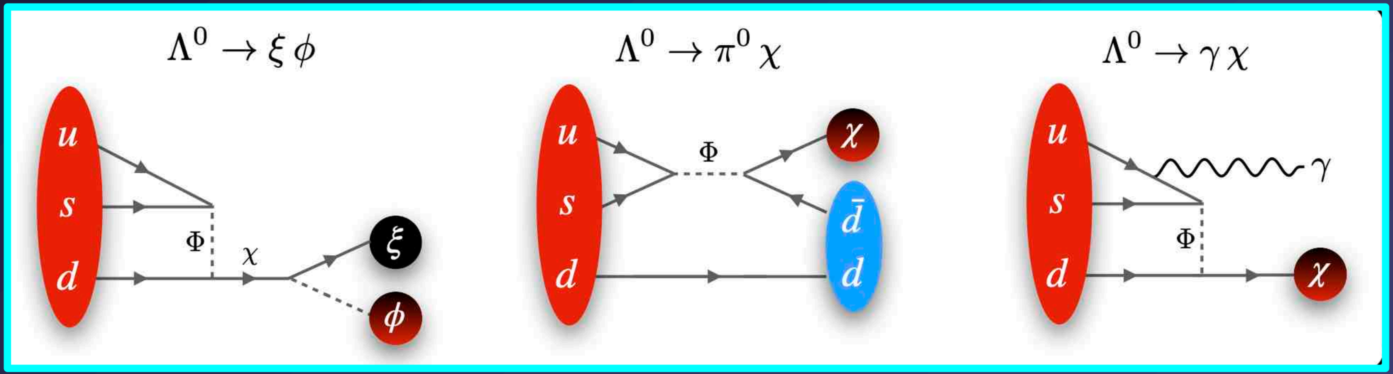
Zhakarov conditions:

1. BNV: dark decay (not really BNV because DM carries baryon-number)
2. Out of equilibrium: late decay
3. CPV: provided by SM!!

Hyperon Dark Decays

Elor et al, in preparation

- As for mesogenesis if *d* quark couples to diquark, so may *s* and *b* quarks.
- Probe this with hyperon decays (BES factory)
- Calculable! (using SU(3) flavor symmetry, ChPT)
- Various modes:



$\Lambda^0 \to \xi\,\phi$ $\quad$ $\Lambda^0 \to \pi^0\,\chi$ $\quad$ $\Lambda^0 \to \gamma\,\chi$

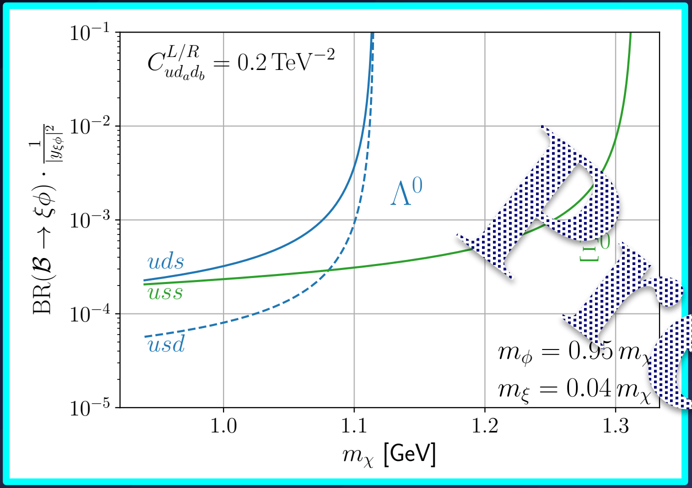
Assume:

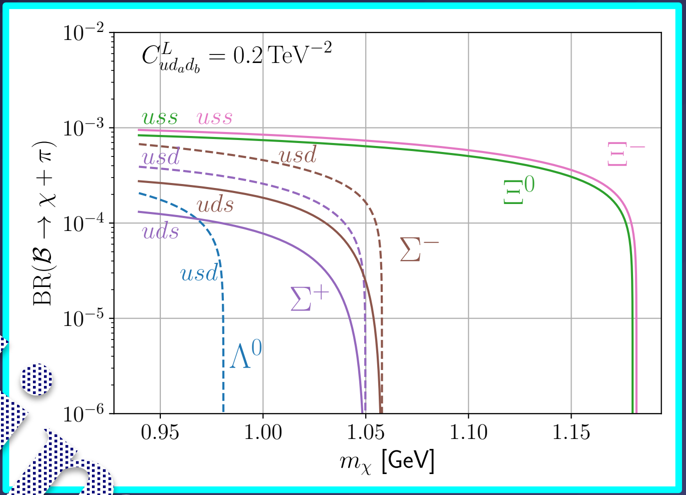
$$\begin{aligned}\mathcal{L}_{\text{eff}} &= C_{ab,c}\mathcal{O}_{ab,c} + C'_{ab,c}\mathcal{O}'_{ab,c},\\ \mathcal{O}_{ab,c} &= \epsilon_{ijk}(u^i_{Ra}d^j_{Rb})(\chi_R d^k_{Rc}),\\ \mathcal{O}'_{ab,c} &= \epsilon_{ijk}\epsilon_{\alpha\beta}(Q^{i\alpha}_{La}Q^{j\beta}_{Lb})(\chi_R d^k_{Rc}),\end{aligned}$$

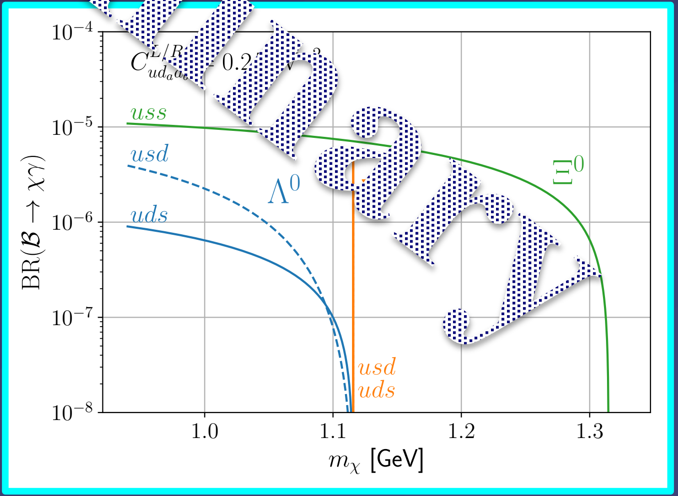
and

$$\mathcal{L} \supset y_{\xi\phi}\bar{\chi}\xi\phi + \text{h.c.}\,,$$

Hyperon Dark Decays







- Large Br’s – possibly
- Complimenarity of modes

LHC signatures

Φ can be singly produced through *u d* → Φ
or pair produced via gluon fusion *g g* → Φ Φ

Collider signatures involve:

→ **2 jets, monojet + MET**

→ **2 jets + MET, 3 jets + MET, 4 jets**

$$\boxed{\frac{M_\Phi}{\sqrt{|\lambda_q \lambda_\chi|}} \approx 400\ \text{TeV}}$$

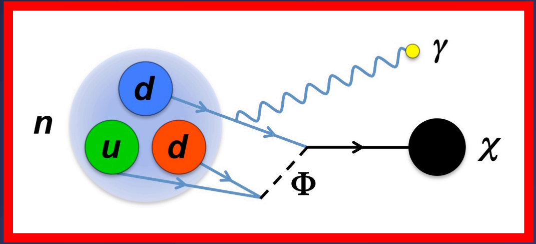
+ absence of signal

$$\longrightarrow \quad \boxed{\sqrt{|\lambda_q \lambda_\chi|} \gtrsim 3 \times 10^{-3}}$$

Dark matter

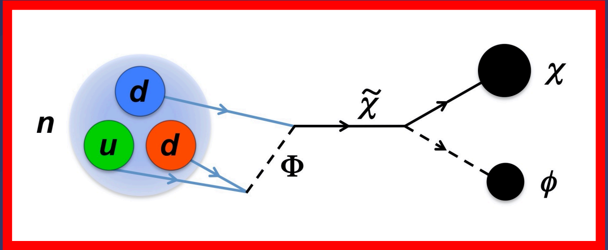
Model 1: non-thermal DM production

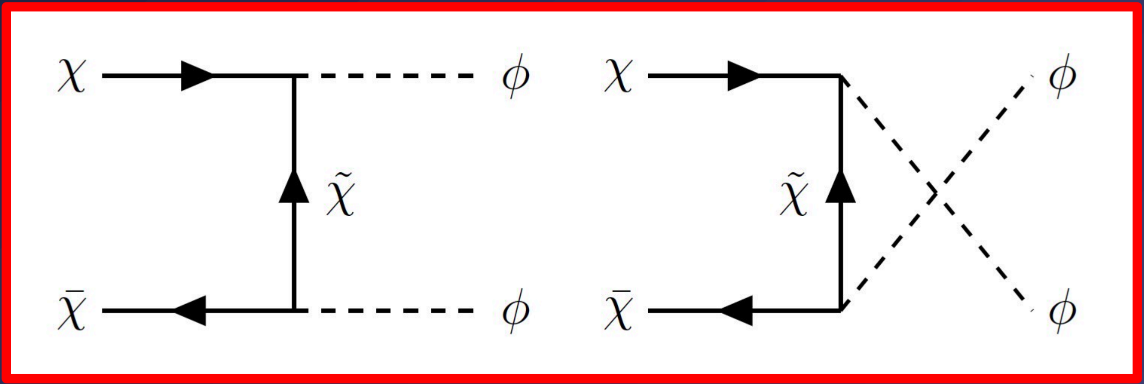
(Allahverdi, Dev, Dutta, PLB 02, 019 (2018))



Model 2: (a) DM non-thermally produced

(b) DM thermally produced





DM annihilation

$$\lambda_\phi \, \bar{\tilde{\chi}} \, \chi \, \phi$$

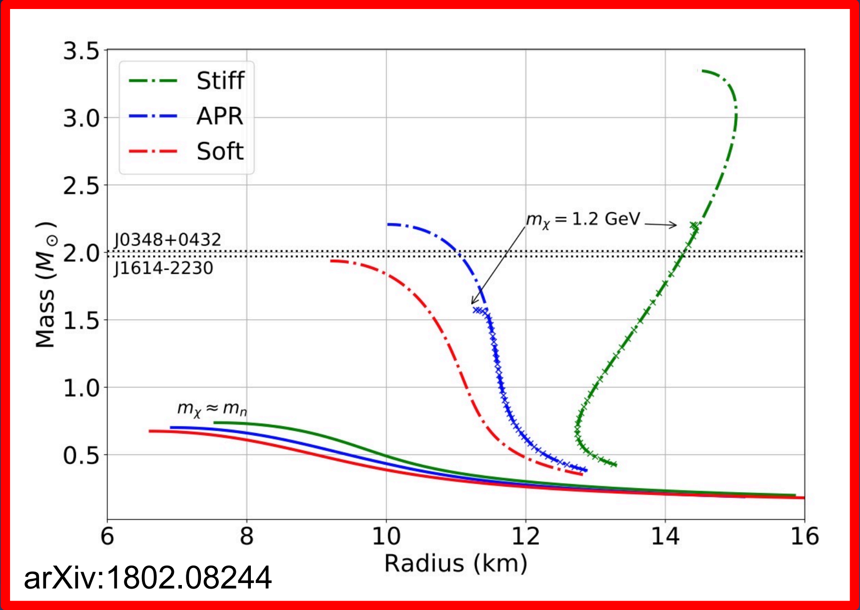
Observed DM relic abundance:

$$\lambda_\phi \approx 0.04$$

(and ~1GeV masses)

Neutron star constraints

- McKeen, Nelson, Reddy & Zhou, arXiv:1802.08244 [hep-ph]
- Baym, Beck, Geltenbort & Shelton, arXiv:1802.08282 [hep-ph]
- Motta, Guichon & Thomas, J. Phys. G 45 05LT01 (2018), arXiv:1802.08427 [nucl-th]



arXiv:1802.08244

➡ **Neutron dark decay channel and no DM self-interactions imply neutron star masses < 0.8 $M_\odot$**

➡ **Dark matter *repulsive* self-interactions can block dark decays and allow the observed neutron star masses**

➡ **DM-neutron cross repulsive interactions, energy cost for converting neutrons to DM**

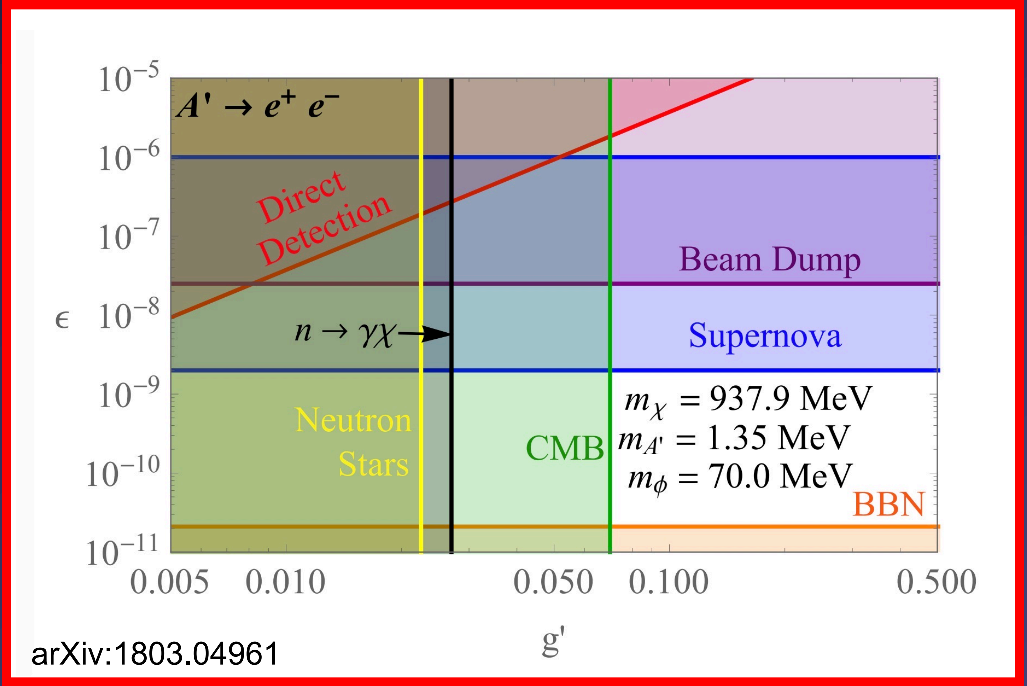
BG, Kouvaris, Nielsen, 1811.06546

Dark decay models with SIDM

➡ **Complete models:** $n \to \chi\, A'$ or $n \to \chi \phi$

✓ **Cline & Cornell, *Dark decay of the neutron*, JHEP 07 (2018) 081**

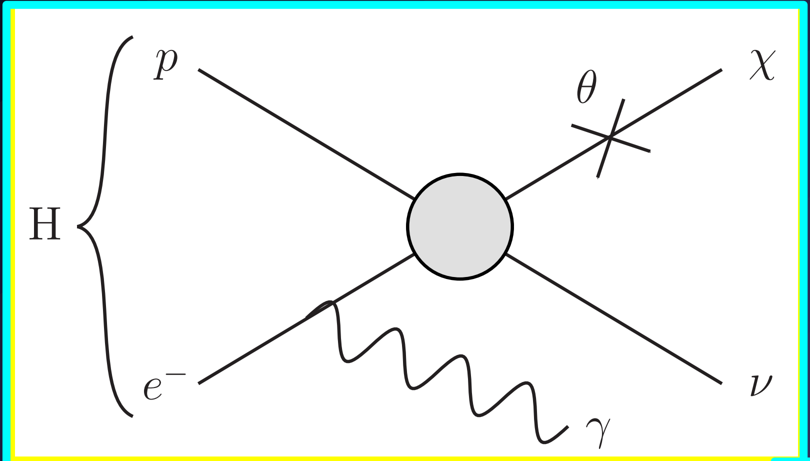
✓ **Karananas & Kassiteridis, *Small-scale structure from neutron dark decay*, JCAP 09 (2018) 036**



arXiv:1803.04961

Hydrogen decay

McKeen, Pospelov, e-Print: 2003.02270
McKeen et al, PRL125 (2020) 231803 , 2006.15140
Fornal et al, PLB 811 (2020) 135869

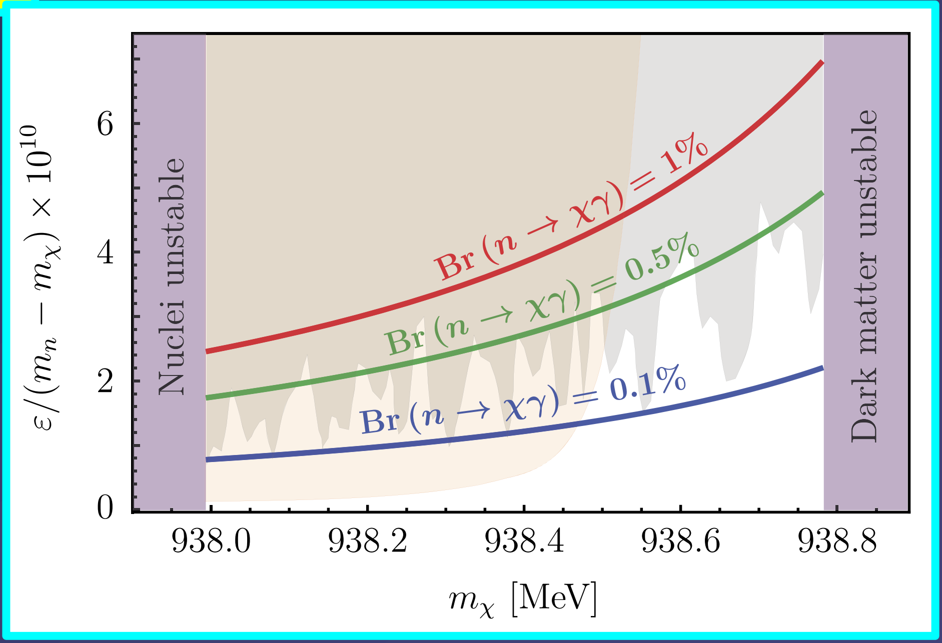


- **Electron capture in H**
- **Probes sensitively same region as DM hypothesis:**

$$m_\chi < m_p + m_e = 938.783 \text{ MeV}$$

Light brown region excluded using Borexino data reinterpreted as

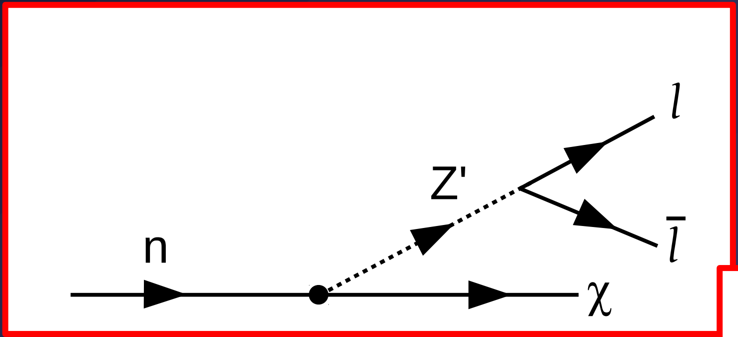
$$H \to \gamma + nothing$$

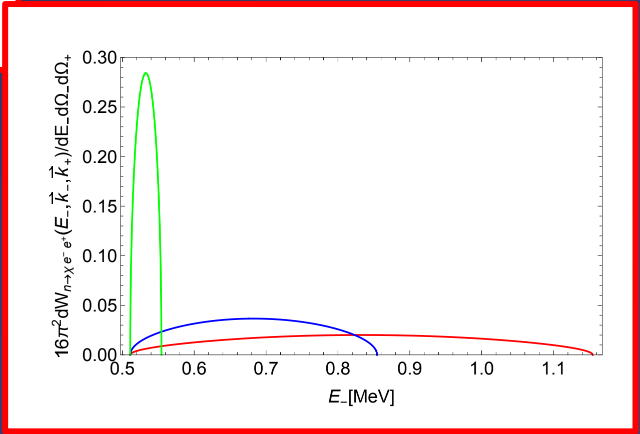


Other solutions to the neutron lifetime puzzle

Ivanov, Hollwieser, Troitskaya, Wellenzohn, Berdnikov, arXiv:1806.10107

→ **Neutrino mode**





$\chi e^+ e^-$ **is phase space suppressed relative to** $\chi \nu \nu$

Search for $e^- n \to e^- \chi$

Other solutions to the neutron lifetime puzzle

→ **Considering only the 2002+ measurements of g_A the bottle neutron lifetime is favored; there is no puzzle, beam is wrong**

Czarnecki, Marciano & Sirlin, PRL 120, 202002 (2018)

→ **Neutron-mirror neutron oscillations enhanced in large magnetic fields**

Z.Berezhiani, Eur.Phys.J.C 79 (2019) 6, 484

→ **Neutron – mirror-neutron rapid oscillations**

W.Tan, Phys.Lett.B 797 (2019) 134921

Back-up slide: Neutron star constraints

(arXiv:1802.08244, 1802.08282, 1802.08427)

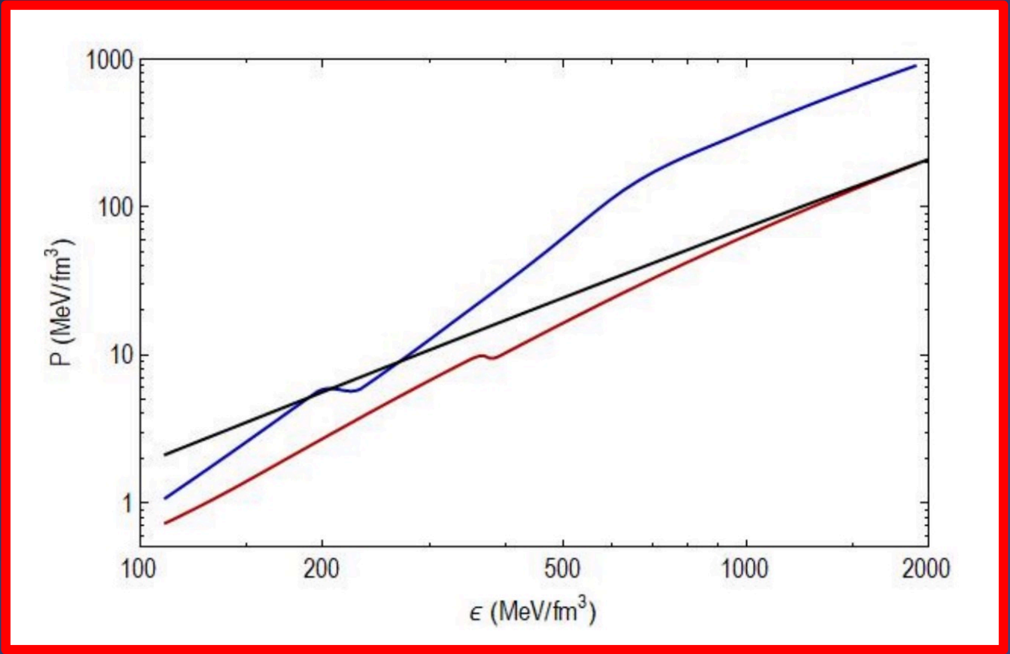
→ Neutron dark decay channel affects the equation of state for neutron stars

Total energy density

$$\epsilon = \epsilon_b(n_b) + m_\chi n_b y^3 + \frac{k_b^5}{10\pi^2 m_\chi} y^5$$

Total pressure

$$P = P_b(n_b) + \frac{k_b^5}{15\pi^2 m_\chi} y^5$$



(arXiv:1802.08282)

Where y relates the Fermi k

$$k_\chi = y k_b$$

DM-neutron cross repulsive interactions, energy cost for converting neutrons to DM

BG, Kouvaris, Nielsen, Phys.Rev.Lett. 123 (2019) 9, 091601 [1811.06546]

Assume potential from exchange of ϕ: $$U = -\frac{g_n g_\chi}{4\pi r}\exp(-m_\phi r)$$

Equation of state: $$\varepsilon(n_n, n_\chi) = \varepsilon_{\rm nuc}(n_n) + \varepsilon_\chi(n_\chi) + \frac{n_\chi n_n}{z^2}$$

$(z = m_\phi/\sqrt{|g_\chi g_n|})$

Fixed total number density: $$n_{\rm F} = n_n + n_\chi$$

Free energy cost @ T=0 to create a DM particle:

$$\Delta E \equiv \frac{\partial\varepsilon(n_{\rm F} - n_\chi, n_\chi)}{\partial n_\chi} = \mu_\chi(n_\chi) - \mu_{\rm nuc}(n_n) + \frac{n_{\rm F} - 2n_\chi}{z^2}$$

Pure n environment: $$\Delta E_0 = \Delta E|_{n_\chi=0} = m_\chi - \mu_{\rm nuc}(n_{\rm F}) + \frac{n_{\rm F}}{z^2}$$

No conversion to χ for small z !!