Flavour from the $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

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Flavour problem

- Why are fermions are hierarchical among and within three families with charged lepton masses being of the same order as down-type quark masses?
- What is the origin of quark-mixing?
- Why is quark mixing hierarchical?
- What is the origin of neutrino masses?
- Why are neutrino masses so small?
- What is the origin of leptonic mixing?
- Why is quark mixing is so different from leptonic mixing?

The Froggart-Nielson mechanism

Solution

- An abelian flavour symmetry $U(1)_F$ is added to the SM in such a way that only top quark acquires its mass through renormalized operator. Froggatt and Nielson1978

- Thus masses of fermions are recovered through higher order effective operators having the following structure :

$$\mathcal{O} = y(\frac{S}{\Lambda})^{(\theta_i+\theta_j)}\bar{\psi}_i\varphi\psi_j,$$

where y is the coupling constant, and S is the flavon field.

The Froggart-Nielson mechanism

- The new physics scale Λ can be anywhere between the weak and the Planck scale.
- The crucial question is how low this scale could be given the present bounds on flavour-changing and CP-violating processes.
- This interesting question depends on the underlying unknown dynamics, for instance whether abelian flavour symmetry $U(1)_F$ is local or global.

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

A novel solution

- We show the minimal realization of the Froggatt-Nielson mechanism where one does not need to impose a continuous $U(1)_F$ symmetry.
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- For achieving this goal, we employ a complex singlet scalar field χ which behaves under the SM symmetry as,

$$\chi : (1, 1, 0),$$

  and impose $\mathcal{Z}_2 \times \mathcal{Z}_5$ on the SM.

- The masses of the three fermionic families appear in terms of the expansion parameter $\langle\chi\rangle/\Lambda$ where Λ is the scale of new physics which renormalizes our model.

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_5$
u_R, c_R, t_R	+	ω^2
$d_R, s_R, b_R, e_R, \mu_R, \tau_R$	-	ω
ν_{e_R}	-	ω^3
ν_{μ_R}	-	ω^2
ν_{τ_R}	+	1
ψ_L^1	+	ω
ψ_L^2	+	ω^4
ψ_L^3	+	ω^2
χ	-	ω
φ	+	1

Table: The charges of left and right-handed fermions of three families of the SM,right-handed neutrinos, Higgs, and singlet scalar fields under $\mathcal{Z}_2$ and $\mathcal{Z}_5$ symmetries where ω is the fifth root of unity.

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

The mass Lagrangian for fermions reads,

$$\begin{aligned}\mathcal{L}_{mass} &= \sum_{n=0}^{2} \left(\frac{\chi}{\Lambda}\right)^{2n} \sum_{i,j=3,2,1} y_{ij}^{u} \bar{\psi}_{L_i}^{q} \tilde{\varphi} \psi_{R_j}^{u} + \sum_{n=0}^{2} \left(\frac{\chi}{\Lambda}\right)^{2n+1} \sum_{i,j=3,2,1} y_{ij}^{d} \bar{\psi}_{L_i}^{q} \varphi \psi_{R_j}^{d} \\ &+ \sum_{n=0}^{2} \left(\frac{\chi}{\Lambda}\right)^{2n+1} \sum_{i,j=3,2,1} y_{ij}^{\ell} \bar{\psi}_{L_i}^{\ell} \varphi \psi_{R_j}^{\ell} + \mathrm{H.c.}.\end{aligned}$$

where $\psi_R^u, \psi_R^d, \psi_R^\ell$ are right-handed up, down type singlet quarks and singlet leptons, ψ_L^q, ψ_L^ℓ are quark and leptonic doublets, i and j are family indices, $\tilde{\varphi} = -i\sigma_2\varphi^*$ conjugate Higgs field and σ_2 is second Pauli matrix. We expand the Lagrangian such that it is invariant under $\mathcal{Z}_2$ and $\mathcal{Z}_5$ symmetries

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

In terms of expansion parameter $\frac{\langle \chi \rangle}{\Lambda} = \frac{f}{\sqrt{2}\Lambda} = \epsilon$, the up-type quark mass matrix is,

$$\mathcal{M}_{\mathcal{U}} = \frac{v}{\sqrt{2}} \begin{pmatrix} y^u_{11}\epsilon^4 & y^u_{12}\epsilon^4 & y^u_{13}\epsilon^4 \\ y^u_{21}\epsilon^2 & y^u_{22}\epsilon^2 & y^u_{23}\epsilon^2 \\ y^u_{31} & y^u_{32} & y^u_{33} \end{pmatrix}, \mathcal{M}_{\mathcal{D}} = \frac{v}{\sqrt{2}} \begin{pmatrix} y^d_{11}\epsilon^5 & y^d_{12}\epsilon^5 & y^d_{13}\epsilon^5 \\ y^d_{21}\epsilon^3 & y^d_{22}\epsilon^3 & y^d_{23}\epsilon^3 \\ y^d_{31}\epsilon & y^d_{32}\epsilon & y^d_{33}\epsilon \end{pmatrix}. \quad (1)$$

The mass matrix of charged leptons can be written as,

$$\mathcal{M}_{\ell} = \frac{v}{\sqrt{2}} \begin{pmatrix} y^\ell_{11}\epsilon^5 & y^\ell_{12}\epsilon^5 & y^\ell_{13}\epsilon^5 \\ y^\ell_{21}\epsilon^3 & y^\ell_{22}\epsilon^3 & y^\ell_{23}\epsilon^3 \\ y^\ell_{31}\epsilon & y^\ell_{32}\epsilon & y^\ell_{33}\epsilon \end{pmatrix}. \quad (2)$$

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

The masses of quarks at leading order are given by,

$$\{m_t, m_c, m_u\} \simeq \{|y^u_{33}|, \left|y^u_{22} - \frac{y^u_{23}y^u_{32}}{|y^u_{33}|}\right| \epsilon^2,$$

$$\left|y^u_{11} - \frac{y^u_{12}y^u_{21}}{|y^u_{22} - y^u_{23}y^u_{32}/y^u_{33}|} - \frac{y^u_{13}|y^u_{31}y^u_{22} - y^u_{21}y^u_{32}| - y^u_{31}y^u_{12}y^u_{23}}{|y^u_{22} - y^u_{23}y^u_{32}/y^u_{33}||y^u_{33}|}\right| \epsilon^4\} v/\sqrt{2},$$

$$\{m_b, m_s, m_d\} \simeq \{|y^d_{33}|\epsilon, \left|y^d_{22} - \frac{y^d_{23}y^d_{32}}{|y^d_{33}|}\right| \epsilon^3, ,$$

$$\left|y^d_{11} - \frac{y^d_{12}y^d_{21}}{|y^d_{22} - y^d_{23}y^d_{32}/y^d_{33}|} - \frac{y^d_{13}|y^d_{31}y^d_{22} - y^d_{21}y^d_{32}| - y^d_{31}y^d_{12}y^d_{23}}{|y^d_{22} - y^d_{23}y^d_{32}/y^d_{33}||y^d_{33}|}\right| \epsilon^5\} v/\sqrt{2},$$

$$\{m_\tau, m_\mu, m_e\} \simeq \{|y^l_{33}|\epsilon, \left|y^l_{22} - \frac{y^l_{23}y^l_{32}}{|y^l_{33}|}\right| \epsilon^3,$$

$$\left|y^l_{11} - \frac{y^l_{12}y^l_{21}}{|y^l_{22} - y^l_{23}y^l_{32}/y^l_{33}|} - \frac{y^l_{13}|y^l_{31}y^l_{22} - y^l_{21}y^l_{32}| - y^l_{31}y^l_{12}y^l_{23}}{|y^l_{22} - y^l_{23}y^l_{32}/y^l_{33}||y^l_{33}|}\right| \epsilon^5\} v/\sqrt{2},$$

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

The quark mixing angles at leading order are found to be,

$$\begin{aligned} \sin\theta_{12} \simeq |V_{us}| &\simeq \left| \frac{y^d_{12}}{y^d_{22}} - \frac{y^u_{12}}{y^u_{22}} \right| \epsilon^2, \sin\theta_{23} \simeq |V_{cb}| \simeq \left| \frac{y^d_{23}}{y^d_{33}} - \frac{y^u_{23}}{y^u_{33}} \right| \epsilon^2, \\ \sin\theta_{13} \simeq |V_{ub}| &\simeq \left| \frac{y^d_{13}}{y^d_{33}} - \frac{y^u_{12} y^d_{23}}{y^u_{22} y^d_{33}} - \frac{y^u_{13}}{y^u_{33}} \right| \epsilon^4. \end{aligned} \tag{3}$$

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

Neutrino masses and oscillations

The tree level Majorana Lagrangian can be written with help of table 1,

$$\mathcal{L}_M = M\bar{\nu^c}_{e_R}\nu_{\mu_R} + M\bar{\nu^c}_{\mu_R}\nu_{e_R} + M\bar{\nu^c}_{\tau_R}\nu_{\tau_R},$$

where M is the Majorana mass scale.
The neutrino Dirac mass matrix is given by,

$$\mathcal{M}_{\mathcal{D}} = \frac{v}{\sqrt{2}} \begin{pmatrix} y^{\nu}_{11}\epsilon^3 & y^{\nu}_{12}\epsilon & y^{\nu}_{13}\epsilon^4 \\ y^{\nu}_{21}\epsilon & y^{\nu}_{22}\epsilon^3 & y^{\nu}_{23}\epsilon^4 \\ y^{\nu}_{31}\epsilon & y^{\nu}_{32}\epsilon^5 & y^{\nu}_{33}\epsilon^2. \end{pmatrix} \quad (4)$$

The neutrino mass matrix after including the Majorana mass terms becomes,

$$\mathcal{M} = \begin{pmatrix} 0 & \mathcal{M}_{\mathcal{D}} \\ \mathcal{M}^T_{\mathcal{D}} & \mathcal{M}_R \end{pmatrix}, \quad (5)$$

where the Majorana mass matrix $\mathcal{M}_R$

$$\mathcal{M}_R = \begin{pmatrix} c^{\nu}_{11}\epsilon^4 & M & c^{\nu}_{13}\epsilon^3 \\ M & c^{\nu}_{22}\epsilon^4 & c^{\nu}_{23}\epsilon^3 \\ c^{\nu}_{13}\epsilon^3 & c^{\nu}_{23}\epsilon^3 & M, \end{pmatrix} \quad (6)$$

The $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

Neutrino masses and oscillations

The masses of neutrinos now can be determined using type-I seesaw mechanism. Assuming $\mathcal{M}_\mathcal{D} << \mathcal{M}_R$, the mass matrix of the light neutrinos reads,

$$\mathcal{M} = -\mathcal{M}_\mathcal{D}\mathcal{M}_R^{-1}\mathcal{M}_\mathcal{D}^T. \tag{7}$$

The light neutrino masses can approximately be written as,

$$m_1 \approx y_{11}^\nu \epsilon^2 \epsilon', \; m_2 \approx y_{22}^\nu \epsilon\, \epsilon', \; m_3 \approx y_{33}^\nu \epsilon\, \epsilon'.$$

where $\epsilon' = \frac{v}{\sqrt{2}M}$.
The leptonic mixing angles approximately can be read as,

$$\sin\theta_{12} \simeq \left|\frac{y_{21}^\nu}{y_{22}^\nu}\right| \epsilon^2, \sin\theta_{23} \simeq \left|\frac{y_{32}^\nu}{y_{33}^\nu}\right|, \sin\theta_{13} \simeq \left|\frac{y_{31}^\nu}{y_{33}^\nu}\right| \epsilon^2.$$

All the masses and mixing angles can be recovered for $\epsilon = 0.1$, $\epsilon' = 1.259 \times 10^{-9}$ and all the couplings in the range of $0.1 - 2\pi$.

Bounds on the flavour scale

In the scenario where there is no Higgs-flavon mixing.

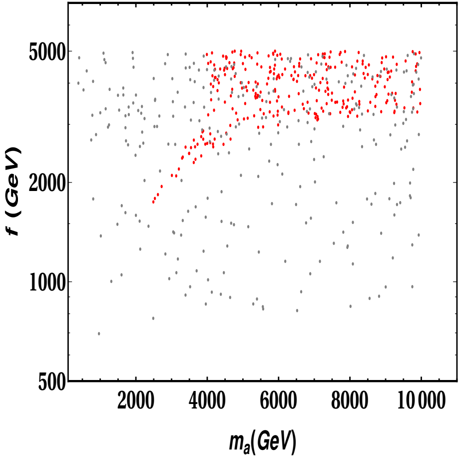

Figure: The allowed parameter space by ϵ_K and Δm_K for $\lambda_\chi = 2$ in the $m_a - f$ plane. The red points represent allowed flavon contribution to ϵ_K, and the allowed contribution to Δm_K is shown by grey points.

Summary

1. We have presented a novel and original idea based on a minimal $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry which is capable of providing a solution for the flavour problem of the SM.

2. The remarkable feature is the emergence of an explanation for the neutrino mixing angles.

3. A partial phenomenological study of the flavour bounds are also presented.

4. An origin of $\mathcal{Z}_2$ and $\mathcal{Z}_5$ may be traced to Abelian or non-Abelian continous symmetries. For instance, $\mathcal{Z}_2$ and $\mathcal{Z}_5$ may be an artefact of spontaneous breaking of $U(1) \times U(1)$ continuous symmetries. This is radically different from the standard mechanism which is based on a continuous $U(1)$ symmetry.