$p \to e^+\gamma$ using LCSR

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(Based on: AB, Namit Mahajan (2111.XXXX))

Outline

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- Light cone sum rules
- Form Factors in LCSR
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Motivation

- Proton decay : Forbidden in the Standard Model
  $\Longrightarrow$ A clear signal of physics beyond the standard model.
- Experimental contraints : $\tau_p > 10^{34}$ years.
- As $p \rightarrow e^+\gamma$ is free from nuclear absorption and complications due to strong interactions compared to the mesonic modes, it is a cleaner channel for experimenatl analysis (experimental partial mean life > 6.7×10^{30} years).
- Helpful in understanding the structure of Proton.
- Can help in constraining the parameter space of various BSM models.

Introduction

- Proton decay is possible via baryon number violating dim-6 operator.

$$\mathcal{O}_{\Gamma\Gamma'} = \epsilon^{abc}(\bar{d}^c_a P_\Gamma u_b)(\bar{e}^c P_{\Gamma'} u_c)$$

Projection operators

- The amplitude for the process is:

$$\mathcal{A}(p \to e^+\gamma) = \sum_{\Gamma,\Gamma'} c_{\Gamma\Gamma'} \left\langle e^+(p_e)\gamma(k) \,|\, \mathcal{O}_{\Gamma\Gamma'} \,|\, p(p_p) \right\rangle = \sum_{\Gamma,\Gamma'} c_{\Gamma\Gamma'} \bar{v}^c_e(p_e) H_{\Gamma\Gamma'}(p_p, p_e) u_p(p_p)$$

- Using gauge invariance, it can be written in terms of two form factors (to be calculated using LCSR):

$$\mathcal{A}(p \to e^+\gamma) = \sum_{\Gamma,\Gamma'} c_{\Gamma\Gamma'} \left\{ i\sigma^{\alpha\beta} k_\beta \epsilon^*_\alpha \left(A_{\Gamma\Gamma'} + B_{\Gamma\Gamma'}\gamma_5 \right) \right\}$$

- Becaue the parity is conserved in QCD,

$A_{LL} = A_{RR}$, $A_{LR} = A_{RL}$, $B_{LL} = B_{RR}$, and $B_{LR} = B_{RL}$

Light Cone Sum Rules

- **Idea**: *To compute hadronic parameters using the analytic properties of the correlation function (treated in the framework of OPE).*

TOOLS TO DERIVE SUM RULES

Dispersion Relation
(relates real part of correlation function to its imaginary part)

Operator Product Expansion
(Enables one to write correlation function as a product of short distance and long distance physics)

Quark Hadron Duality
(Relates the non-perturbative spectral function to the perturbatively calculated amplitude function)

Borel Transformation
(To supress the effect of continuum and higher resonances)

Light Cone Sum Rules for $p \to e^{+}\gamma$

- The hadronic matrix element to be calculated is,

$$H_{\Gamma\Gamma'}u_p(p_p) = \left\langle \gamma(k) \middle| \epsilon^{abc}(d_a^T C P_\Gamma u_b) P_{\Gamma'} u_c \middle| p(p_p) \right\rangle$$

- Two possibilities:
  1. Interpolating proton current and using photon distribution amplitudes.
  2. Interpolating electromagnetic current and using proton distribution amplitudes.

Distibution amplitude: The probability amplitude for finding meson (baryon) as a two (three) quark state with the momentum factions u and $(1-u)$ $(\alpha_1,\ \alpha_2,\ \text{and}\ \alpha_3)$.

- To obtain parametrisation for hadronic matrix element $H_{\Gamma\Gamma'}u_p(p_p)$; insert a complete set of states with same quantum numbers as proton.
- We isolate the contribution due to proton state.

$$\Pi^{Had}_{\Gamma\Gamma'} = \frac{-m_p}{p_p^2 - m_P^2 + i\epsilon}\lambda_p H_{\Gamma\Gamma'}(p_p^\mu \gamma_\mu + m_p) + \ldots$$

- The continuum and excited states (spectral densities) can be approximated using global quark hadron duality.

$$\int_{s_0}^{\infty} ds \frac{\rho^{cont}_{\Gamma\Gamma'}(s, Q^2)}{s - p_p^2} \simeq \int_{s_0}^{\infty} ds \frac{\rho^{QCD}_{\Gamma\Gamma'}(s, Q^2)}{s - p_p^2} \qquad (Q^2 = -p_e^2)$$

Photon Distribution Amplitude & Proton interpolation current

- The leading twist (twist-2) DA for photon is:

$$\left\langle \gamma(k) \middle| \bar{q}(x)\sigma_{\mu\nu}q(0) \middle| 0 \right\rangle = -ie_q \langle \bar{q}q \rangle (\epsilon^*_\mu k_\nu - \epsilon^*_\nu k_\mu) \int_0^1 du e^{iq.x\bar{u}} \chi \phi_\gamma(u)$$

Quark Condensate

Magnetic Susceptibility

Twist-2 DA

$$\phi_\gamma(u) = 6u\bar{u}\left(1 + \sum_{n=2,4,..}^{\infty} L^{(\gamma_n - \gamma_0)/b} \phi_n c_n^{3/2}(u - \bar{u})\right)$$

- The proton interpolation current is chosen to be,

$$\eta(x) = 2\epsilon^{abc}(u_a^T(x) C\gamma_5 d_b(x)) u_c(x)$$

Such that $\left\langle 0 \middle| \eta(0) \middle| p(p_p) \right\rangle = m_p \lambda_p u_p(p_p)$.

Form Factors in QCD

- In QCD,

Charge Conjugation matrix

$$\Pi_{\Gamma\Gamma'} = i\int d^4x e^{ip_e.x}\left\langle \gamma(k)\left| \underbrace{T\left\{\epsilon^{abc}(d_A^T(x)CP_\Gamma u_b(x))P_{\Gamma'}u_c(x)\times 2\epsilon^{ijk}\bar{u}_c(0)(u_a^T(0)C\gamma_5 d_b(x))\right\}} \right| 0\right\rangle$$

$$-\frac{1}{2}\epsilon_{ijk}\epsilon_{abc}P_{\Gamma'}\{(\bar{u}^a(0)\Gamma_A u^i(x))\Big(s_u^{kc}(x)\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma\Gamma^A + s_u^{kc}(x)Tr(\Gamma^A\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma) + \Gamma^A\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma s_u^{kc}(x) + \Gamma^A Tr(s_u^{kc}\gamma_5\tilde{s}_d^{jb}(x)P_\Gamma)\Big)$$

$$+(\bar{d}^a(0)\Gamma_A d^i(x))\Big(s_u^{kc}(x)\gamma_5\tilde{\Gamma}^A P_\Gamma s_u^{jb}(x) + s_u^{kc}(x)Tr(s_u^{jb}(x)\gamma_5\tilde{\Gamma}^A P_\Gamma)\Big)\}$$

$$\Gamma_A = \left\{1,\gamma_5,\gamma^\rho, i\gamma_\rho\gamma_5, \frac{1}{\sqrt{2}}\sigma^{\rho\sigma}\right\}$$

$$s^{ij}(x) = \frac{ix_\mu\gamma^\mu}{2\pi^2x^4}$$

$$\tilde{\Gamma}_A = C\Gamma_A C$$

- At leading twist, only $\Gamma_A = \frac{1}{\sqrt{2}}\sigma^{\rho\sigma}$ will contribute.

- Upto leading twist (twist-2),

$$A_{LL}^{QCD} = \frac{3i}{32\pi^2}\int_0^1 d\alpha e_u < \bar{u}u > P^2 ln(-P^2)\sigma_{\alpha\beta}\epsilon^{*\alpha}k^\beta \chi\phi_\gamma(\alpha)$$

$$B_{LL}^{QCD} = \frac{-3i}{32\pi^2}\int_0^1 d\alpha e_u < \bar{u}u > P^2 ln(-P^2)\sigma_{\alpha\beta}\epsilon^{*\alpha}k^\beta \chi\phi_\gamma(\alpha)$$

$$A_{RL}^{QCD} = \frac{i}{32\pi^2}\int_0^1 d\alpha (3e_u < \bar{u}u > -\frac{1}{3}e_d < \bar{d}d >)P^2 ln(-P^2)\sigma_{\alpha\beta}\epsilon^{*\alpha}k^\beta \chi\phi_\gamma(\alpha)$$

$$B_{RL}^{QCD} = \frac{i}{32\pi^2}\int_0^1 d\alpha (3e_u < \bar{u}u > -\frac{1}{3}e_d < \bar{d}d >)P^2 ln(-P^2)\sigma_{\alpha\beta}\epsilon^{*\alpha}k^\beta \chi\phi_\gamma(\alpha)$$

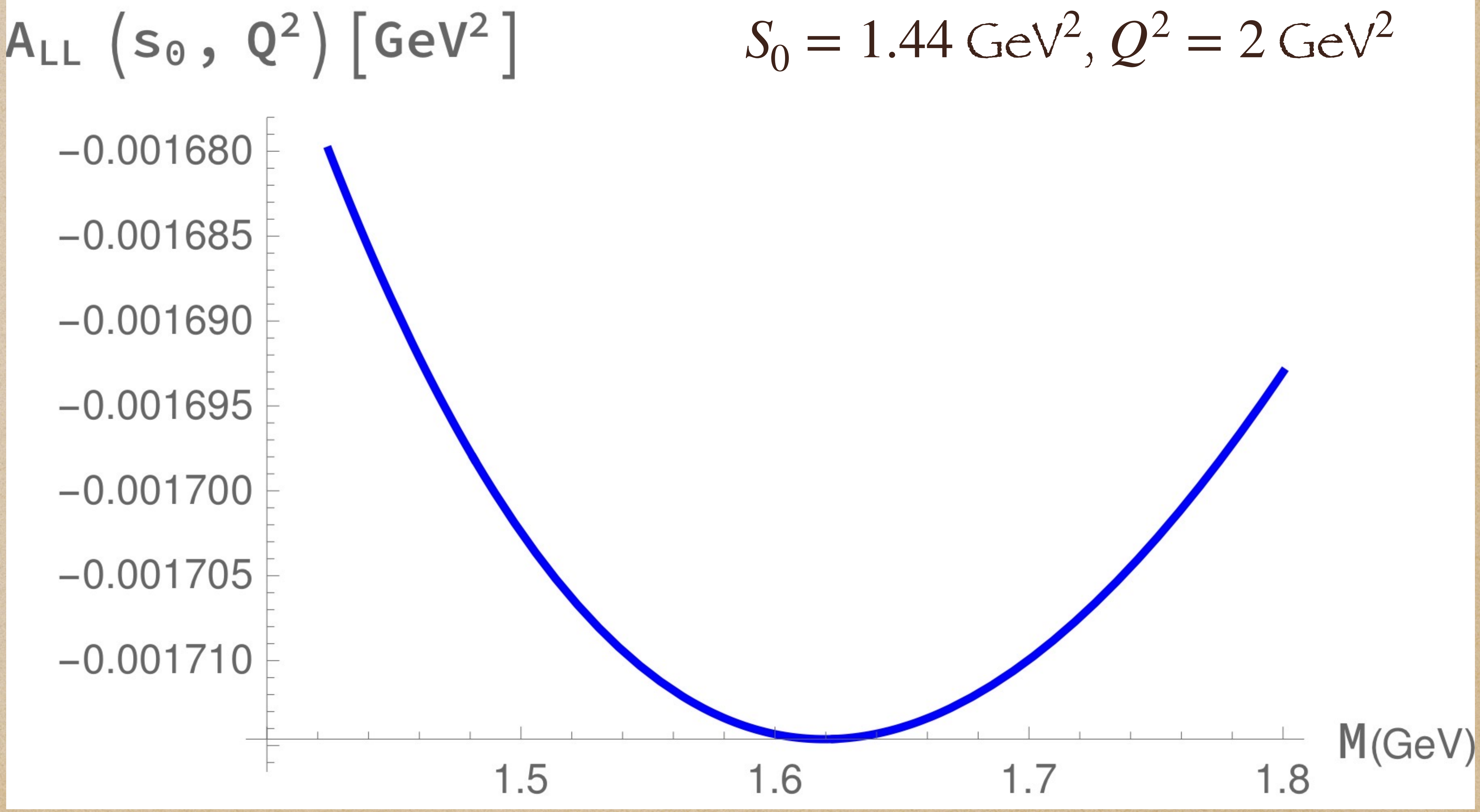
where, $P^2 = (p_e + \alpha k)^2 = -\alpha P_p^2 - \bar{\alpha}Q^2$.

Results

- The analytical form of the form factors using LCSR is given by,

$$i\lambda_p m_p^2 e^{\frac{-m_p^2}{M^2}} A_{\Gamma\Gamma'}(s_0, Q^2) = \frac{1}{\pi}\int_0^{s_0} ds e^{\frac{-s}{M^2}} Im\left(A_{\Gamma\Gamma'}^{QCD}(s, Q^2)\right)$$

M is the Borel parameter.



- Similar results for other form factors.

Summary & Conclusions

- $p \to e^+\gamma$ decay involves 2 FFs: calculated in the fracmework of LCSR.
- FFs presented upto twist-2 accuracy using photon DA.
- Using proton DA, it is found that there is no contribution from the leading twist (twist-3) DAs.
- Higher twist three-particel DAs (upto twist-6) are required to get estimates for the form factor involved.

Thank You