Probing Lorentz Invariance Violation with Atmospheric Neutrinos at INO-ICAL

(India-based Neutrino Observatory)

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Probing Lorentz Invariance Violation with Atmospheric Neutrinos at INO-ICAL

(India-based Neutrino Observatory)

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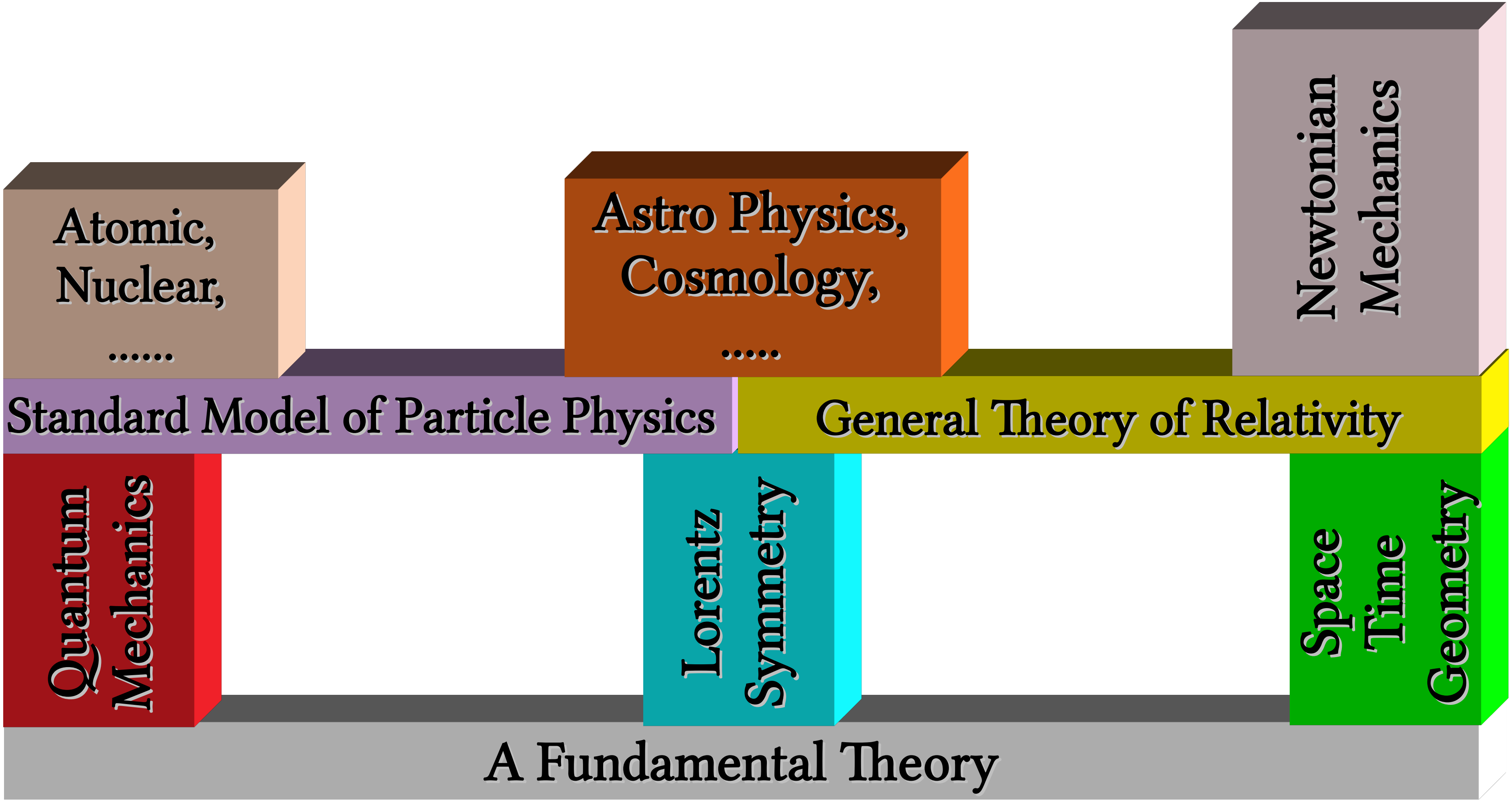
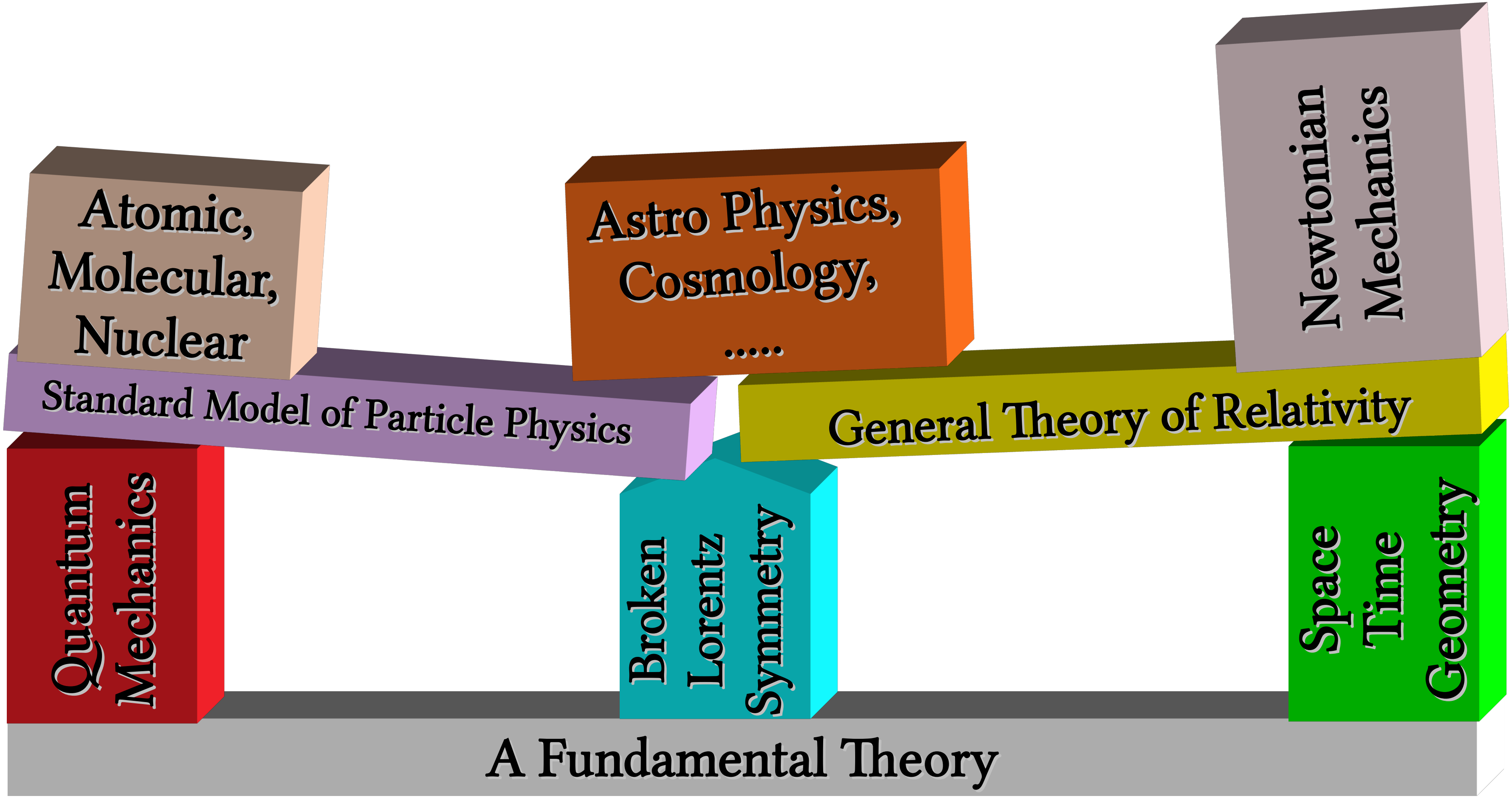
: Plan of Presentation :

- Motivation to Lorentz Invariant Violation
- Brief Discussion on LIV studies using Atmospheric Neutrinos
- Exploring CPT-Violating LIV parameters at INO-ICAL
- Effects of non-zero CPT-Violating LIV parameters on M.O. determination
- Summary & Remark

Motivation to Lorentz Invariant Violation



Original: M. Mathew



Original: M. Mathew

Lorentz Symmetry Breaking
&
Standard Model Extension

Spontaneous Lorentz Symmetry Breaking

$$\Rightarrow \mathcal{L} \equiv \mathcal{L}_0 - \mathcal{L}'$$

Spontaneous Lorentz Symmetry Breaking

$$\Rightarrow \mathcal{L} \equiv \boxed{\mathcal{L}_0} - \boxed{\mathcal{L}'}$$

Weak Interaction in SM

New Interaction induced due to LIV

Spontaneous Lorentz Symmetry Breaking

$$\Rightarrow \mathcal{L}' \supseteq \frac{\lambda}{(M_p)^k} \langle T \rangle \, \overline{\psi} \, \Gamma \, (i\partial)^k \, \psi + h.c., \qquad k = 0,\, 1$$

CPT violation and the standard model
Don Colladay and V. Alan Kostelecký
Phys. Rev. D 55, 6760 – Published 1 June 1997

Spontaneous Lorentz Symmetry Breaking

Dimension-less Quantity

$$\Rightarrow \mathcal{L}' \supseteq \frac{\lambda}{(M_p)^k} \langle T \rangle \overline{\psi}\, \Gamma\, (i\partial)^k \psi + h.c., \qquad k = 0,\, 1$$

mass dimension

possible Lorentz bases

Planck Mass ~ 10^{19} GeV

neutrino field

Non-zero Tensor VEVs

Spontaneous Lorentz Symmetry Breaking

$$\Rightarrow \mathcal{L} \equiv \mathcal{L}_0 - \mathcal{L}'$$

$$\Rightarrow \mathcal{L}' \supseteq \frac{\lambda}{(M_p)^k} \langle T \rangle \, \overline{\psi} \, \Gamma \, (i\partial)^k \, \psi + h.c., \qquad k = 0, 1$$

$$\mathrm{k} = 0, \ \langle T \rangle \sim \left(\frac{m^2}{M_p} \right); \ (\,\text{leads to CPT} - \text{violating LIV})$$

$$\mathrm{k} = 1, \ \langle T \rangle \sim m; \qquad (\,\text{leads to CPT} - \text{conserving LIV})$$

$$\mathcal{L}' = \frac{1}{2} \left[a_\mu \overline{\psi} \gamma^\mu \psi + b_\mu \overline{\psi} \gamma_5 \gamma^\mu \psi - i c_{\mu\nu} \overline{\psi} \gamma^\mu \partial^\nu \psi - i d_{\mu\nu} \overline{\psi} \gamma_5 \gamma^\mu \partial^\nu \psi \right] + h.c.$$

Spontaneous Lorentz Symmetry Breaking

$$\mathcal{L}' = \frac{1}{2}\left[a_\mu \overline{\psi}\gamma^\mu\psi + b_\mu\overline{\psi}\gamma_5\gamma^\mu\psi - ic_{\mu\nu}\overline{\psi}\gamma^\mu\partial^\nu\psi - id_{\mu\nu}\overline{\psi}\gamma_5\gamma^\mu\partial^\nu\psi\right] + h.c.$$

Can't Measure Individually

Can't Measure Individually

$$\mathrm{a}_L = a + b$$

$$\mathrm{a}_R = a - b$$

Interference Effect

$$\mathrm{c}_L = c + d$$

$$\mathrm{c}_R = c - d$$

$$\mathrm{a}_R = -\mathrm{a}_L$$

$$c_R = \mathrm{c}_L$$

Hamiltonian in Standard Model Extension

$$H_{ij} = E\delta_{ij} + \frac{m^2_{ij}}{2E} + \frac{1}{E}\left(\mathrm{a}^{\mu}_{L} p_{\mu} - c^{\mu\nu}_{L} p_{\mu} p_{\nu}\right)_{ij}$$

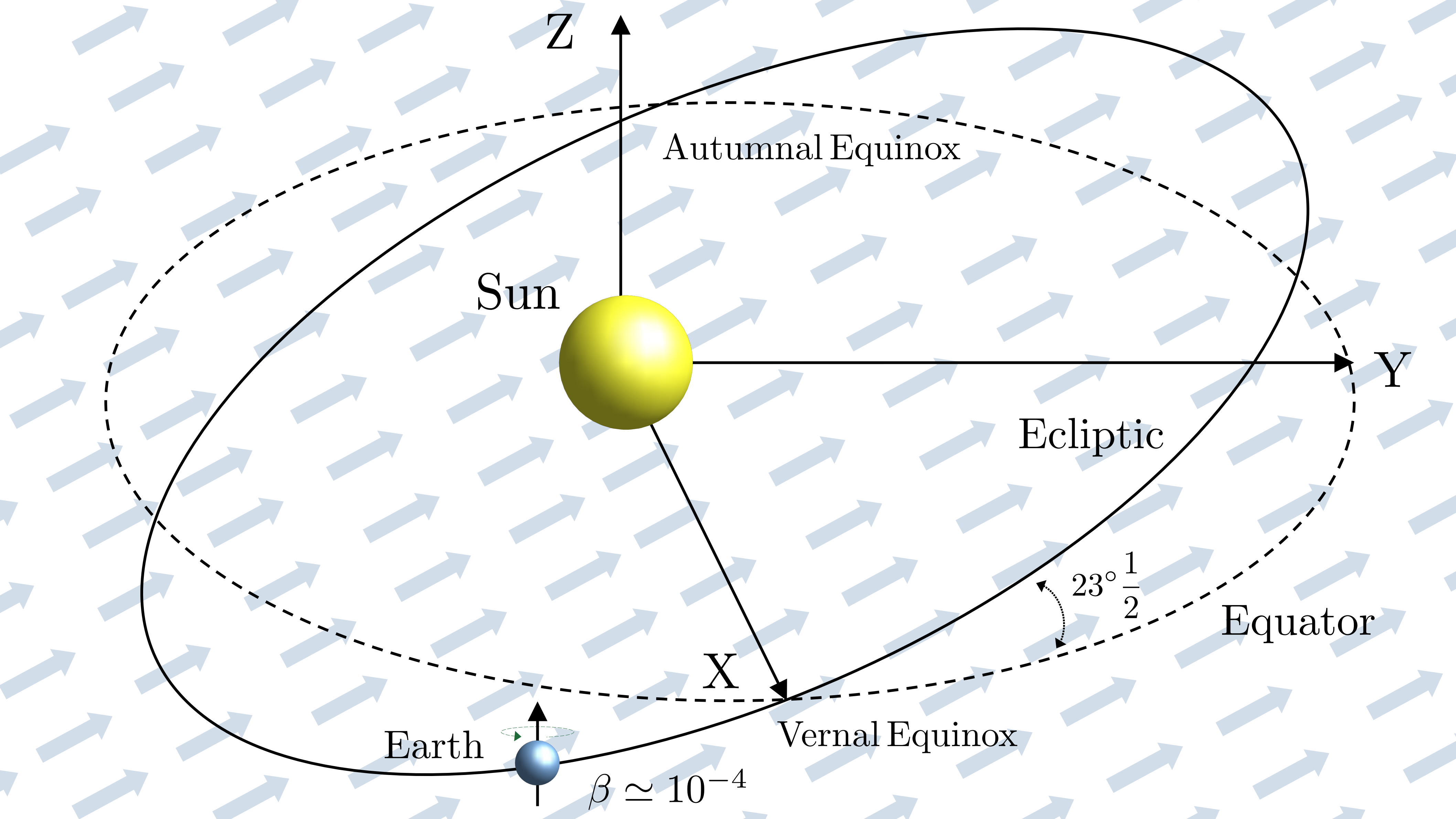
$p \equiv (E, -E\hat{p})$

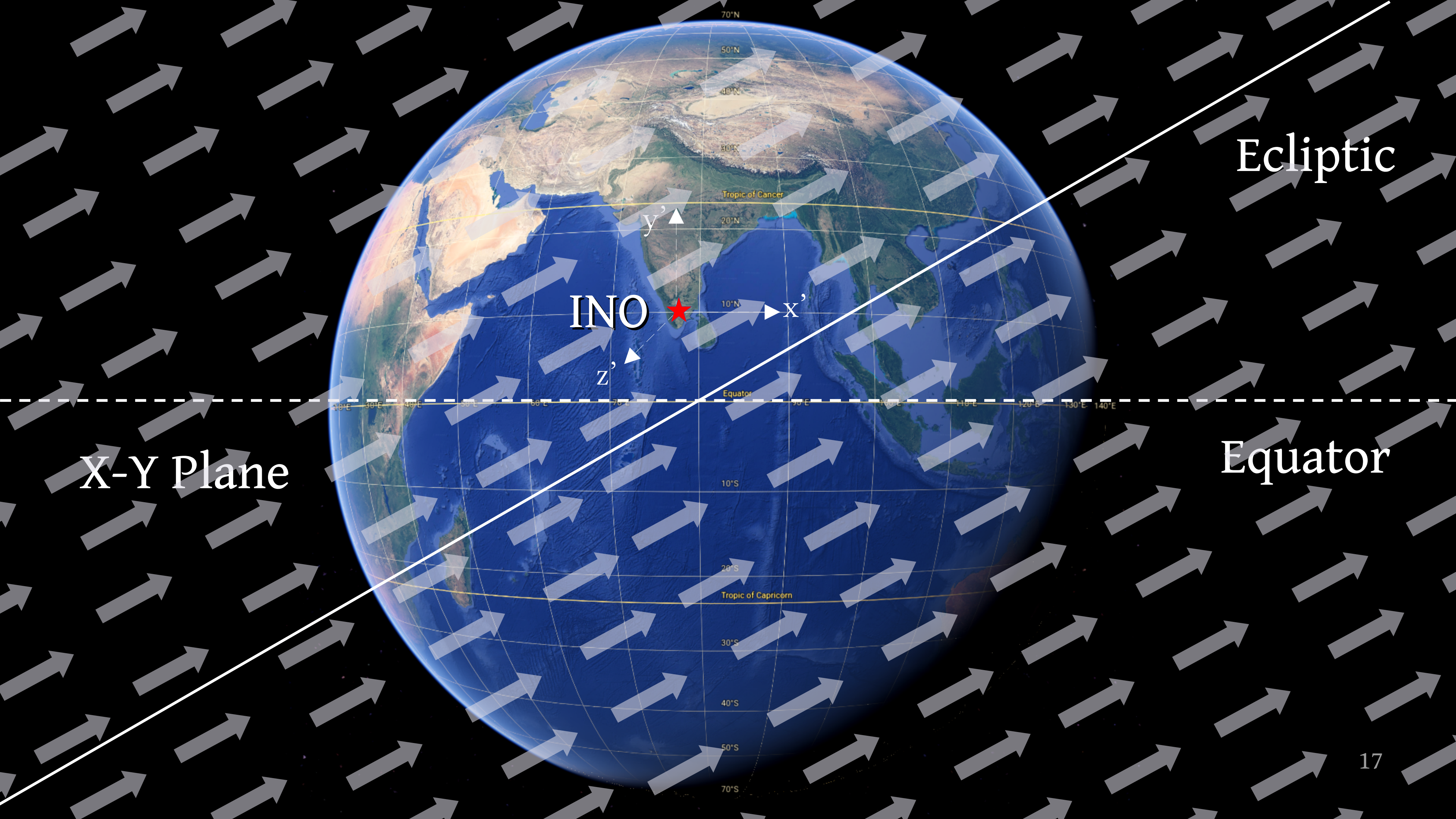
$i, j \rightarrow$ flavour indices

$\mu, \tau \rightarrow$ space time indices

$m^2_{ij} \rightarrow$ mass squared splitting in flavour indices

Choice of Inertial Frame of Reference


Z
Autumnal Equinox
Sun
Y
Ecliptic
$23^\circ\frac{1}{2}$
Equator
X
Vernal Equinox
Earth
$\beta \simeq 10^{-4}$

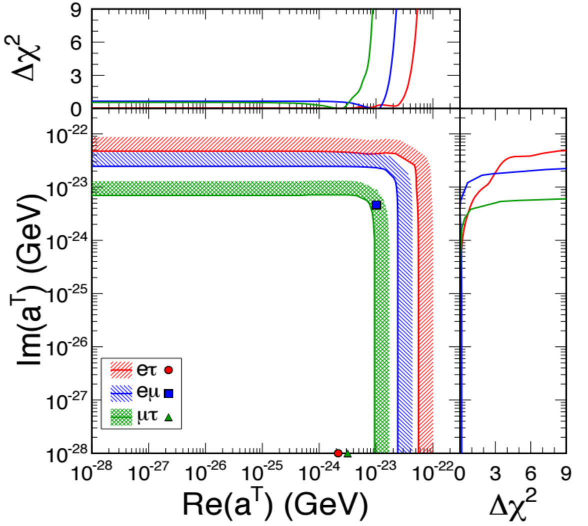

Ecliptic
INO
y'
x'
z'
X-Y Plane
Equator

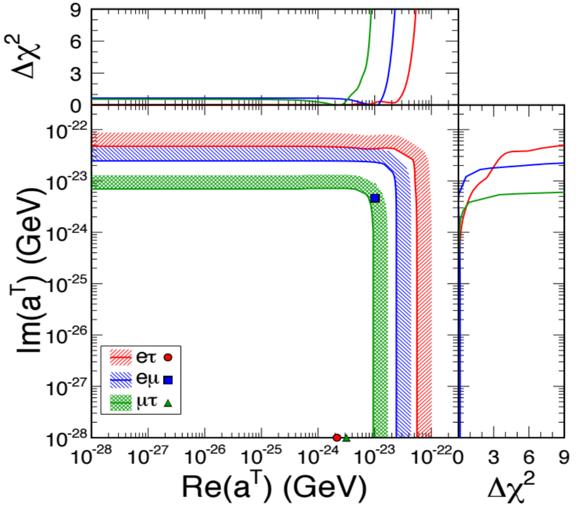
Brief Discussion on LIV studies using Atmospheric Neutrinos

Results From Super-K with Atmospheric Neutrino

$$H_{LIV} = \pm \begin{bmatrix} 0 & a_{e\mu} & a_{e\tau} \\ a^*_{e\mu} & 0 & a_{\mu\tau} \\ a^*_{e\tau} & a^*_{\mu\tau} & 0 \end{bmatrix} - \frac{4}{3}E \begin{bmatrix} 0 & c_{e\mu} & c_{e\tau} \\ c^*_{e\mu} & 0 & c_{\mu\tau} \\ c^*_{e\tau} & c^*_{\mu\tau} & 0 \end{bmatrix}$$

LV Parameter		Limit at 95% C.L.	Best Fit	No LV $\Delta\chi^2$	Previous Limit	
$e\mu$	$\mathrm{Re}\left(a^T\right)$	1.8×10^{-23} GeV	1.0×10^{-23} GeV	1.4	4.2×10^{-20} GeV	[58]
	$\mathrm{Im}\left(a^T\right)$	1.8×10^{-23} GeV	4.6×10^{-24} GeV			
	$\mathrm{Re}\left(c^{TT}\right)$	8.0×10^{-27}	1.0×10^{-28}	0.0	9.6×10^{-20}	[58]
	$\mathrm{Im}\left(c^{TT}\right)$	8.0×10^{-27}	1.0×10^{-28}			
$e\tau$	$\mathrm{Re}\left(a^T\right)$	4.1×10^{-23} GeV	2.2×10^{-24} GeV	0.0	7.8×10^{-20} GeV	[59]
	$\mathrm{Im}\left(a^T\right)$	2.8×10^{-23} GeV	1.0×10^{-28} GeV			
	$\mathrm{Re}\left(c^{TT}\right)$	9.3×10^{-25}	1.0×10^{-28}	0.3	1.3×10^{-17}	[59]
	$\mathrm{Im}\left(c^{TT}\right)$	1.0×10^{-24}	3.5×10^{-25}			
$\mu\tau$	$\mathrm{Re}\left(a^T\right)$	6.5×10^{-24} GeV	3.2×10^{-24} GeV	0.9	–	
	$\mathrm{Im}\left(a^T\right)$	5.1×10^{-24} GeV	1.0×10^{-28} GeV			
	$\mathrm{Re}\left(c^{TT}\right)$	4.4×10^{-27}	1.0×10^{-28}	0.1	–	
	$\mathrm{Im}\left(c^{TT}\right)$	4.2×10^{-27}	7.5×10^{-28}			





[Phys. Rev. D 91, 052003]

Test of LIV with Atmospheric Neutrino @ IceCube

nature physics

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Neutrino interferometry for high-precision tests of Lorentz symmetry with IceCube

The IceCube Collaboration*

Lorentz symmetry is a fundamental spacetime symmetry underlying both the standard model of particle physics and general relativity. This symmetry guarantees that physical phenomena are observed to be the same by all inertial observers. However, unified theories, such as string theory, allow for violation of this symmetry by inducing new spacetime structure at the quantum gravity scale. Thus, the discovery of Lorentz symmetry violation could be the first hint of these theories in nature. Here we report the results of the most precise test of spacetime symmetry in the neutrino sector to date. We use high-energy atmospheric neutrinos observed at the IceCube Neutrino Observatory to search for anomalous neutrino oscillations as signals of Lorentz violation. We find no evidence for such phenomena. This allows us to constrain the size of the dimension-four operator in the standard-model extension for Lorentz violation to the 10^{-28} level and to set limits on higher-dimensional operators in this framework. These are among the most stringent limits on Lorentz violation set by any physical experiment.

Very small violations of Lorentz symmetry, or Lorentz violation (LV), are allowed in many ultrahigh-energy theories, including string theory[1], non-commutative field theory[2] and supersymmetry[3]. The discovery of LV could be the first indication of such new physics. Worldwide efforts are therefore underway to search for evidence of LV. The standard-model extension (SME) is an effective-field-theory framework to systematically study LV[4]. The SME includes all possible types of LV that respect other symmetries of the standard model such as energy–momentum conservation and coordinate independence. Thus, the SME can provide a framework to compare results of LV searches from many different fields such as photons[5–8], nucleons[9–11], charged leptons[12–14] and gravity[15]. Recently, neutrino experiments have performed searches for LV[16–18]. So far, all searches have obtained null results. The full list of existing limits from all sectors and a brief overview of the field are available elsewhere[19,20]. Our focus here is to present the most precise test of LV in the neutrino sector.

The fact that neutrinos have mass has been established by a series of experiments[21–26]. The field has incorporated these results into the neutrino standard model (νSM)—the standard model with three massive neutrinos. Although the νSM parameters are not yet fully determined[27], the model is rigorous enough to be brought to bear on the question of LV. In the Methods, we briefly review the history of neutrino oscillation physics and tests of LV with neutrinos.

To date, neutrino masses have proved to be too small to be measured kinematically, but the mass differences are known via neutrino oscillations. This phenomenon arises from the fact that production and detection of neutrinos involves the flavour states, while the propagation is given by the Hamiltonian eigenstates. Thus, a neutrino with flavour $|\nu_\alpha\rangle$ can be written as a superposition of Hamiltonian eigenstates $|\nu_i\rangle$; that is, $|\nu_\alpha\rangle = \sum_{i=1}^{3} V_{\alpha i}(E)|\nu_i\rangle$, where V is the unitary matrix that diagonalizes the Hamiltonian and, in general, is a function of neutrino energy E. When the neutrino travels in vacuum without new physics, the Hamiltonian depends only on the neutrino masses, and the Hamiltonian eigenstates coincide with the mass eigenstates. That is, $H=\frac{1}{2E}U^\dagger \mathrm{diag}(m_1^2,m_2^2,m_3^2)U=\frac{m^2}{2E}$, where m_i are the neutrino masses and U is the Pontecorvo–Maki–Nakagawa–Sakata matrix that diagonalizes the mass matrix m (ref. [27]).

A consequence of the flavour misalignment is that a neutrino beam that is produced purely of one flavour will evolve to produce other flavours. Experiments measure the number of neutrinos of different flavours, observed as a function of the reconstructed energy of the neutrino, E, and the distance the beam has travelled, L. The microscopic neutrino masses are directly tied to the macroscopic neutrino oscillation length. In this sense, neutrino oscillations are similar to photon interference experiments in their ability to probe very small scales in nature.

Lorentz-violating neutrino oscillations

Here, we use neutrino oscillations as a natural interferometer with a size equal to the diameter of Earth. We look for anomalous flavour-changing effects caused by LV that would modify the observed energy and zenith angle distributions of atmospheric muon neutrinos observed in the IceCube Neutrino Observatory[28] (see Fig. 1). Beyond flavour change due to small neutrino masses, any hypothetical LV fields could contribute to muon neutrino flavour conversion. We therefore look for distortion of the expected muon neutrino distribution. As this analysis does not distinguish between a muon neutrino (ν_μ) and its antineutrino ($\overline{\nu}_\mu$), when the word 'neutrino' is used, we are referring to both.

Past searches for LV have mainly focused on the directional effect in the Sun-centred celestial-equatorial frame[19] by looking only at the time dependence of physics observables as direction-dependent physics appears as a function of Earth's rotation. However, in our case, we assume no time dependence, and instead look at the energy distribution distortions caused by direction- an time-independent isotropic LV. Isotropic LV may be a factor ~10 larger than direction-dependent LV in the Sun-centred celestial equatorial frame if we assume that the new physics is isotropic i the cosmic microwave background frame[20]. It would be most opti mal to simultaneously look for both effects, but our limited statis tics do not allow for this.

*A full list of authors and affiliations appears in the online version of this paper.

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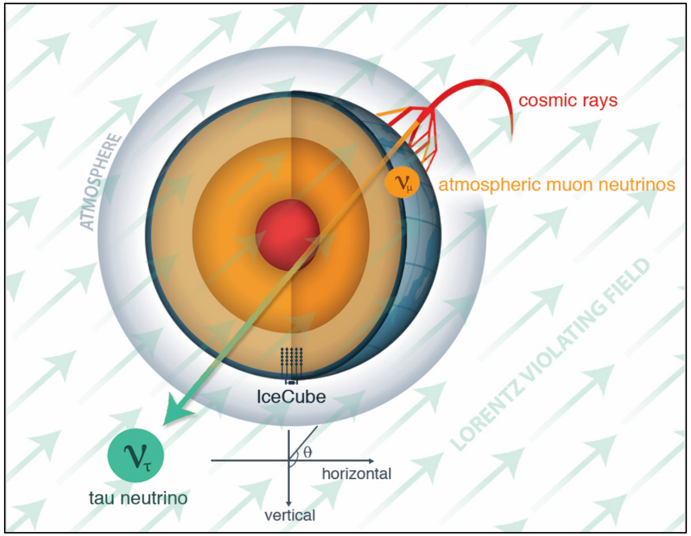
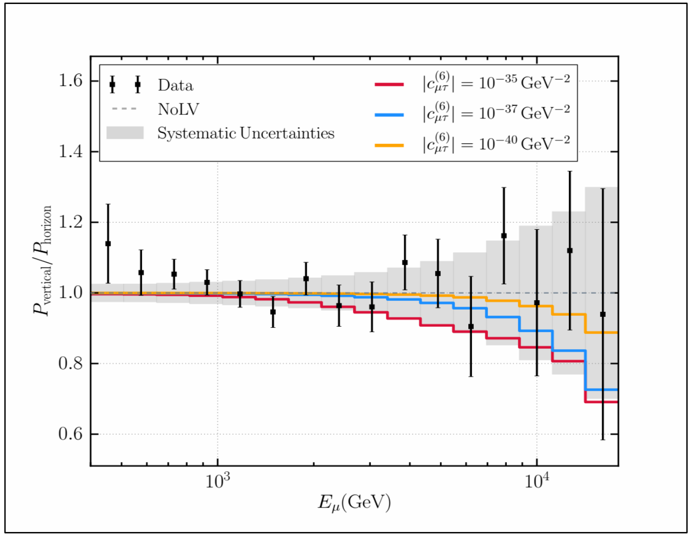




$$\mathrm{H} \sim \frac{m^2}{2E} + \mathring{a}^{(3)} - E \cdot \mathring{c}^{(4)} + E^2 \cdot \mathring{a}^{(5)} - E^3 \cdot \mathring{c}^{(6)} \cdots$$

$$|\mathrm{Re}\,(\mathring{a}_{\mu\tau}^{(3)})|, |\mathrm{Im}\,(\mathring{a}_{\mu\tau}^{(3)})| \begin{array}{l} < 2.9 \times 10^{-24}\ \mathrm{GeV}\ (99\%\ \mathrm{C.L.}) \\ < 2.0 \times 10^{-24}\ \mathrm{GeV}\ (90\%\ \mathrm{C.L.}) \end{array}$$

[Nature Physics vol 14, p961–966(2018)]

Exploring LIV at INO-ICAL

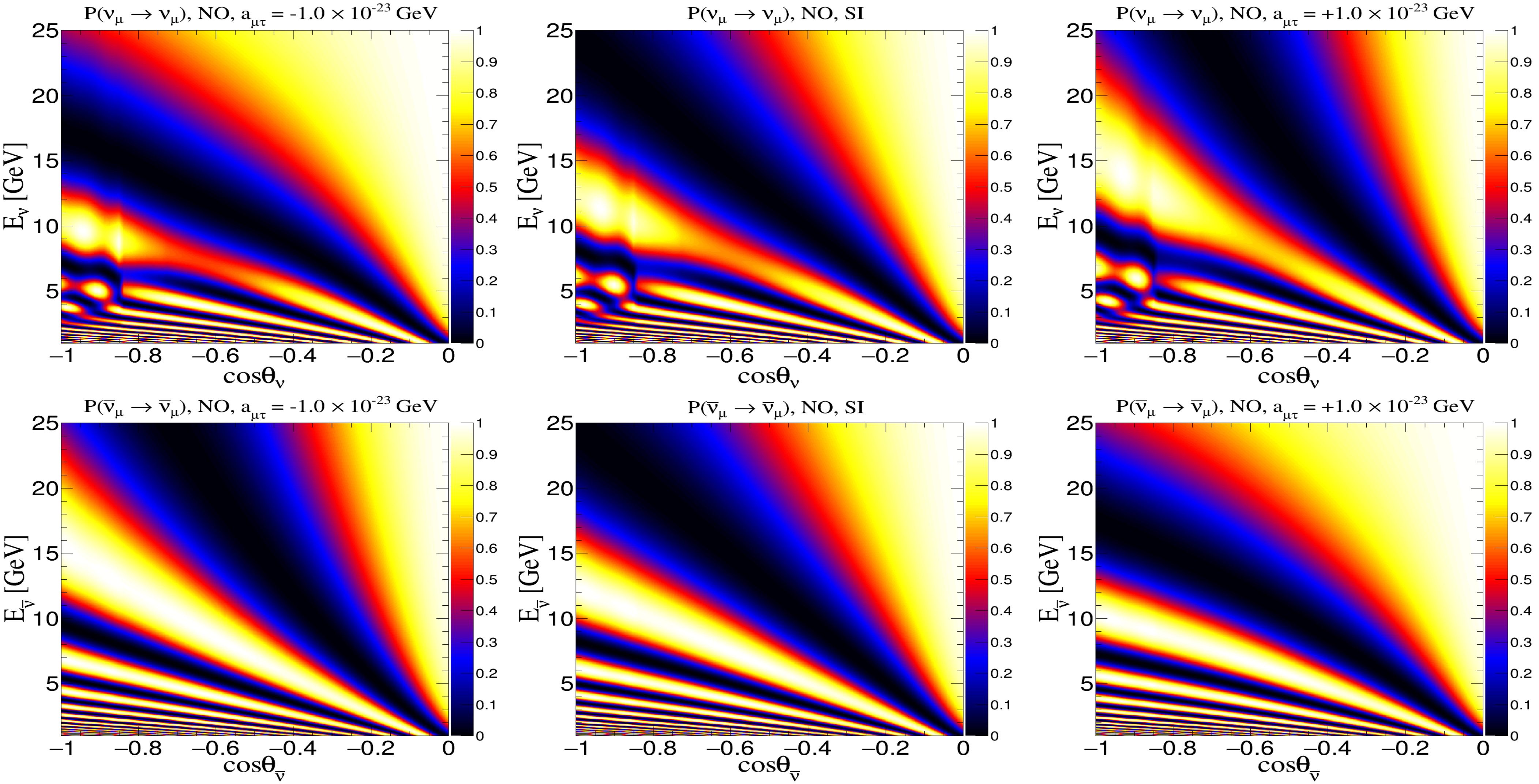
$$H = \frac{1}{2E} U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger \pm \begin{bmatrix} \mathrm{a}_{ee} & \mathrm{a}_{e\mu} & \mathrm{a}_{e\tau} \\ \mathrm{a}^*_{e\mu} & \mathrm{a}_{\mu\mu} & \mathrm{a}_{\mu\tau} \\ \mathrm{a}^*_{e\tau} & \mathrm{a}^*_{\mu\tau} & \mathrm{a}_{\tau\tau} \end{bmatrix}_{00} \pm \sqrt{2} G_F \begin{bmatrix} N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- CPT-Violating parameter "a" with isotropic components
- "+" sign is assigned for neutrino and "-" sign for antineutrino
- $\sqrt{2} G_F N_e$ is standard matter interaction potential of neutrino and antineutrino

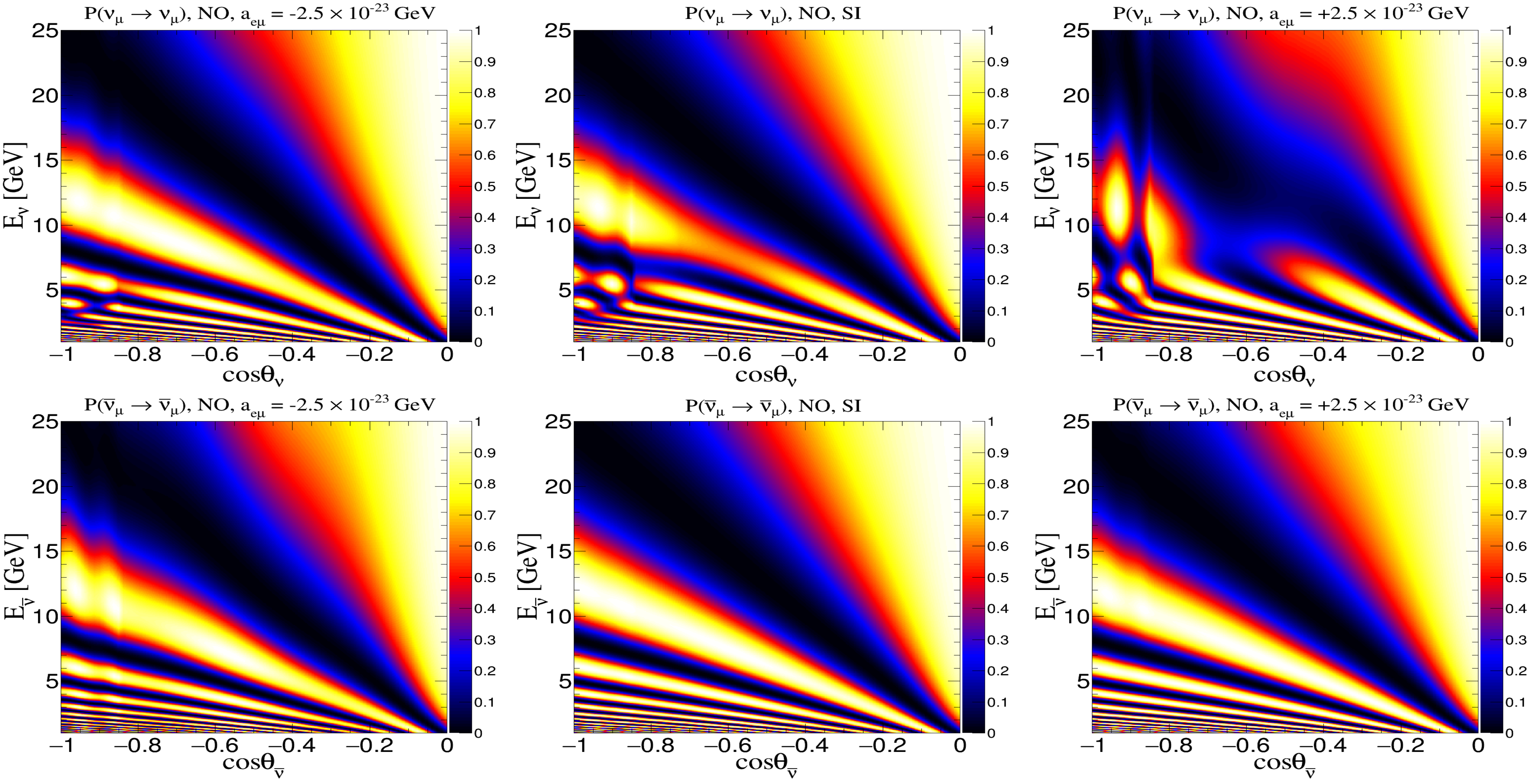
Benchmark Oscillation Parameters

$\sin^2 2\theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 2\theta_{13}$	$\Delta m^2_{\rm eff}$ (eV^2)	Δm^2_{21} (eV^2)	$\delta_{\rm CP}$	Mass Ordering
0.855	0.5	0.0875	2.49×10^{-3}	7.4×10^{-5}	0	Normal (NO)

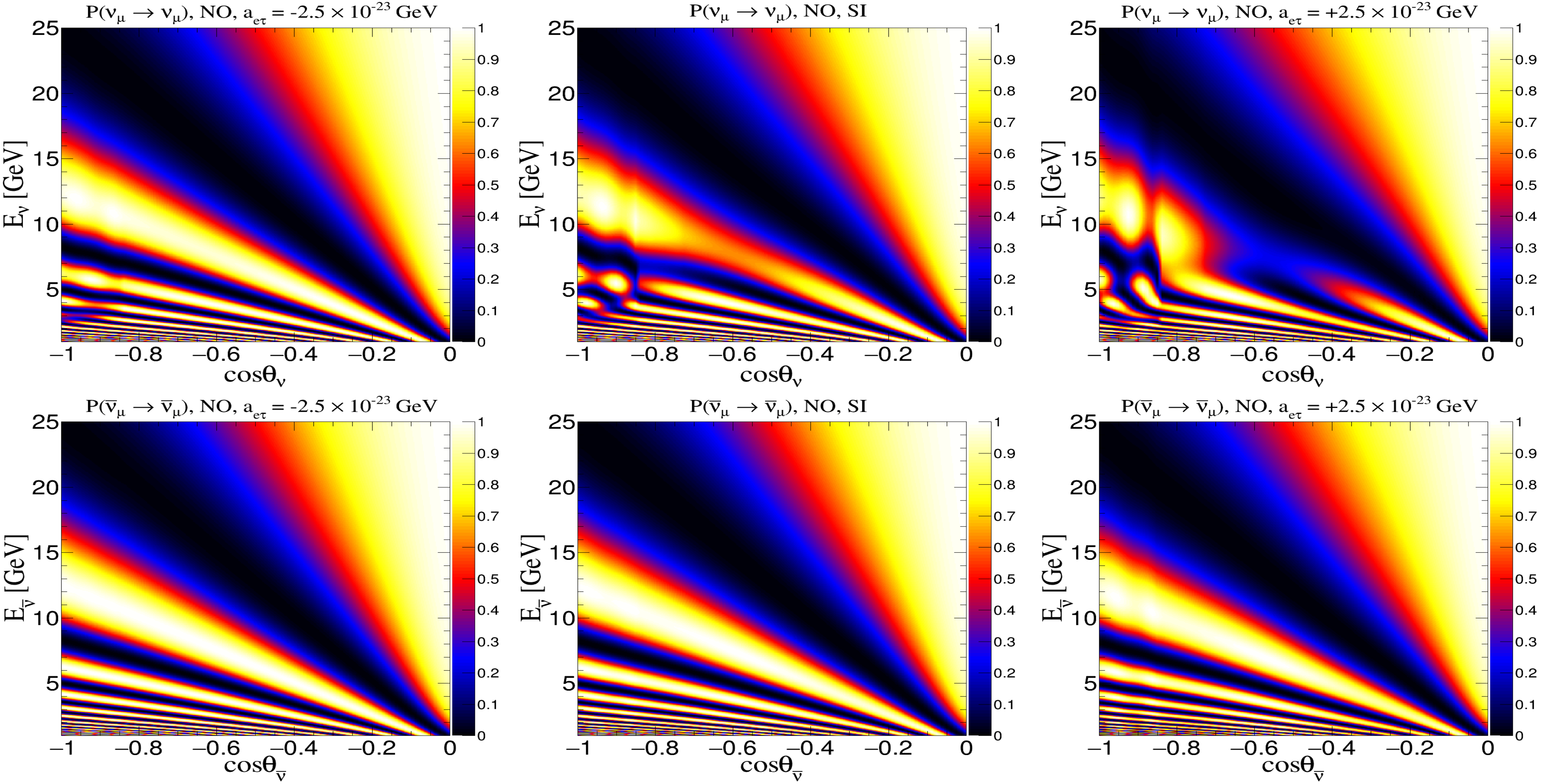
Effect of $a_{\mu\tau} = \pm 1.0 \times 10^{-23}$ GeV on Muon Survival Channel



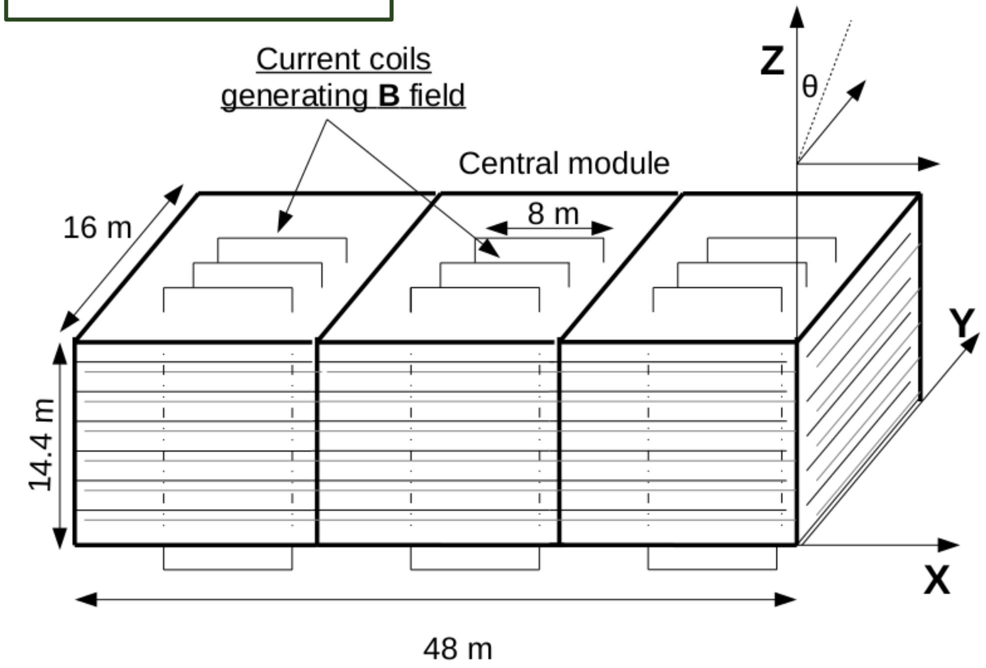
Effect of $a_{e\mu} = \pm 2.5 \times 10^{-23}$ GeV on Muon Survival Channel

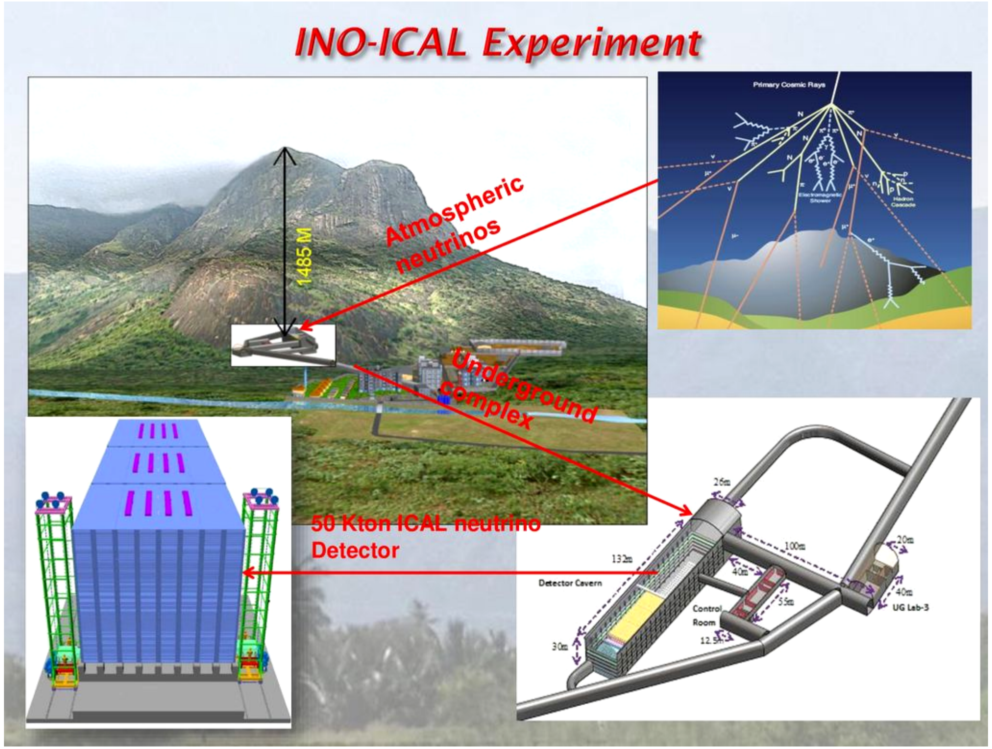


Effect of $a_{e\tau} = \pm 2.5 \times 10^{-23}$ GeV on Muon Survival Channel



INO-ICAL :





- 50 kt Magnetized Iron Calorimeter (ICAL) of Field strength ~ 1.3 Tesla, enables to distinguish atmospheric neutrino and antineutrino events, separately.
- It has ~10% resolution of muon momentum ranging 1-25 GeV and ~1° zenith angle resolution over 15-12800 km range of baselines

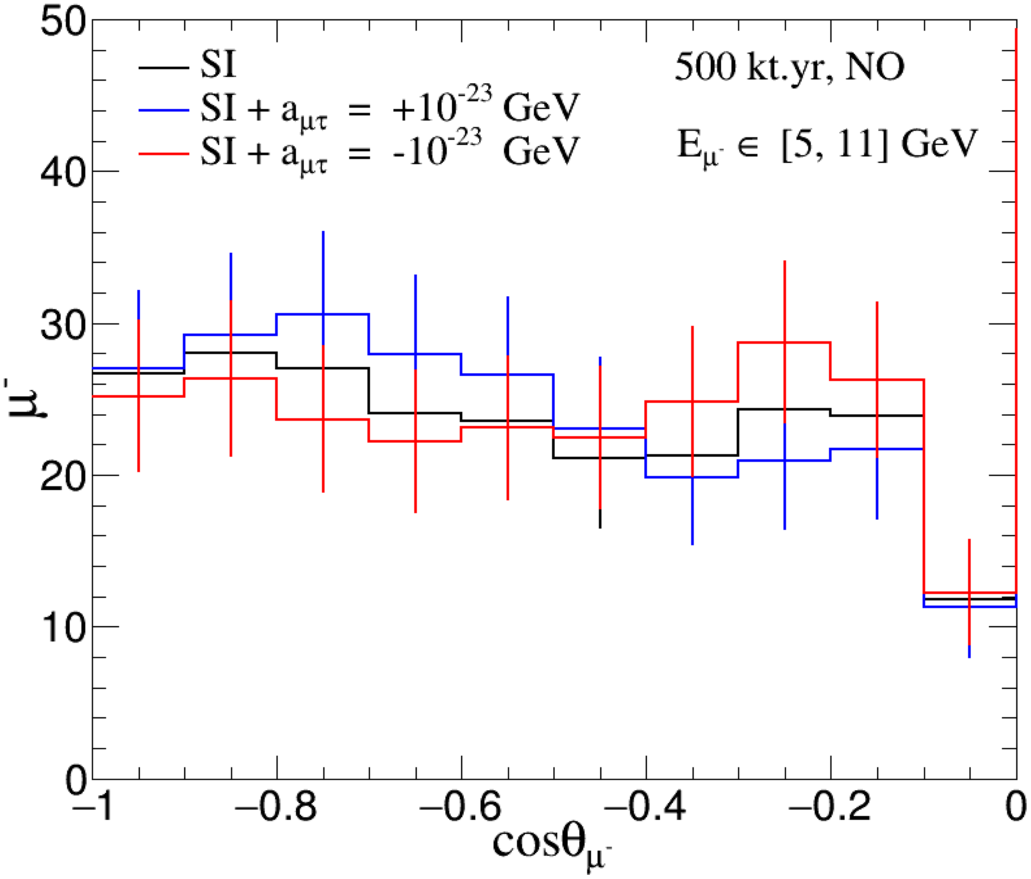
INO-ICAL :

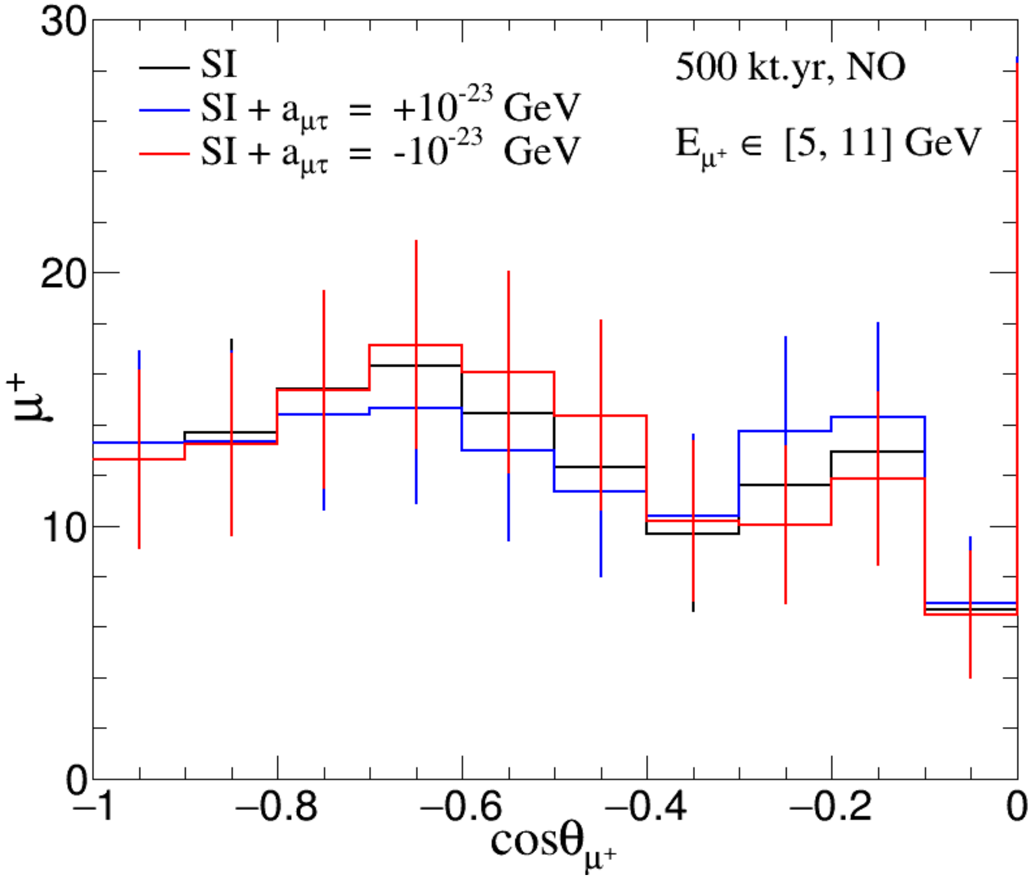
- NUANCE MC Generator using Neutrino Flux (Honda) at INO site
- Three-Flavour Oscillation Framework; PREM profile; 500 kt·yr (10 yr)
- Migration matrices from ICAL-Geant4 simulation [arXiv:1304.5115, 1405.7243]

Observable	Range	Bin width	Total bins	
$E_\mu^{\rm rec}$ (GeV)	$[1, 11]$	1	10	13
	$[11, 21]$	5	2	
	$[21, 25]$	4	1	
$\cos\theta_\mu^{\rm rec}$	$[-1.0, 0.0]$	0.1	10	15
	$[0.0, 1.0]$	0.2	5	
$E'^{\rm rec}_{\rm had}$ (GeV)	$[0, 2]$	1	2	4
	$[2, 4]$	2	1	
	$[4, 25]$	21	1	

Impact of non-zero $a_{\mu\tau}$ (1 d.o.f) on Events

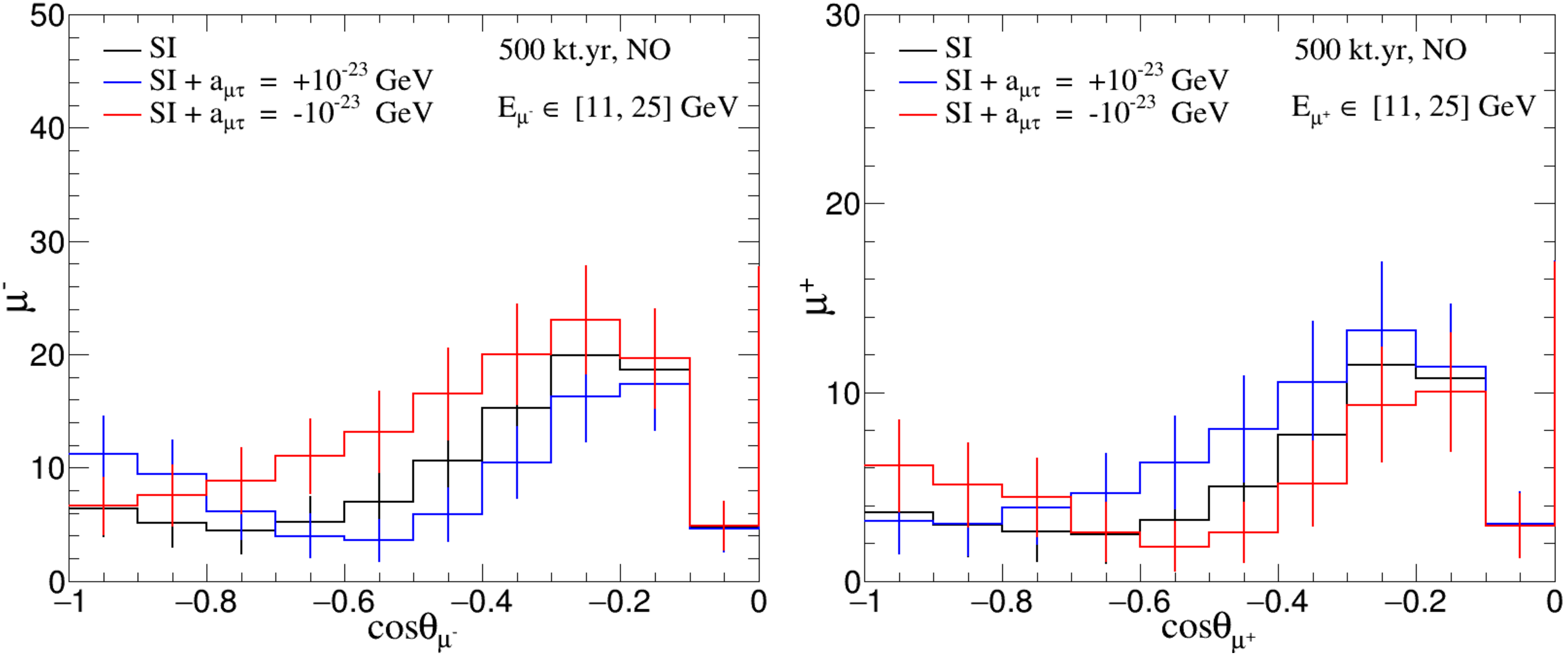
Impact of non-zero ($a_{\mu\tau}$) on Event Distribution :





ICAL unique capability of Charged Identification (CID) helps to probe the properties of $a_{\mu\tau}$ via observing μ^- and μ^+ events separately

Impact of non-zero ($a_{\mu\tau}$) on Event Distribution :



ICAL unique capability of Charged Identification (CID) helps to probe the properties of $a_{\mu\tau}$ via observing μ^- and μ^+ events separately

Method of χ^2 Analysis:

$$\chi^2_- = \min_{\zeta_l} \sum_{i=1}^{N_{E_{\text{had}}}} \sum_{j=1}^{N_{E_{\mu^-}}} \sum_{k=1}^{N_{\cos\theta_\mu}} 2\left[N_{ijk}^{\text{theory}} - N_{ijk}^{\text{data}} - N_{ijk}^{\text{data}} \ln\left(\frac{N_{ijk}^{\text{theory}}}{N_{ijk}^{\text{data}}}\right)\right] + \sum_{l=1}^{5} \zeta_l^2$$

$$\chi^2_+ = \min_{\zeta_l} \sum_{i=1}^{N_{E_{\text{had}}}} \sum_{j=1}^{N_{E_{\mu^+}}} \sum_{k=1}^{N_{\cos\theta_\mu}} 2\left[N_{ijk}^{\text{theory}} - N_{ijk}^{\text{data}} - N_{ijk}^{\text{data}} \ln\left(\frac{N_{ijk}^{\text{theory}}}{N_{ijk}^{\text{data}}}\right)\right] + \sum_{l=1}^{5} \zeta_l^2$$

$$N_{ijk}^{\text{theory}} = N_{ijk}^{0}\left(1 + \sum_{l=1}^{5} \pi_{ijk}^{l} \zeta_l\right);$$

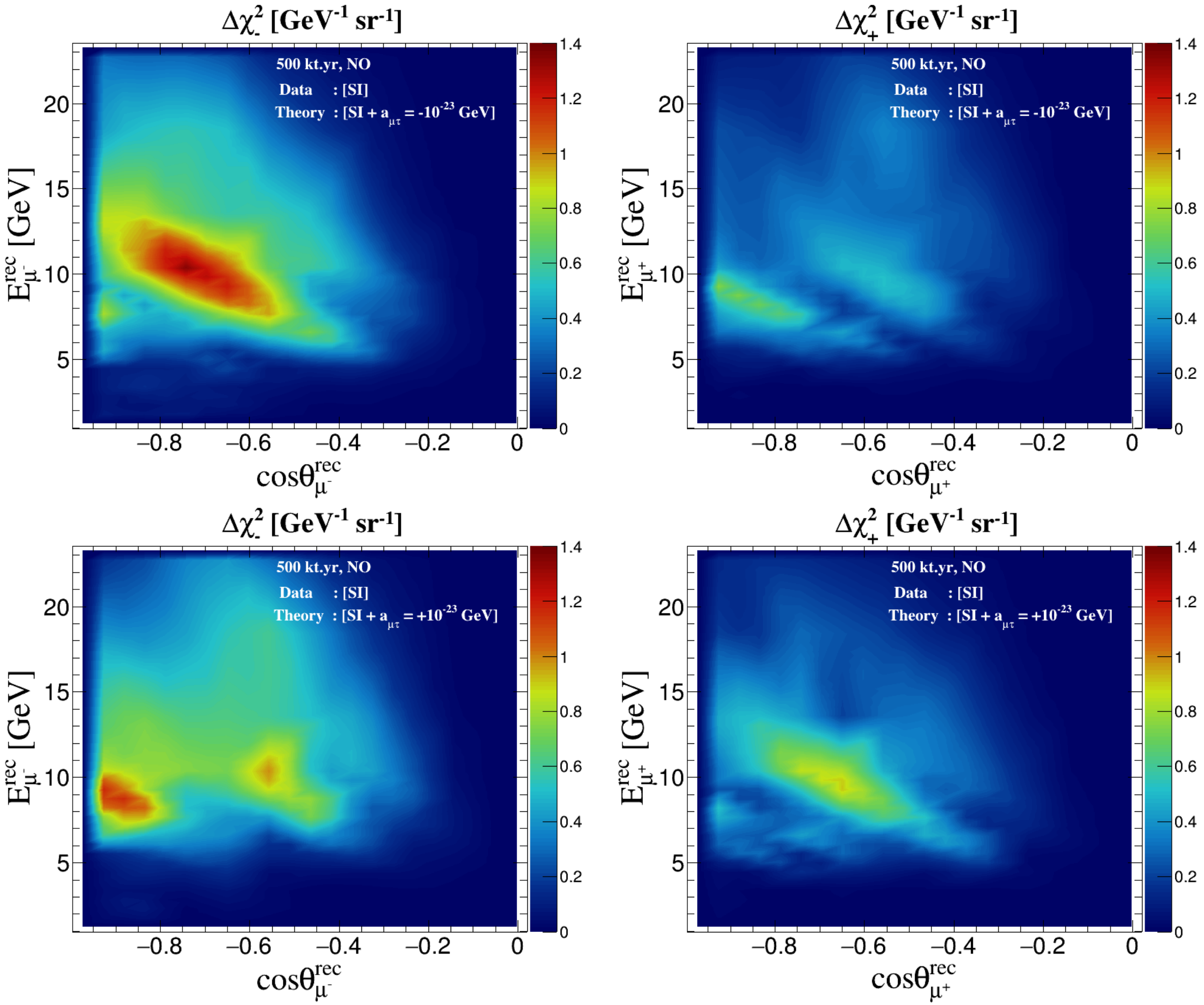
$$\chi^2 = \chi^2_- + \chi^2_+$$

$$\Delta\chi^2 = \chi^2_{\text{std+liv}} - \chi^2_{\text{std}}$$

- Flux Normalization Error = 20%
- Interaction Cross-section Error = 10%
- Tilt Error = 5%
- Zenith Error = 5%
- Overall Systematic Error = 5%

arXiv:1406.3689v1 (Moon Moon Devi, Tarak Thakore, Sanjib Kumar Agarwalla, Amol Dighe)

$\Delta\chi^2$ [GeV^{-1} sr^{-1}] distribution :



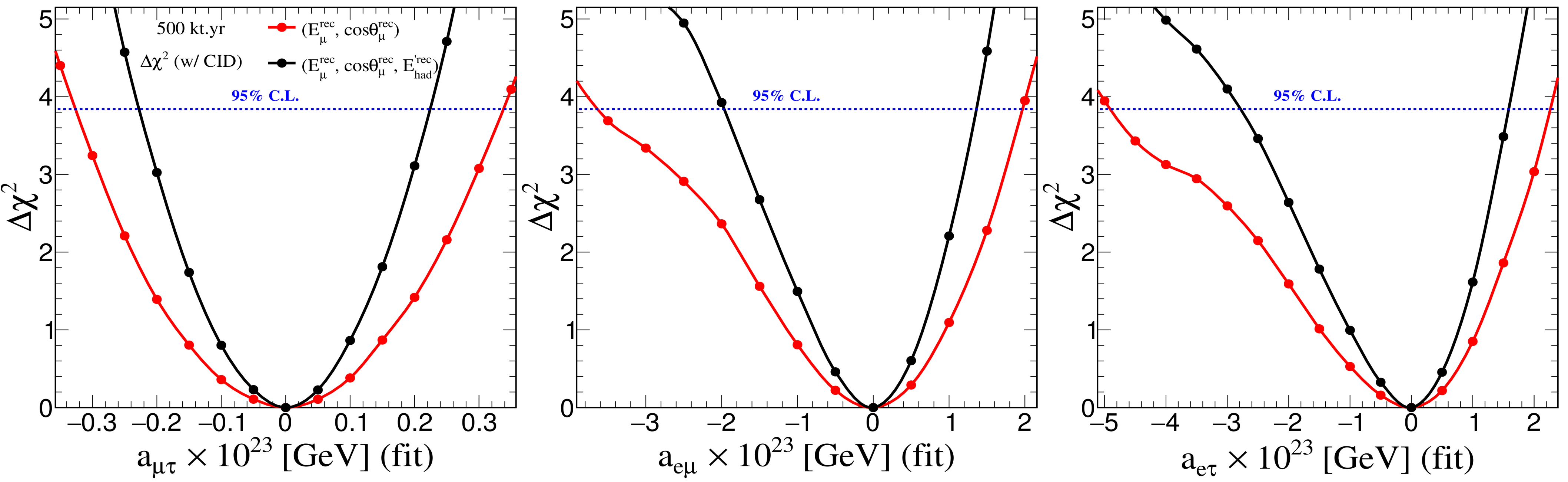
- Fixed-Parameter case
- Data : [SI]
- Theory : [SI + $a_{\mu\tau}$ = $\pm$ 1.0 x 10^{-23} GeV]
- Without pull penalty term (ζ^2)

Observation

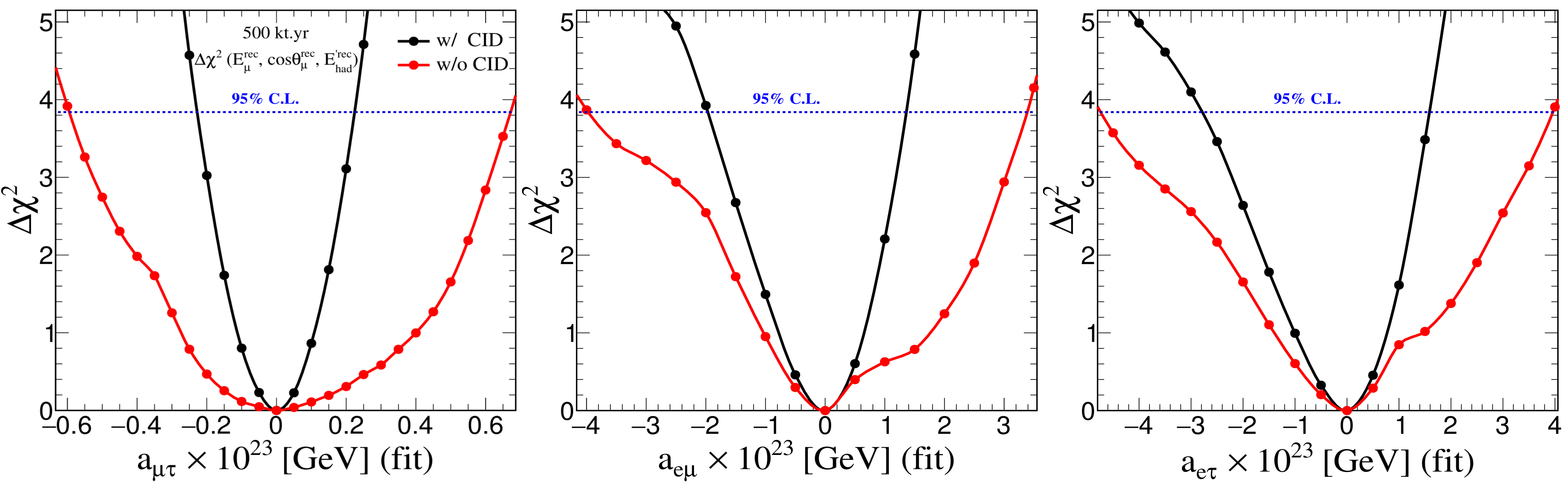
- $\Delta\chi^2$ contribution from μ^- is more than the μ^+
- $\Delta\chi^2$ for ($- a_{\mu\tau}$) is greater than ($+a_{\mu\tau}$)

Constraining CPT-Violating LIV parameters (one-at-a-time)

Improvement of Bounds of CPT-Violating par. (1dof) due to Hadron Information:



Improvement of Bounds of CPT-Violating par. (1dof) due to CID :



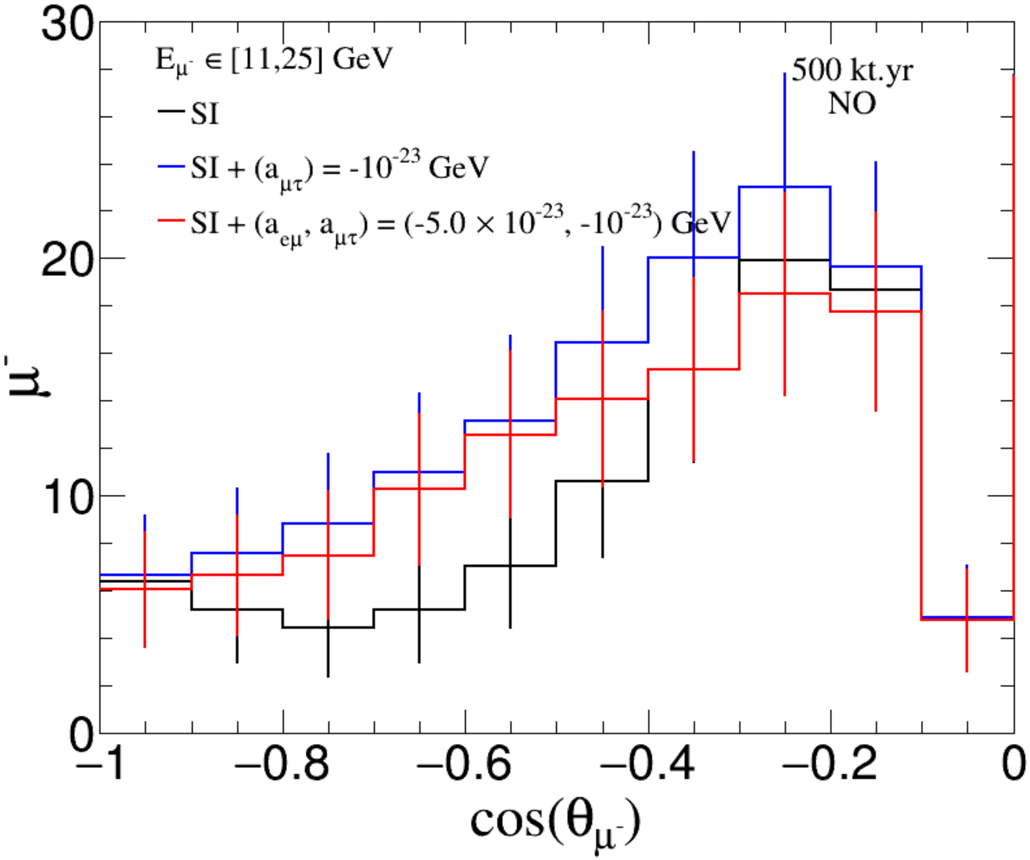
Constraints on CPT-violating LIV parameters				
Experiments		$a_{\mu\tau}[10^{-23}$ GeV]	$a_{e\mu}[10^{-23}$ GeV]	$a_{e\tau}[10^{-23}$ GeV]
IceCube (99% C.L.)		$\lvert\mathrm{Re}(a_{\mu\tau})\rvert < 0.29$ $\lvert\mathrm{Im}(a_{\mu\tau})\rvert < 0.29$	–	–
Super-K (95% C.L.)		$\mathrm{Re}(a_{\mu\tau}) < 0.65$ $\mathrm{Im}(a_{\mu\tau}) < 0.51$	$\mathrm{Re}(a_{e\mu}) < 1.8$ $\mathrm{Im}(a_{e\mu}) < 1.8$	$\mathrm{Re}(a_{e\tau}) < 4.1$ $\mathrm{Im}(a_{e\tau}) < 2.8$
ICAL (95% C.L.)	w/o CID	$-0.59 \leq \mathrm{Re}(a_{\mu\tau}) \leq 0.67$	$-3.97 \leq \mathrm{Re}(a_{\mu\tau}) \leq 3.37$	$-4.71 \leq \mathrm{Re}(a_{\mu\tau}) \leq 3.96$
	w/ CID	$-0.23 \leq \mathrm{Re}(a_{\mu\tau}) \leq 0.22$	$-1.97 \leq \mathrm{Re}(a_{\mu\tau}) \leq 1.34$	$-2.80 \leq \mathrm{Re}(a_{\mu\tau}) \leq 1.58$

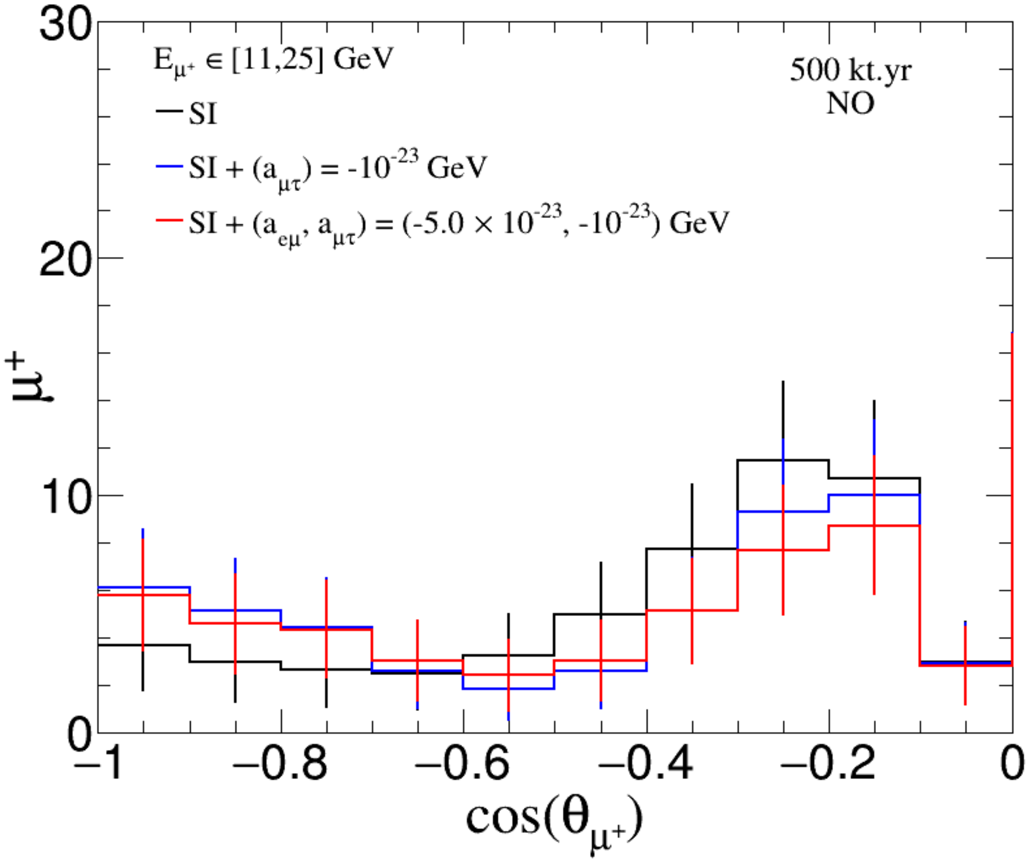
Impact of non-zero CPT-Violating LIV parameters on M.O. determination

Cases	NO (true)		IO (true)	
	$\Delta\chi^2_{\mathrm{ICAL-MO}}$	Deterioration	$\Delta\chi^2_{\mathrm{ICAL-MO}}$	Deterioration
SI	7.55	–	7.48	–
SI + $a_{\mu\tau}$	6.27	16.8 %	6.34	15.2 %
SI + $a_{e\mu}$	5.08	32.7 %	3.90	47.9 %
SI + $a_{e\tau}$	5.23	30.7 %	4.24	15.2 %

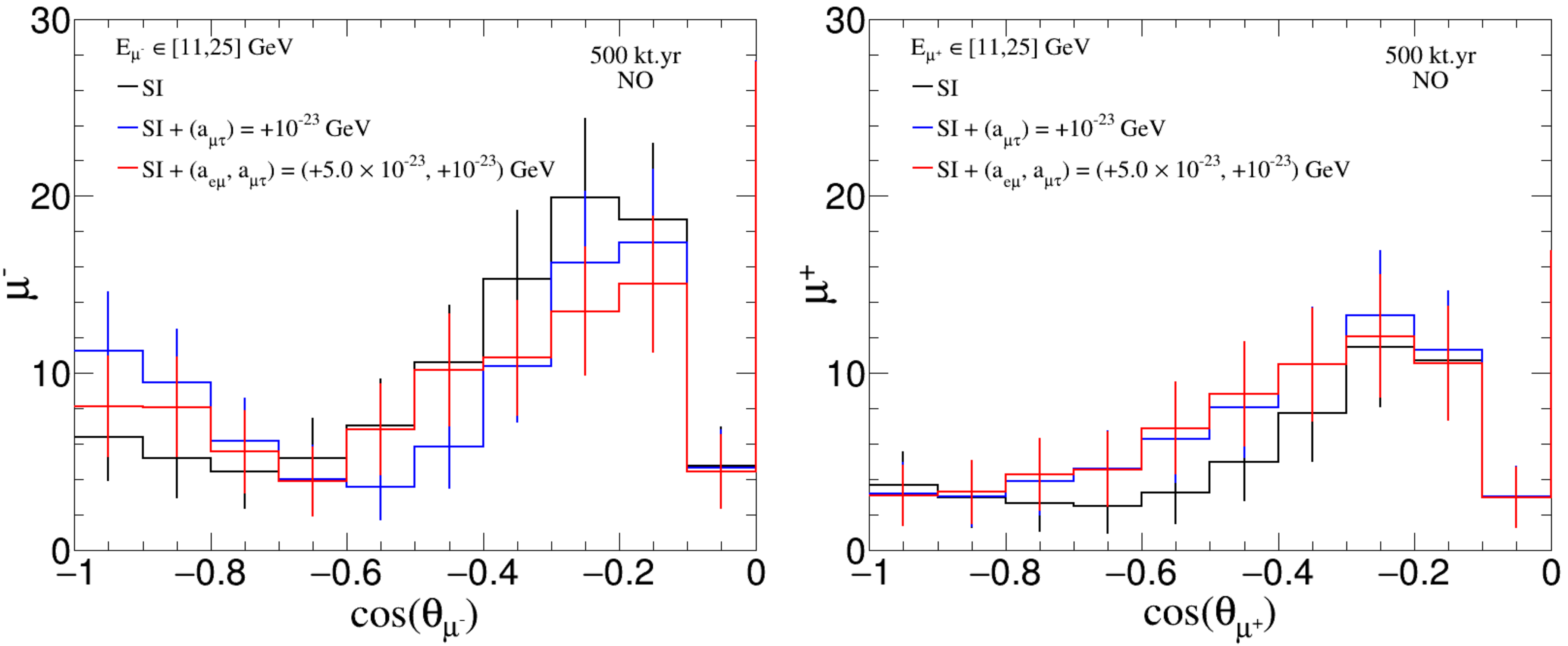
Impact of non-zero ($a_{e\mu}$, $a_{\mu\tau}$) on Events

Impact of non-zero ($a_{\mu\tau}$) on Event Distribution :

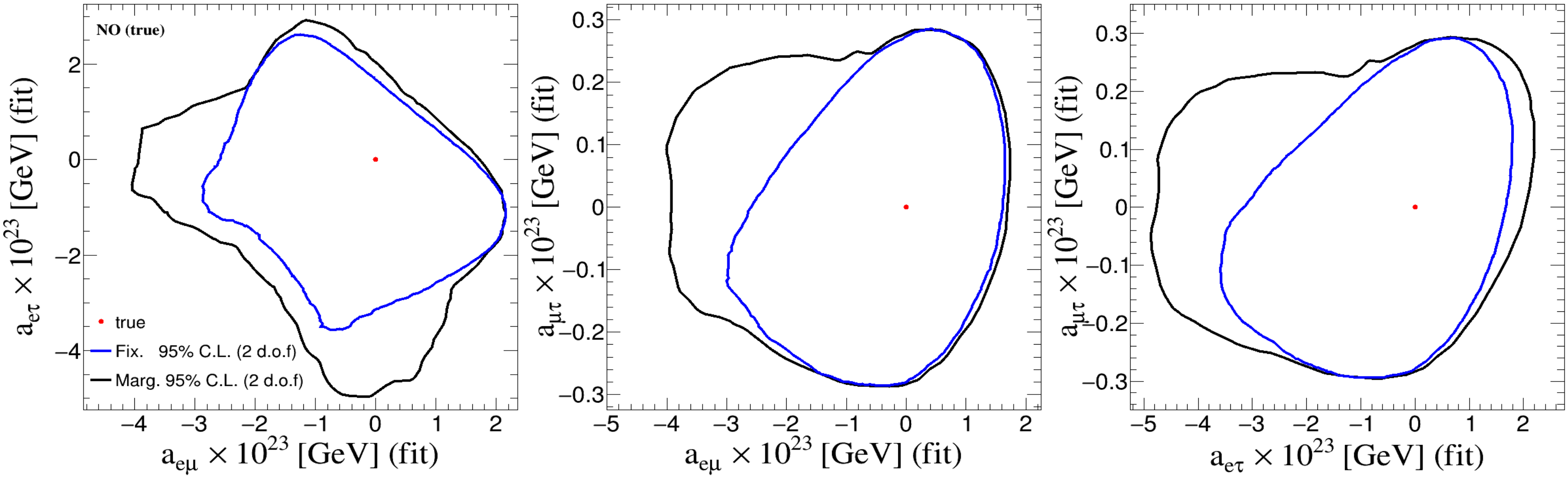




Impact of non-zero ($a_{\mu\tau}$) on Event Distribution :



Constraining CPT-Violating LIV parameters (two-at-a-time)


NO (true)
true
Fix. 95% C.L. (2 d.o.f)
Marg. 95% C.L. (2 d.o.f)
$a_{e\tau} \times 10^{23}$ [GeV] (fit)
$a_{e\mu} \times 10^{23}$ [GeV] (fit)
$a_{\mu\tau} \times 10^{23}$ [GeV] (fit)
$a_{e\mu} \times 10^{23}$ [GeV] (fit)
$a_{\mu\tau} \times 10^{23}$ [GeV] (fit)
$a_{e\tau} \times 10^{23}$ [GeV] (fit)

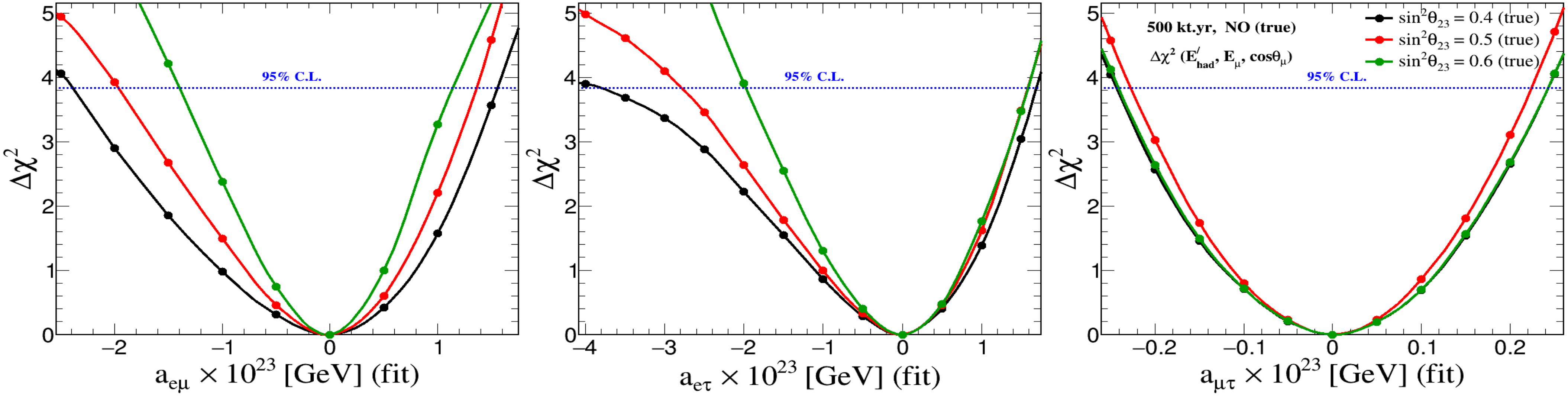
Summary & Remark

- The upcoming magnetised ICAL detector at INO can play a crucial role to establish three-flavour neutrino oscillation framework by observing atmospheric neutrino and antineutrino separately, in the multi-GeV energy range over a wide range of baselines.
- The prime goals of ICAL are to determine M.O. and precise measurement of oscillation parameter at 2-3 sector.
- Using its excellent muon detection sensitivity, for an exposure of 500 kt·yr we place stringent limits on CPT-violating LIV parameters ($a_{\mu\tau}$, $a_{e\mu}$, $a_{e\tau}$) one-at-a-time at 95% C.L. (1 d.o.f), which is slightly better than the current Super-K limits.
- For the first time, we constraint the region of CPT-violating LIV parameters (two-at-a-time) at 95% C.L. (2 d.o.f).
- We also study the effect of non-zero CPT-violating LIV parameters on the M.O. determination.

Thanking You

Back Up

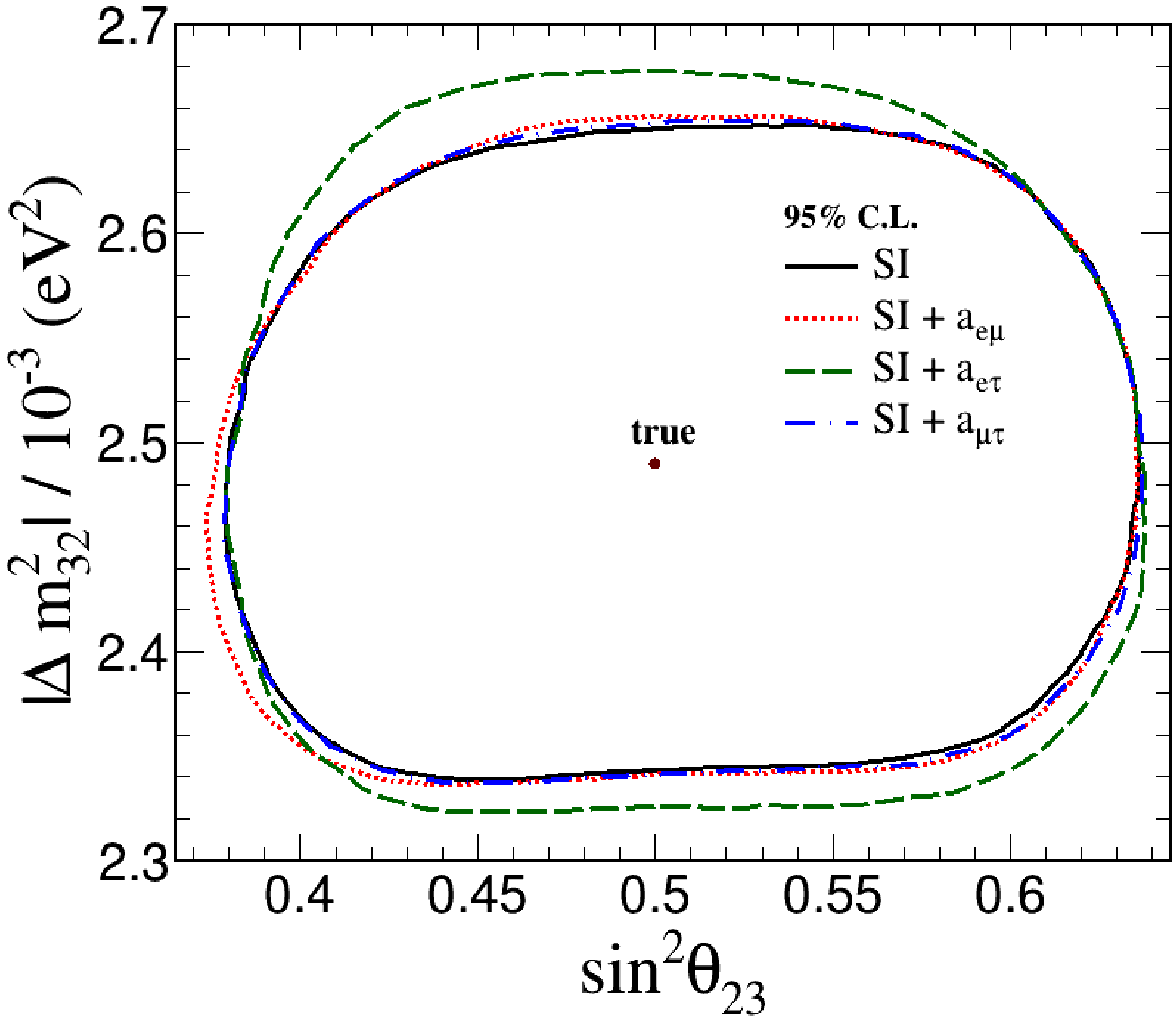
Impact of $\sin^2\theta_{23}$ on LIV CPT-Violating bounds :



Observation:

- For $a_{e\mu}$ and $a_{e\tau}$, $\Delta\chi^2$ [Marg.] is proportional to $\sin^2\theta_{23}$
- For $a_{\mu\tau}$ $\Delta\chi^2$ [Marg.] is proportional to $\sin^2 2\theta_{23}$

Impact of LIV CPT-Violation on Precision Measurement of $(\Delta m^2_{32}, \sin^2\theta_{23})$



In Data :

$(a_{e\mu}, a_{e\tau}, a_{\mu\tau}) = 0$,

$\sin^2\theta_{23} = 0.5$, $\Delta m^2_{32} = 2.49 \times 10^{-3}$ eV^2

In Theory :

$\sin^2\theta_{23} \in [0.3, 0.7]$,

$\Delta m^2_{32} \in [2.3, 2.7] \times 10^{-3}$ eV^2

(SI + $a_{e\mu}$) : $a_{e\mu}$ $[-2.0, 1.5] \times 10^{-23}$ GeV

(SI + $a_{e\tau}$) : $a_{e\tau}$ $[-2.8, 1.6] \times 10^{-23}$ GeV

(SI + $a_{\mu\tau}$) : $a_{\mu\tau}$ $[-2.3, 2.3] \times 10^{-24}$ GeV