Statistical Analysis at e^+e^- Colliders using Optimal Observable Technique (OOT)

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Outlines

⋆ Probing heavy charged fermions at e^+e^- collider using Optimal Observable Technique, S. Bhattachrya, **S. Jahedi**, and J. Wudka. [arXiv : 2106.02846]

- Optimal Observable Technique
- Parametrization of Z coupling with the charged fermions
- 1σ uncertainties on NP parameters
- Distinction of hypotheses from a base model
- A model example
- Summary

Statistical Analysis Without Data

Decomposion of any observable ($\mathcal{O}$) : $\mathcal{O} = \frac{d\sigma}{d\phi} = \sum_{i=1}^{n} c_i f_i$

c_i : Function of new physics (NP) parameters

f_i : **Linearly Independent** functions of phase space variables

ϕ : $\cos\theta$ (For $2 \to 2$ process)

Covariance matrix (V_{ij}) : $\frac{M_{ij}^{-1}}{\mathcal{L}}$ (which is **Minimum**)

Where, $M_{ij} = \int \frac{f_i f_j}{\mathcal{O}} d\phi$

$\mathcal{L}$ = Luminosity

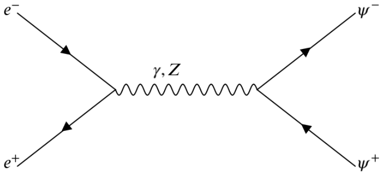
The usual definition of χ^2 function is given by,

$$\chi^2 = \epsilon \sum_{\{i,j\}=1}^{n} (c_i - c_i^0)(c_j - c_j^0) V_{ij}^{-1}$$

$$\text{Efficiency factor } (\epsilon) = \frac{\sigma^{\texttt{sig}}}{\sigma^{prod}}$$

$$c_i^0 = c_i(a^0, b^0) \rightarrow \text{Seed values}$$

$\chi^2 = 1 \rightarrow$ **1σ uncertainties**



Parametrization of charged scalar with Z

$$\psi^+\psi^- Z : -\frac{e_0}{\sin 2\theta_w}\gamma^\mu(a + b\gamma 5)$$

where, e_0 : Electromagnetic coupling constant

θ_w : Weak-mixing angle

a, b : NP parameters

Different hypotheses

Photon coupling

$$\psi^+\psi^-\gamma : -\ ie_0\gamma^\mu$$

Possible **Hypotheses**

- $a = \pm 1, b = \pm 1$ (Mixed coupling)
- $a = \pm 1, b = 0$ (Purely vector like coupling)
- $a = 0, b = \pm 1$ (Purely axial-vector like coupling)

Differential Cross-section

$$\frac{d\sigma(P_{e^-}, P_{e^+})}{d\Omega} = \frac{1}{4}\left\{\left(1 + P_{e^-}\right)\left(1 - P_{e^+}\right)\left(\frac{d\sigma}{d\Omega}\right)_{\rm RL} + \left(1 - P_{e^-}\right)\left(1 + P_{e^+}\right)\left(\frac{d\sigma}{d\Omega}\right)_{\rm LR}\right\}$$

Choice of Polarization (ILC TDR, arXiv: 1306:6352)

$P_{e^-} \rightarrow$ Electron beam (e^-) polarization.
$P_{e^+} \rightarrow$ Positron beam (e^+) polarization.
$P_{\tt eff} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} P_{e^+}}$

$$\{P_{e^-} : P_{e^+}\} : \{-80\%, +20\%\}$$

$+ :\rightarrow$ Right polarized $\qquad - :\rightarrow$ Left polarized

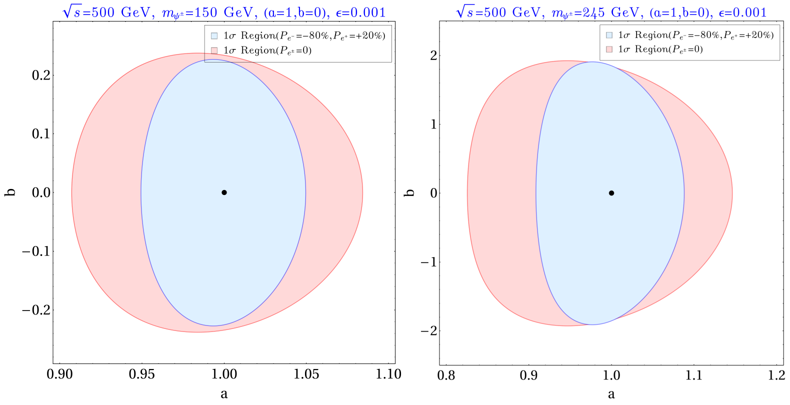
$$
\begin{aligned}
c_1 =& \alpha_0^2 \frac{(1 - P_{e^-} P_{e^+})}{2} \left[1 + 2\xi_1 a + (\xi_1^2 + \xi_2^2) \left(a^2 + \frac{\beta_\psi^2}{2 - \beta_\psi^2} b^2 \right) \right. \\
& \left. - 2P_{\texttt{eff}} \left\{ \xi_2 a + \xi_1 \xi_2 a^2 + \frac{\beta_\psi^2}{2 - \beta_\psi^2} \xi_1 \xi_2 b^2 \right\} \right] \\
c_2 =& \alpha_0^2 \frac{(1 - P_{e^-} P_{e^+})}{2} \left[2\xi_2 b + 4\xi_1 \xi_2 ab - P_{\texttt{eff}} \left\{ 2\xi_1 b + (\xi_1^2 + \xi_2^2) ab \right\} \right] \\
c_3 =& \alpha_0^2 \frac{(1 - P_{e^-} P_{e^+})}{2} \left[1 + 2\xi_1 a + (\xi_1^2 + \xi_2^2)(a^2 + b^2) \right. \\
& \left. - 2P_{\texttt{eff}} \left\{ \xi_2 a + \xi_1 \xi_2 (a^2 + b^2) \right\} \right]
\end{aligned}
$$

$$
\{f_1,\ f_2,\ f_3\} = \frac{\beta_\psi}{2s} \left\{ (2 - \beta_\psi^2),\ \beta_\psi \cos\theta,\ \beta_\psi^2 \cos^2\theta \right\}
$$

$$
\xi_1 = \frac{C_v}{s_{2w}^2 (1 - m_Z^2/s)}; \quad \xi_2 = \frac{C_a}{s_{2w}^2 (1 - m_Z^2/s)}; \quad \beta_\psi = \sqrt{1 - 4 m_{\psi^\pm}^2 / s}
$$

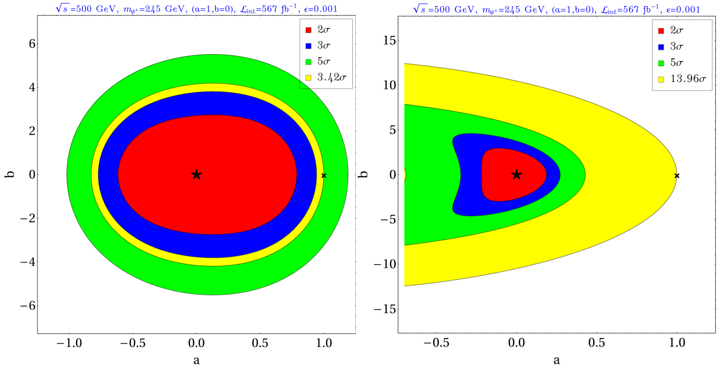
1σ uncertainties on NP parameters

a=1, b=0 (Purely vector like coupling)



- For lower mass, the NP parameters have better precision.
- Polarized beams help us to achieve better precision.

Unpolarized beam | **Polarized beam** ($P_{e^\pm} = ^{+20\%}_{-80\%}$)



Polarized beam ($P_{e^\pm} = ^{+20\%}_{-80\%}$) helps to distinguish the model ($a = 1, b = 0$) from the base model ($a = 0, b = 0$).

A model example (Singlet-doublet fermionic DM)

Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y \times \mathcal{Z}_2$			
$\psi = \begin{pmatrix} \psi^0 \\ \psi^- \end{pmatrix}$	1	2	-1	-
χ	1	1	0	-
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	1	2	1	$+$

The Lagrangian of the model is:

$$\mathcal{L}^{VF} = \bar{\psi}\ [i\gamma^\mu(\partial_\mu - ig\frac{\sigma^a}{2}W^a_\mu - ig'\frac{Y}{2}B_\mu) - m_\psi]\ \psi \\ +\bar{\chi}\ (i\gamma^\mu\partial_\mu - m_\chi)\ \chi - (Y_1\bar{\psi}\widetilde{H}\chi + h.c)$$

After EWSB, $H = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}(v+h) \end{pmatrix}^T$ where $v = 246$ GeV.

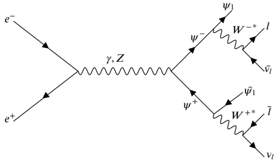
The interaction lagrangian is given by:

$$\mathcal{L}_{int}^{VF} = \left(\frac{e_0}{2\sin\theta_W\cos\theta_W}\right)\Big[\sin^2\theta\bar{\psi}_1\gamma^\mu Z_\mu\psi_1 + \cos^2\theta\bar{\psi}_2\gamma^\mu Z_\mu\psi_2$$
$$+\sin\theta\cos\theta(\bar{\psi}_1\gamma^\mu Z_\mu\psi_2 + \bar{\psi}_2\gamma^\mu Z_\mu\psi_1)\Big]$$
$$+\frac{e_0}{\sqrt{2}\sin\theta_W}\sin\theta\bar{\psi}_1\gamma^\mu W_\mu^+\psi^- + \frac{e_0}{\sqrt{2}\sin\theta_W}\cos\theta\bar{\psi}_2\gamma^\mu W_\mu^+\psi^-$$
$$+\frac{e_0}{\sqrt{2}\sin\theta_W}\sin\theta\psi^+\gamma^\mu W_\mu^-\psi_1 + \frac{e_0}{\sqrt{2}\sin\theta_W}\cos\theta\psi^+\gamma^\mu W_\mu^-\psi_2$$
$$-\left(\frac{e_0}{2\sin\theta_W\cos\theta_W}\right)\cos 2\theta_W\psi^+\gamma^\mu Z_\mu\psi^- - e_0\psi^+\gamma^\mu A_\mu\psi^-$$
$$-\frac{Y_1}{\sqrt{2}}h\Big[\sin 2\theta(\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2) + \cos 2\theta(\bar{\psi}_1\psi_2 + \bar{\psi}_2\psi_1)\Big]$$

θ : Singlet-doublet mixing angle

This model contains **vector like** $(a = 0.54, b = 0)$ interaction of charged fermion with Z.

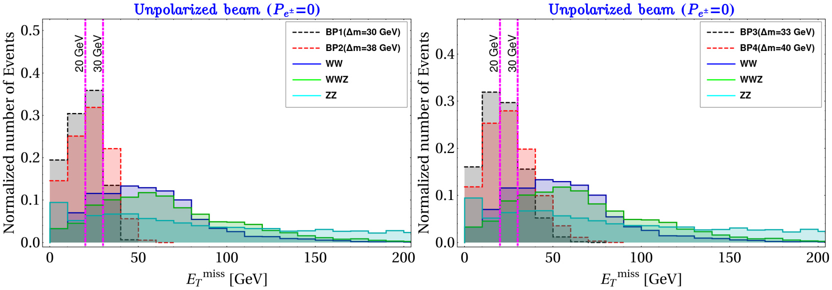
Collider simulation of the signal events



Signal: $\ell^+\ell^- + \not{E}_T$, SM background : WW, WWZ, ZZ.

Benchmark Points	$m_{\psi^\pm}$(GeV)	$m_{\psi 1}$(GeV)	Δm(GeV)
BP1	245	215	30
BP2	245	207	38
BP3	150	117	25
BP4	150	110	40

Missing energy distribution



$$\text{Efficiency factor } (\epsilon) = \frac{\sigma^{\text{OSD}}}{\sigma^{\text{prod}}} \sim 0.005.$$

$$\text{This analysis : } \epsilon \sim 0.001$$

Summary

- OOT guides us to extract the uncertainties to new physics (NP) parameters and helps us to distinguish one model from another model.
- Vector like coupling can be extracted more precisely than other hypotheses.
- The uncertainties and the segregation depend on ϵ, higher value of ϵ always indicate better sensitivity and better segregation.
- The f_i's always have to be linearly independent.

Thank You