[arXiv:2004.09440]

Direct CP violation and the $\Delta I=1/2$ rule in $K\to\pi\pi$ decay from the Standard Model

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Motivation

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \rightarrow \pi\pi$:

$$\eta_{00} = \frac{A(K_{\mathrm{L}} \to \pi^0\pi^0)}{A(K_{\mathrm{S}} \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_{\mathrm{L}} \to \pi^+\pi^-)}{A(K_{\mathrm{S}} \to \pi^+\pi^-)}.$$

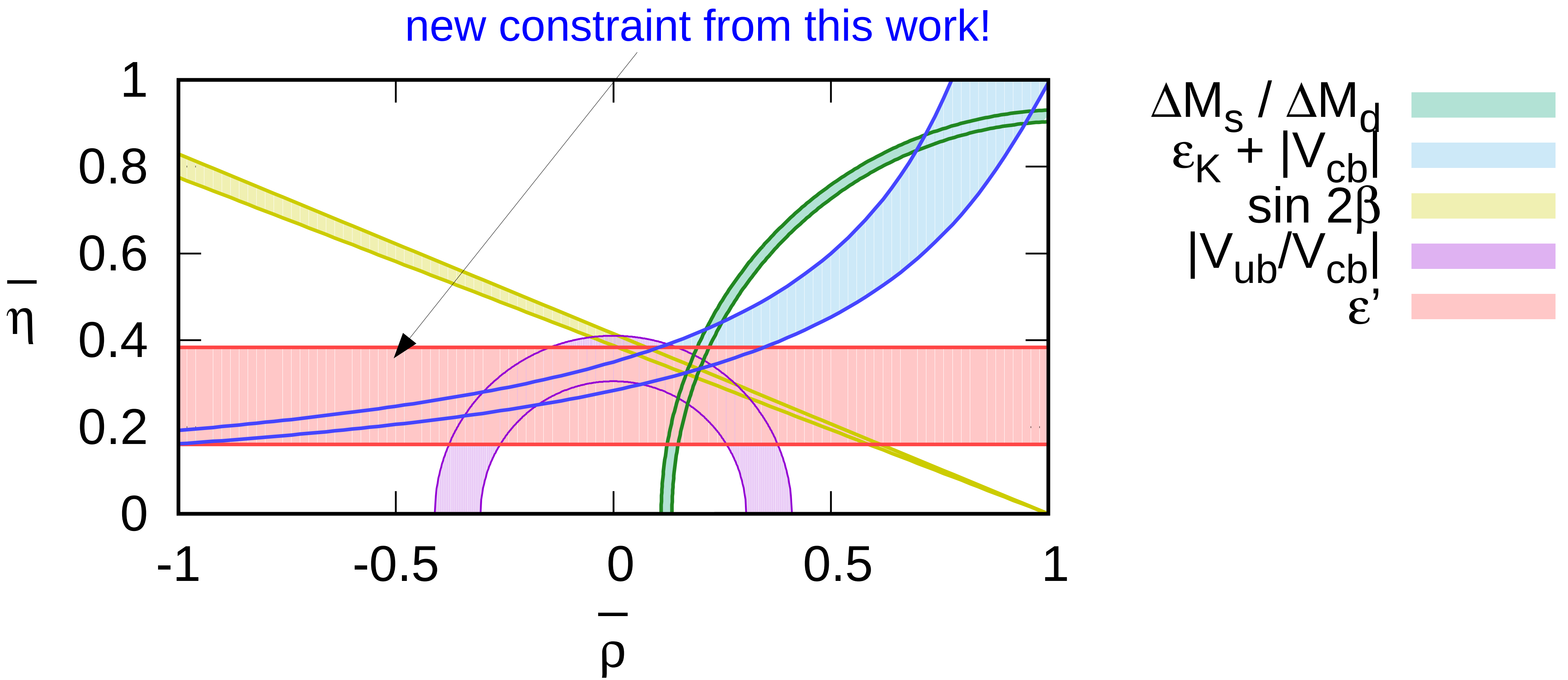
$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by many BSM models.
- Looking for deviations from experiment may help shed light on origin of M/AM asymmetry.

- A Standard Model prediction of ε' also provides a new horizontal band constraint on CKM matrix in ρ-η plane:



- While underlying weak process occurs at high energies ~M_W=80 GeV, K→ππ decays receive large corrections from low-energy hadronic physics $O(\Lambda_{QCD})$~250 MeV.

- Lattice QCD is the only known *ab initio*, **systematically improvable** technique for studying non-perturbative QCD.

Overview of calculation

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\ ,$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}\ .$$

ππ phase shifts

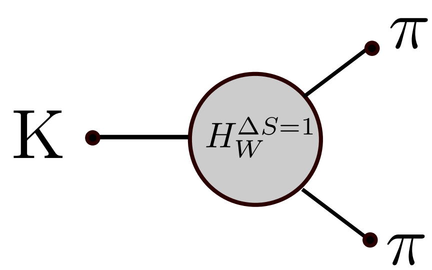
$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

I=2 decay

I=0 decay

- **Hadronic energy scale << M_W – use weak effective theory (3 flavors)**



LL finite-volume correction

renormalization matrix (mixing)
Use RI-SMOM
convert to MSbar perturbatively

$$A^I = F\frac{G_F}{\sqrt{2}}V_{ud}^* V_{us}\sum_{i=1}^{10}\sum_{j=1}^{7}\left[(z_i(\mu)+\tau y_i(\mu))\, Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}} M_j^{I,\,\mathrm{lat}}\right]$$

perturbative Wilson coeffs.

$M_j^{I,lat} = \langle(\pi\pi)_I|Q_j|K\rangle$ (lattice)

10 effective four-quark operators

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

Anatomy of a lattice calculation

- Lattice QCD uses Monte Carlo techniques to sample the discretized (Euclidean) Feynman path integral directly, generating an ensemble of N "gauge configurations".
- Expectation value of some Green's function computed over the ensemble converges to path integral value in large N limit.
- Green's function composed of *operators* created from quark fields that create/destroy states of interest.
- Operators create all states with same quantum numbers, eg $\bar{u}\gamma^5 d$ creates pions and all excited pion-like states.
- Contributions of each state i decay exponentially in time as exp($-E_i$ t) due to Euclidean time.
- Extract contributions of lightest states by fitting large time dependence.
- Challenges:
  - Computationally expensive, requiring months to years of running on the world's fastest supercomputers.
  - Much like experiment, have both statistical and systematic errors.
  - Systematic errors (e.g. from discretization or from fitting) require careful analysis and treatment.

Lattice QCD for K→ππ

operator with ππ q. numbers.

operator with kaon q. numbers.

$$\langle 0|\mathcal{O}_{\mathrm{snk}}(t_{\mathrm{snk}})H_W(t)\mathcal{O}_{\mathrm{src}}(t_{\mathrm{src}})|0\rangle$$

state contributions exponentially falling according to their energy (Euclidean time!)

$$= \sum_{n,m}\langle 0|\mathcal{O}_{\mathrm{snk}}(t_{\mathrm{snk}})|n\rangle\langle n|H_W(t)|m\rangle\langle m|\mathcal{O}_{\mathrm{src}}(t_{\mathrm{src}})|0\rangle$$

$$\times e^{-E_n(t_{\mathrm{snk}}-t)}e^{-E_m(t-t_{\mathrm{src}})}$$

many states contribute

- Extract matrix elements by fitting time dependence in limit of large (t_{snk}-t), (t_{src}- t) at which lower-energy states dominate.
- Series is necessarily truncated for fit: Systematic errors arise if excited state effects not properly taken into account.

I=2 calculation

[Phys.Rev. D91 (2015) no.7, 074502]

- A_2 can be measured very precisely using “standard” lattice techniques.
- Most recent result (2015):
  - Computed with large, ~ $(5.5\ \mathrm{fm})^3$ volumes
  - Physical quark masses
  - Two lattice spacings (2.36 GeV and 1.73 GeV) → Continuum limit taken.
- <1% statistical error!
- 10% and 12% total errors on $\mathrm{Re}(A_2)$ and $\mathrm{Im}(A_2)$ resp.
- Dominant sys. errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

ΔI=1/2 rule

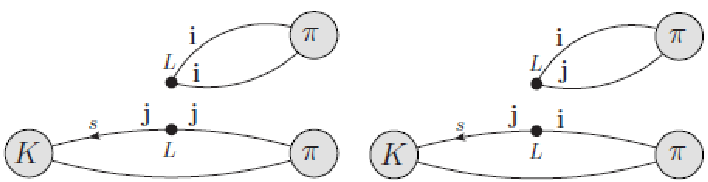
- In experiment kaons ~450x (!) more likely to decay into I=0 pi-pi states than I=2.

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 22.45(6) \quad \text{(the } \Delta I=1/2 \text{ rule)}$$

- Perturbative running to charm scale accounts for about a factor of 2. Where does the remaining 10x come from? New Physics?
- The answer is **low-energy *QCD*!**

RBC/UKQCD [arXiv:1212.1474, arXiv:1502.00263]

Strong cancellation between the two dominant contractions not predicted by naive factorization:

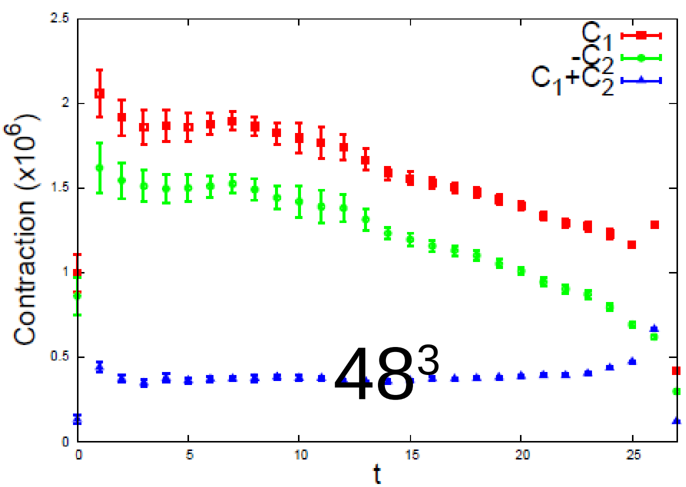


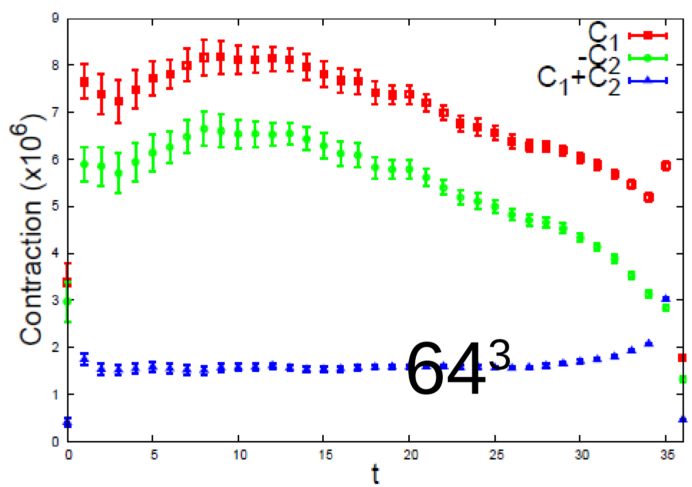
Contraction ①.　　Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

find $② \approx -0.7①$ heavily suppressing $\mathrm{Re}(A_2)$.

[Phys.Rev. D91 (2015) no.7, 074502]





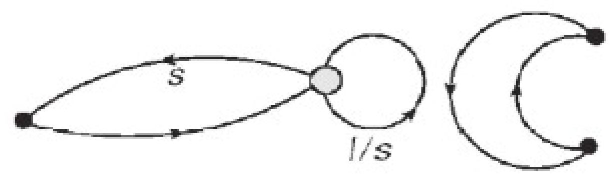
Pure-lattice calculation

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} = 19.9(5.0)$$

[$\mathrm{Re}(A_0)$ agrees with expt.]

I=0 Calculation

- A_0 is more difficult than A_2, primarily because I=0 ππ state has *vacuum quantum numbers.*
- *"Disconnected diagrams"* dominate statistical noise



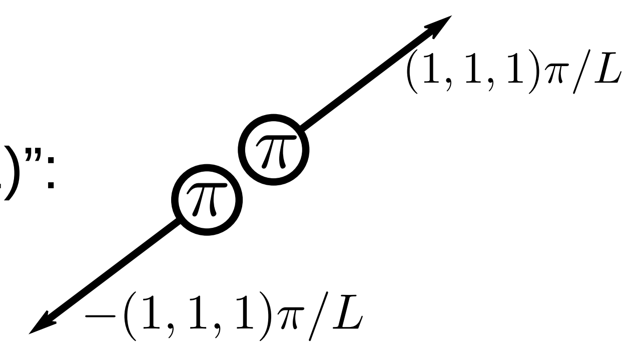
"type4"

2015 calculation

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- Physical quark masses on single, coarse lattice (a^{-1}= 1.38 GeV) but with large (4.6 fm)^3 physical volume to control FV errors.
- Single ππ operator:

"ππ(111)":



21% and 65% stat errors on Re(A_0) and Im(A_0) due to disconn. diagrams and, for Im(A_0) a strong cancellation between Q_4 and Q_6.

- Dominant, 15% systematic error due again to PT truncation errors.

2015 calculation: ε'

- Re(A_0) and Re(A_2) from expt.
- Lattice values for Im(A_0), Im(A_2) and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{ll} = \quad 1.38(5.15)(4.43) \times 10^{-4}, & \text{(our result)} \\ \quad 16.6(2.3) \times 10^{-4} & \text{(experiment)} \end{array}}$$

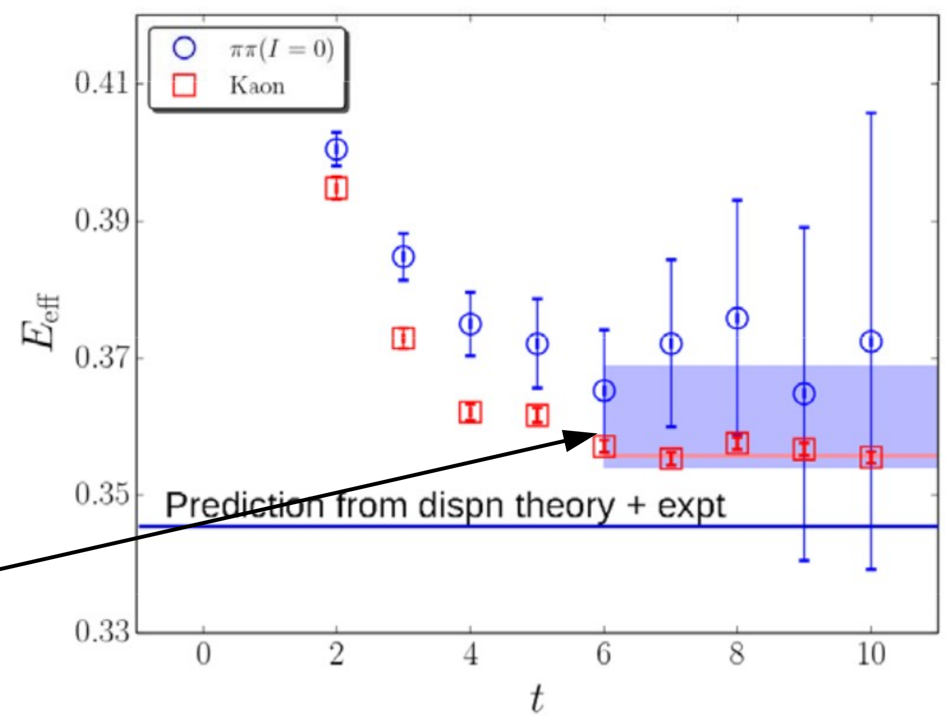
- Total error on Re(ε'/ε) is ~3x the experimental error
- Find reasonable (2.1σ) consistency with Standard Model
- “This is now a quantity accessible to lattice QCD”!
- Focus since has been to improve statistics and reduce / improve understanding of systematic errors.

The “ππ puzzle”

- Essential to understand ππ system:
  - Energy needed to extract ground-state matrix element
  - Energy also needed to compute phase-shift (Luscher)
  - Derivative of phase-shift w.r.t. energy is required for Lellouch-Luscher finite-volume correction (F)
- 2015 calculation phase shift $\delta_0(E_{\pi\pi} \approx m_K) = 23.8(5.0)^\circ$ substantially smaller than prediction obtained by combining dispersion theory with experimental input, 36°.

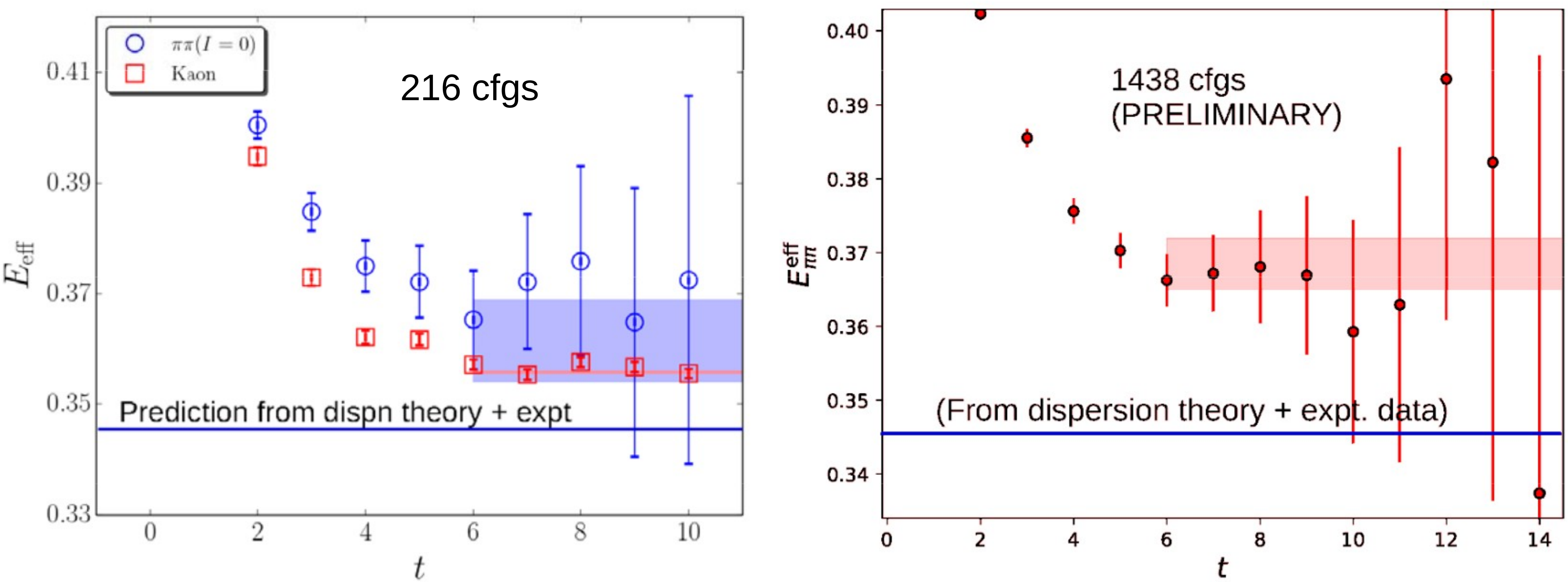
- Result was very stable under varying fit range and also with 2-state fits.
- Nevertheless, most likely explanation is excited-state contamination hidden by rapid reduction in signal/noise.

rapid error growth



Increased statistics

- To resolve the "pi-pi puzzle" we increased statistics from 216 to 1438 (a 6.6x increase!).
- However this did not resolve the situation:

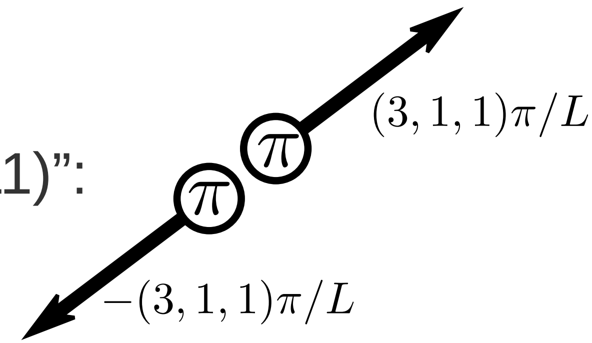


$$\delta_0 = 23.8(4.9)(2.2)^\circ \rightarrow 19.1(2.5)(1.2)^\circ$$

Resolving the ππ puzzle

- To better resolve the ground-state we have introduced 2 more ππ operators:

"ππ(311)":



$$\sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

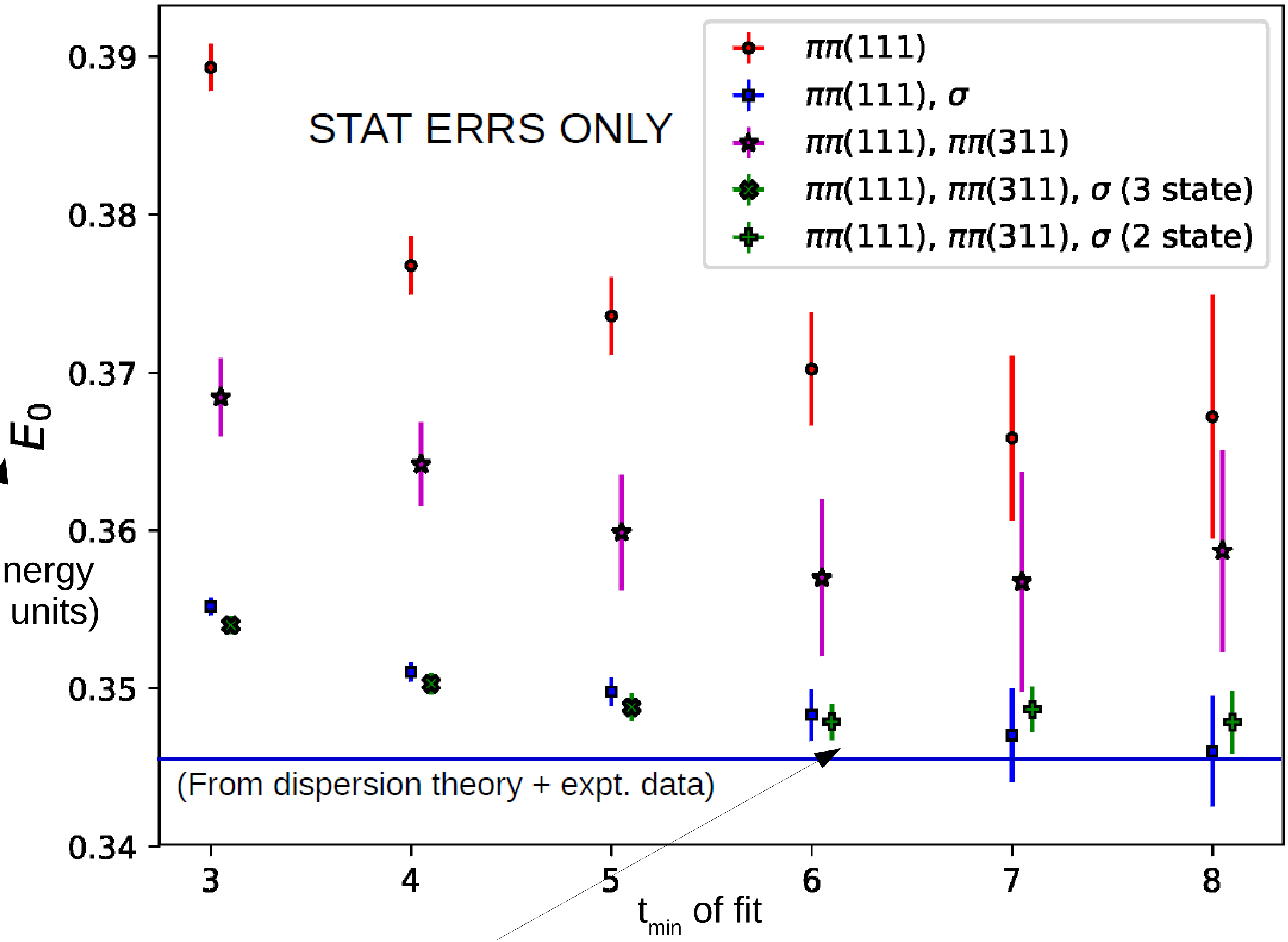
- Obtain parameters by simultaneous fitting to matrix of correlation functions, eg for pipi 2pt Green's function:

$$C_{ij}(t) = \langle 0|O_i^\dagger(t)O_j(0)|0\rangle = C + \sum_\alpha A_{i,\alpha}A_{j,\alpha}e^{-E_\alpha t}$$

round-the-world single pion propagation
small compared to errors - drop

- A far more powerful technique than just increasing statistics alone.
- 741 configurations measured with 3 operators.

Effect of multiple operators on ππ



fitted energy (lattice units)

Result compatible with dispersive value: $\delta_0(479.5\ \mathrm{MeV}) = 32.3(2.1)^\circ$

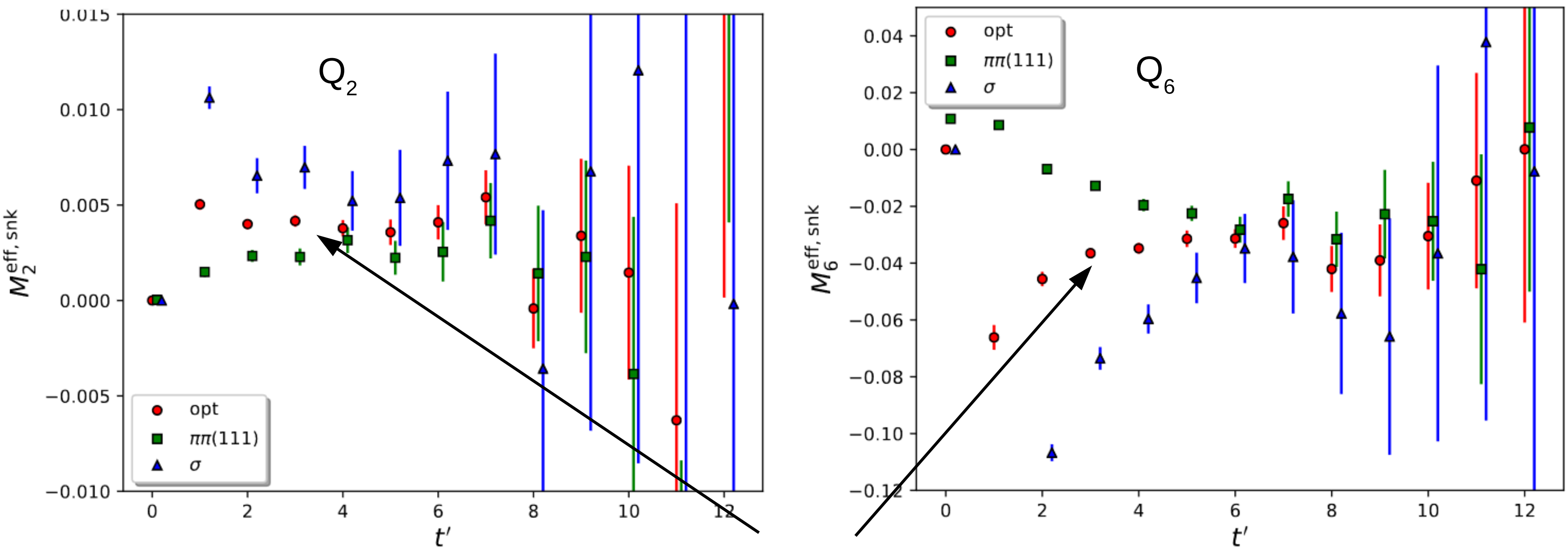
Effect of multiple operators on K→ππ

- Convenient to visualize data by taking “optimal” linear combination of the two most important operators that best projects onto ground-state.

$$\mathcal{O}_{\rm opt} = r_1 \mathcal{O}_{\pi\pi(111)} + r_2 \mathcal{O}_\sigma$$

$$r_1 = 5.24(18) \times 10^{-7}$$
$$r_2 = -2.86(17) \times 10^{-4}$$

using ππ fits

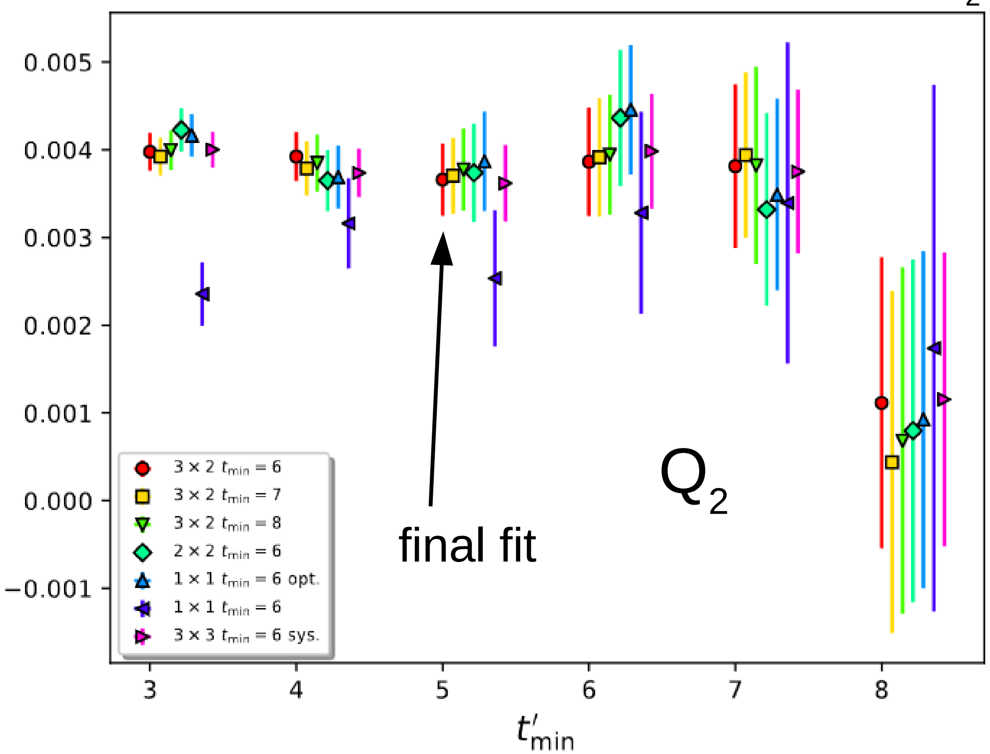


strong, clear plateau + improved precision

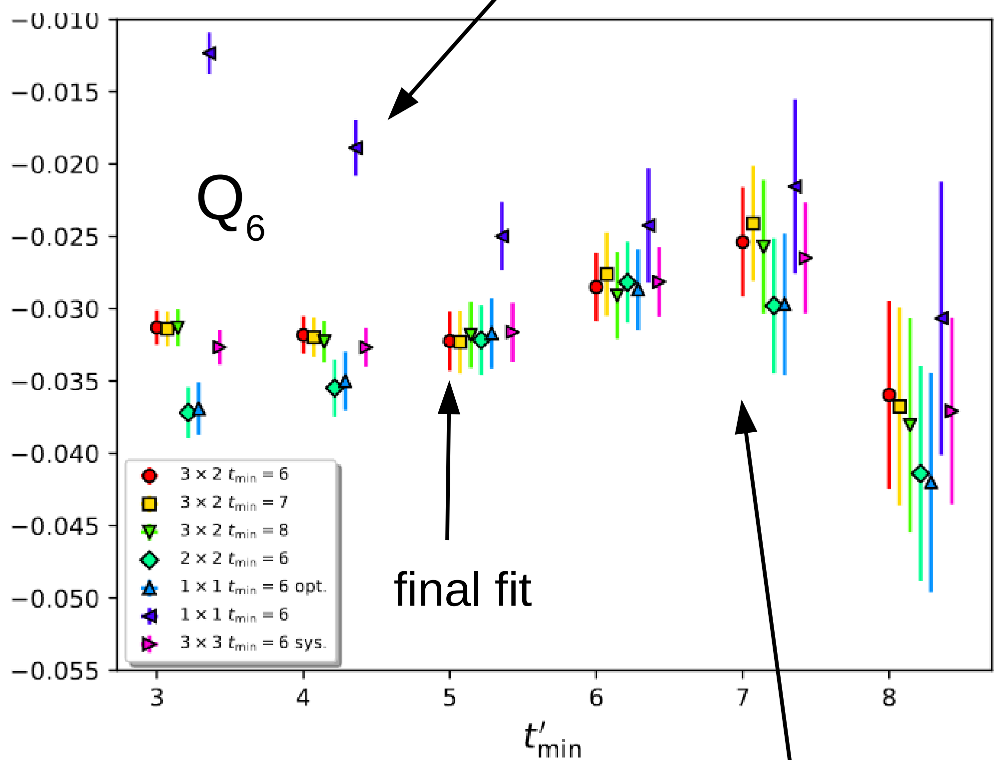
K→ππ fit results

- Examine many fit ranges, #states and #operators

little indication of exc. stat. cont. for Q_2



exc. state contam. with 1op
(this was t'_{min} for 2015 work)

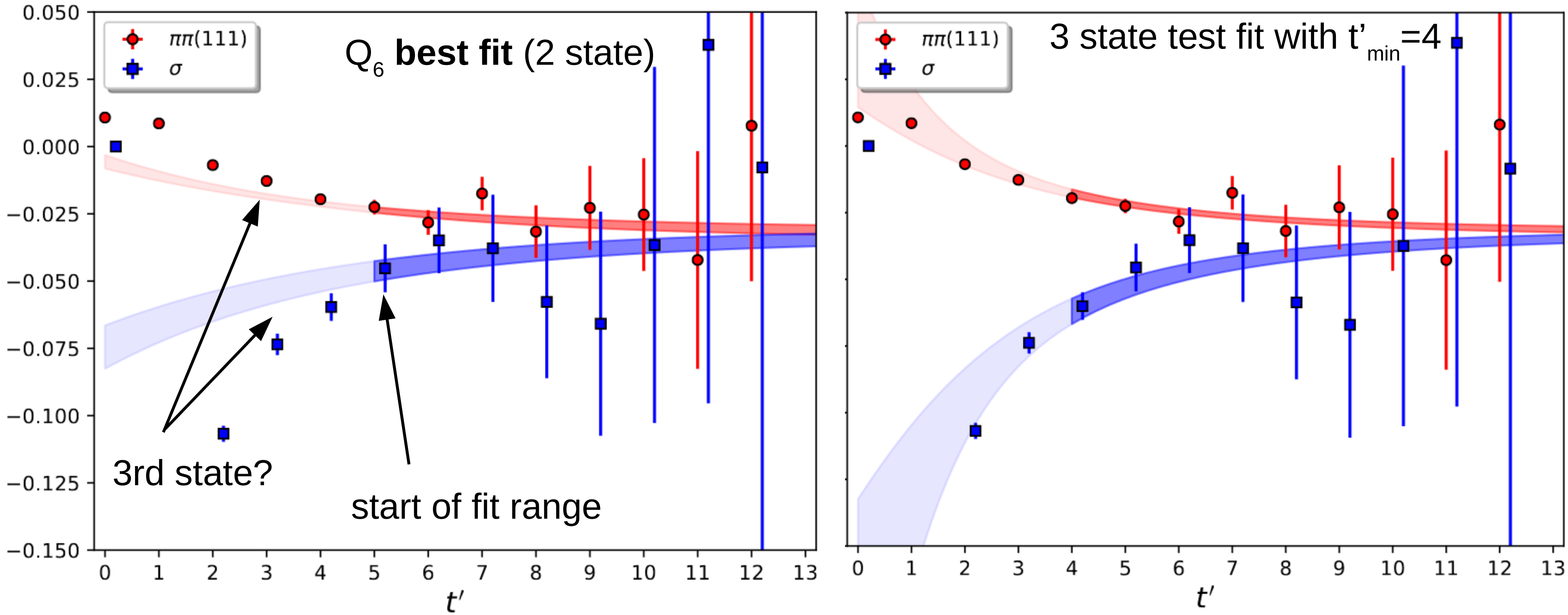


“bump” appears to be statistical

- Adopt uniform fit t'_{min}=5 which is stable for all Q_i
- Evidence that excited state error was significantly underestimated in 2015 work

Excited state contamination

- Primary concern is residual excited-state contamination
- See excellent consistency and strong plateaus among fits for 2+ operators with $t'_{min} \geq 4$
- Also examine 2 and 3-state fits with 3 ops:



- 3-state fit with lower t_{min}=4 describes data well outside fit range.
  Complete consistency in gnd-state matrix elem of best fit

Systematic error budget

- Primary systematic errors of 2015 work:
  - Finite lattice spacing: 12%
  - Wilson coefficients: 12%
  - Renormalization (mostly PT matching): 15%
  - Excited-state: ≤ 5% but now known to be significantly underestimated
  - Lellouch-Luscher factor (derivative of $\pi\pi$ phase shift wrt. energy): 11%
- In our new work we have used step-scaling to raise the renormalization scale from 1.53 → 4.00 GeV: 15% → 5%
- 3 operators have dramatically improved understanding of $\pi\pi$ system: Lellouch-Luscher factor 11% → 1.5%
- Detailed analysis shows no evidence of remaining excited-state contamination: Excited state error now negligible!
- Still single lattice spacing: Discretization error unchanged.
- Evidence that Wilson coefficient systematics are driven by using PT for 3-4f matching, not improved by higher μ: Wilson coeff error unchanged.

Isospin breaking + EM effects

- Our simulation does not include effects of isospin breaking or EM effects. Did not attempt to account for in 2015 analysis.
- While these effects are typically small O(1%), heavy suppression of A_2 ($\Delta I=1/2$ rule) means relative effect on A_2 and ε' could be O(20%).

[JHEP 02 (2020) 032]

- Current best determination of effect uses NLO χPT and $1/N_c$ expansion predicts 23% correction to our result:

  Include as separate systematic error.

Final result for ε'

- Combining our new result for $\mathrm{Im}(A_0)$ and our 2015 result for $\mathrm{Im}(A_2)$, and again using expt. for the real parts, we find

$$\boxed{\begin{aligned}\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) &= \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\} \\ &= 0.00217(\underset{\text{stat}}{26})(\underset{\text{sys}}{62})(\underset{\text{IB + EM}}{50})\end{aligned}}$$

Consistent with experimental result:

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{expt}} = 0.00166(23)$$

Conclusions

- Completed update on our 2015 lattice determination of A_0 and ε’
  - 3.2x increase in statistics.
  - Improved systematic errors, notably use of multi-operator techniques essentially removes excited-state systematic.
- Reproduce experimental value for $\Delta I=1/2$ rule, demonstrating that QCD sufficient to solve this decades-old puzzle.
- Result for ε’ consistent with experimental value.
- Total error is ~3.6x that of experiment.
- ε’ remains a promising avenue to search for new physics, but greater precision is required.

The road ahead

- We intend to perform measurements on two larger lattices with different lattice spacings to perform continuum limit.
- Due to computational expense this will require next-gen supercomputers (Perlmutter, Aurora).
- More readily addressed is reliance on PT to perform 3-4f matching in the Wilson coefficients. Project presently underway to compute these non-perturbatively. [PoS LATTICE2018 (2019) 216]
- Also working on laying the groundwork for the lattice calculation of isospin+EM effects. [EPJ Web Conf. 175 (2018) 13016]