Results and challenges in the $B_{(s)}$-sector from lattice QCD

Justus Tobias Tsang

tsang@imada.sdu.dk

The University of Southern Denmark
Centre for Cosmology and Particle Physics Phenomenology

Anomalies 2020, Hyderabad, India

11 September 2020

DEPARTMENT OF MATHEMATICS
AND COMPUTER SCIENCE

Outline

b-decays as "sweet spot" for experiments

Properties of b-decays [PDG'20]

1. $\overline{m}_b(\overline{m}_b) = 4.18(3)\,\mathrm{GeV} \gg \overline{m}_c(\overline{m}_c) = 1.27(2)\,\mathrm{GeV} \gg m_s, m_u, m_d$
   $\rightarrow$ many different decay products
2. b hadrons have *relatively long* lifetime of $\tau_b \sim 10^{-12}s$ ($\tau_t \sim 10^{-25}s$)
   $\rightarrow$ b hadronises and b-jets travel some distance before decaying
   $\rightarrow$ but not far enough to escape the detector
   $\rightarrow$ allows for $b-$**tagging**

$\Rightarrow$ **Plethora of <u>accessible</u> decay channels for hadrons with b-quarks**

b-decays as "sweet spot" for experiments

Properties of b-decays [PDG'20]

1. $\overline{m}_b(\overline{m}_b) = 4.18(3)\,\mathrm{GeV} \gg \overline{m}_c(\overline{m}_c) = 1.27(2)\,\mathrm{GeV} \gg m_s, m_u, m_d$
   $\rightarrow$ many different decay products
2. b hadrons have *relatively long* lifetime of $\tau_b \sim 10^{-12}s$ ($\tau_t \sim 10^{-25}s$)
   $\rightarrow$ b hadronises and b-jets travel some distance before decaying
   $\rightarrow$ but not far enough to escape the detector
   $\rightarrow$ allows for $b-$**tagging**

$\Rightarrow$ **Plethora of accessible decay channels for hadrons with b-quarks**

Distinguish two categories:

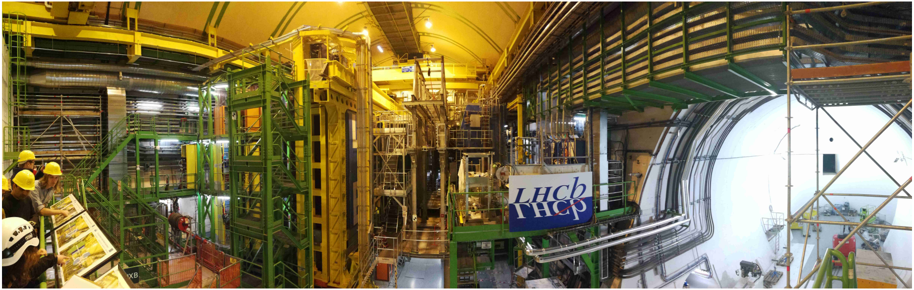
Charged currents

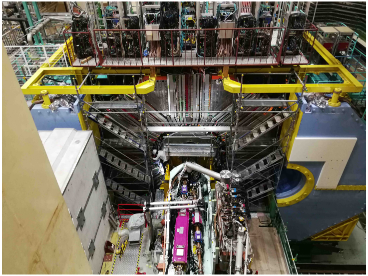
- Present at *tree level* in the SM
  e.g. $B^0 \rightarrow D^+ \ell^- \nu_\ell$
  $\Rightarrow$ Precision tests of the SM

Flavour changing neutral currents

- Only at *loop level* in the SM
  e.g. $B \rightarrow K \ell^+ \ell^-$
  $\Rightarrow$ Sensitive to NP searches

Search for New Physics: (flavour) experiments





top: LHCb at LHC, CERN

left: Belle II at SuperKEKb, KEK

⇒ Huge experimental efforts!
  + BES-III and other LHC experiments

⇒ B-**factory** vs **hadron machine**
  Very complementary

- "Old" data from BaBar, Belle, Cleo, ...

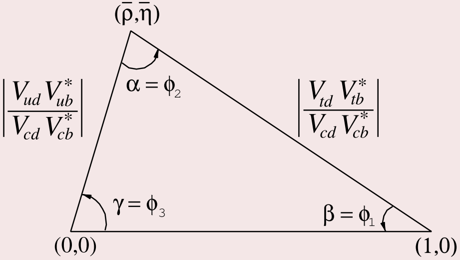
CKM

CKM Matrix

- 3 generations
- appears whenever *u*-type quark changes into *d*-type or vice versa
- complex
  $\Rightarrow$ allows for $\not{C}\not{P}$
  via a single phase
- unitary
  e.g. 2nd row:
  $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 \stackrel{?}{=} 1$

Unitarity Triangle

[PDG]



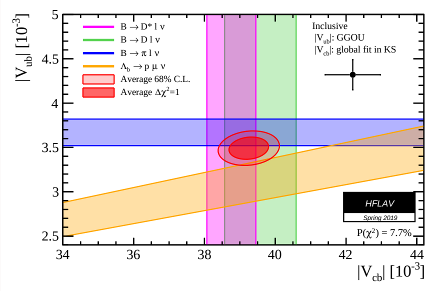
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = -1.$$

$\Rightarrow$ Test SM by determining CKM matrix elements

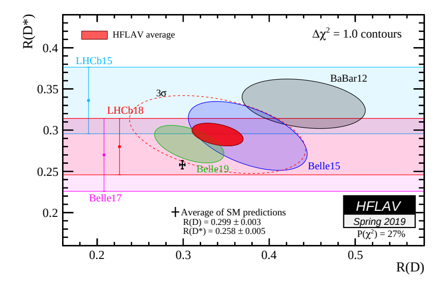
Indirect searches for New Physics in $B_{(s)}$ decays

Inclusive vs Exclusive



exclusive: specific (e.g. $B \to D\ell\nu$)
inclusive: general (e.g. $B \to X_c\ell\nu$)

Lepton Flavour Universality



$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

- More experimental data soon - **need to sharpen theory predictions**!

⇒ **Further work is needed to clarify theory uncertainties!**

Relating experiment and theory

- Experiment measures differential decay rates, mass differences, branching fractions, ...,
- In the SM predictions are parameterised as a sum of products between known/calculable coefficients and low energy matrix elements.

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to P\ell\nu_\ell)}{\mathrm{d}q^2} = |V_{qb}|^2 \frac{G_F^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \times$$

$$\left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) \left|f_+(q^2)\right|^2 + \frac{3m_\ell^2}{8q^2} \left(m_{B_{(s)}}^2 - m_P^2\right)^2 \left|f_0(q^2)\right|^2 \right]$$

Relating experiment and theory

- Experiment measures differential decay rates, mass differences, branching fractions, ...,
- In the SM predictions are parameterised as a sum of products between known/calculable coefficients and low energy matrix elements.

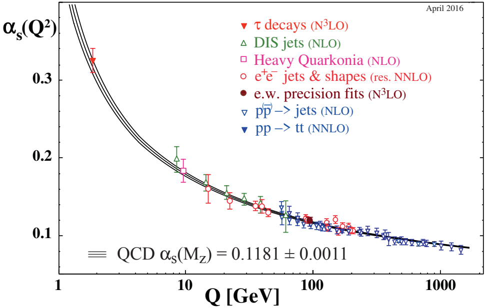
$$\frac{\mathrm{d}\Gamma(B_{(s)} \to P\ell\nu_\ell)}{\mathrm{d}q^2} = |V_{qb}|^2 \frac{G_F^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \times$$

$$\left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) \left|f_+(q^2)\right|^2 + \frac{3m_\ell^2}{8q^2}\left(m_{B_{(s)}}^2 - m_P^2\right)^2 \left|f_0(q^2)\right|^2\right]$$

Or in short

$$\overbrace{\frac{\mathrm{d}\Gamma(B_{(s)} \to P\ell\nu_\ell)}{\mathrm{d}q^2}}^{\text{experiment}} \approx \underbrace{\left|V_{qb}\right|^2}_{\text{CKM−element}} \times \left[\overbrace{\underbrace{\left|f_+(q^2)\right|^2}_{\text{non−pert.}} \mathcal{K}_1 + \underbrace{\left|f_0(q^2)\right|^2}_{\text{non−pert.}} \mathcal{K}_2}^{\text{theory}}\right]$$

Theory predictions for non-perturbative physics



Source: PDG

At *low energy scales*:
perturbative methods fail.
Require non-perturbative methods, e.g.

- Effective theories
- 'AdS/CFT-like' correspondences
- Sum rules
- Lattice QFT

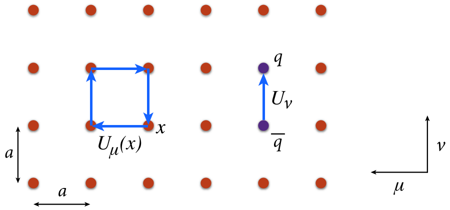
- Lattice QCD simulations provide **first principle precision predictions** for phenomenology
- Calculations need to be improved for observables where the error is dominated by **non-perturbative physics**...

Lattice QCD methodology

Wick rotate ($t \to i\tau$) Path Integral to Euclidean space:

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

Introducing lattice renders PI large **but finite** dimensional.



PDG

- Finite lattice spacing a
  $\Rightarrow$ UV regulator
- Finite Box of length $L^3 \times T$
  $\Rightarrow$ IR regulator
  $\Rightarrow$ Discretised momenta
- $\int \to \sum$, $\partial \to$finite differences

$\Rightarrow$ **The Path Integral is now (large but) finite dimensional.**

$\Rightarrow$ Need to discretise the action (S_G and S_F) and any operators $\mathcal{O}$...

A Lattice Computation

Lattice vs Continuum

We simulate:

- at finite lattice spacing a
- in finite volume L^3
- lattice regularised
- Some bare input quark masses
  am_l, am_s, am_h
  In general: $m_\pi \neq m_\pi^{\mathrm{phys}}$
- Vacuum contributions of
  $N_f = 2$: $2l(u = d)$,
  $N_f = 2 + 1(+1)$: $2l + s(+c)$

We want:

- $a = 0$
- $L = \infty$
- some continuum scheme
- $m_u = m_u^{\mathrm{phys}}$
- $m_d = m_d^{\mathrm{phys}}$
- $m_s = m_s^{\mathrm{phys}}$
- $m_h = m_c^{\mathrm{phys}}, m_b^{\mathrm{phys}}$
- $N_f = 1 + 1 + 1 + 1 + 1 + 1$

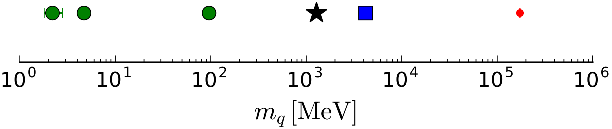
$\Rightarrow$ Need to control all limits!

$\rightarrow$ particularly simultaneously control FV and discretisation

Multiple scale problem on the lattice: back of the envelope

Control effects of IR (finite volume) and UV (discretisation) regulators:

$$m_\pi L \gtrsim 4 \qquad\qquad a^{-1} \gg \text{Mass scale of interest}$$



For $m_\pi = m_\pi^{\mathrm{phys}} \sim 140\,\mathrm{MeV}$ and $\overline{m}_b(\overline{m}_b) \approx 4.2\,\mathrm{GeV}$:

$$L \gtrsim 5.6\,\mathrm{fm} \qquad\qquad a^{-1} \gg 4.2\,\mathrm{GeV} \approx (0.05\,\mathrm{fm})^{-1}$$

Requires $N \equiv L/a \gg 120 \Rightarrow N^3 \times (2N) \gg 4 \times 10^8$ lattice sites.

VERY EXPENSIVE to satisfy both constraints simultaneously...

... needs to be repeated for different values of a.

How to simulate the b-quark?

Many different actions which differ in

- computational cost
- chirality
- tuning errors
- systematic errors
- cut off effects
- renormalisation

For now choose between:

Effective action for b

- Can tune to m_b
- comes with systematic errors which are hard to estimate/reduce

Relativistic action for b

- Theoretically cleaner and systematically improvable
- Need to control extrapolation in heavy quark mass

How to simulate the b-quark?

Many different actions which differ in

- computational cost
- tuning errors
- cut off effects
- chirality
- systematic errors
- renormalisation

For now choose between:

Effective action for b

- Static quarks
- Non-Relativistic QCD
- Fermilab action
- Relativistic heavy quarks

Relativistic action for b

- Wilson, twisted mass
- Domain Wall Fermions
- Overlap
- Staggered (asqtad, HISQ)

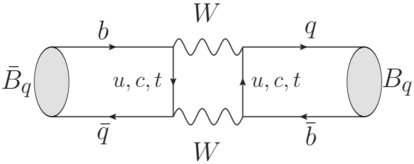
BUT SOON:

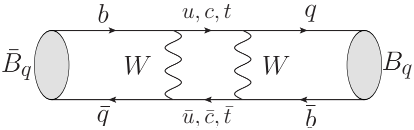
Huge efforts in the community to produce **very fine lattice spacings**:

$\Rightarrow$ **Direct simulation of $\approx m_b^{\text{phys}}$ will become possible!**

Neutral $B_{(s)}$ meson mixing - background

Neutral mesons oscillate:





where $q = d, s$

mass eigenstate $\neq$ flavour eigenstate

$$|B_{L,H}\rangle = p\,|B_q^0\rangle \pm q\,|\bar{B}_q^0\rangle$$

- $\Delta m_q \equiv m_H - m_L$
- $\Delta\Gamma_q \equiv \Gamma_L - \Gamma_H$
- $\Gamma_q \equiv (\Gamma_L + \Gamma_H)/2$

Time dependence:

$$|B_q^0(t)\rangle = g_+(t)\,|B_q^0\rangle + \frac{q}{p} g_-(t)\,|\bar{B}_q^0\rangle$$

$$|\bar{B}_q^0(t)\rangle = g_+(t)\,|\bar{B}_q^0\rangle + \frac{p}{q} g_-(t)\,|B_q^0\rangle$$

Neutral $B_{(s)}$ Meson Mixing - experiment

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos\left(\Delta m_q t\right)\right]$$

Δm can be measured very precisely as a frequency!

B_d^0: Many results

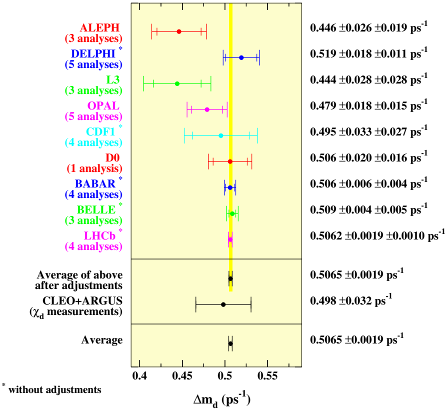
B_s^0: "Only" CDF and LHCb

$$\Delta m_d = 0.5065(19)\mathrm{ps}^{-1}$$

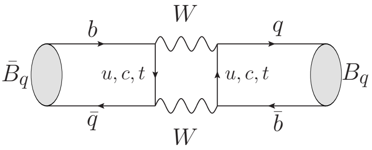
$$\Delta m_s = 17.757(21)\mathrm{ps}^{-1}$$

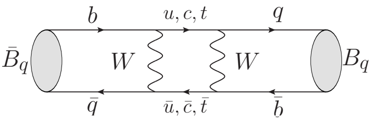
Well below per cent level!

[HFLAV]



Neutral $B_{(s)}$ Meson Mixing - theory





$$\Delta m \propto \underbrace{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=2} \middle| \bar{B}^0_{(s)} \right\rangle}_{\text{Short distance}} + \underbrace{\sum_n \frac{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=1} \middle| n \right\rangle \left\langle n \middle| \mathcal{H}^{\Delta b=1} \middle| \bar{B}^0_{(s)} \right\rangle}{E_n - M_{B_{(s)}}}}_{\text{Long distance}}$$

$$\text{short distance} \propto \left| \sum_{q'=u,c,t} \frac{m^2_{q'}}{M^2_W} V_{q'b} V^*_{q'q} \right|^2 \approx \frac{m^4_t}{M^4_W} \left| V_{tb} V^*_{tq} \right|^2$$

SD: Top enhanced: $m_t^2 V_{tb} V_{tq}{}^* \gg m_c^2 V_{cb} V^*_{cq} \gg m_u^2 V_{ub} V^*_{uq}$
LD: Only m_c, m_u in intermediate states: no top + CKM suppressed
$\Rightarrow$ **Short distance dominated** $\Rightarrow$ Can do it on the lattice!

Operator Product Expansion

$\Lambda_{\rm QCD} \sim 1\,\mathrm{GeV} \ll m_{EW} \sim 100\,\mathrm{GeV} \Rightarrow$ OPE factorises this into

- Perturbative model-dependent Wilson coefficients $C_i(\mu)$
- **Non-perturbative model-independent matrix elements**

$$\left\langle B^0_{(s)} \right| \mathcal{H}^{\Delta b=2} \left| \bar{B}^0_{(s)} \right\rangle = \sum_i C_i(\mu) \left\langle B^0_{(s)} \right| \mathcal{O}_i^{\Delta b=2}(\mu) \left| \bar{B}^0_{(s)} \right\rangle$$

- 5 independent (parity even) operators $\mathcal{O}_i$.
- Only $\mathcal{O}_1$ is relevant for Δm:

$$\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$$

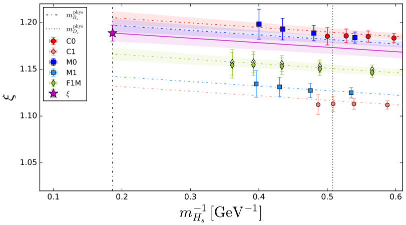
- Define bag parameters: $B_i = \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle / \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle_{VSA}$

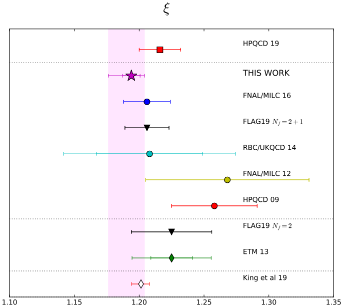
$$\Delta m_P = |V^*_{tb} V_{tq}| \times f_P^2 \hat{B}_P \times m_P \frac{G_F^2 m_W^2}{6\pi^2} \mathcal{K}$$

Computing ξ gives access to $|V_{td}/V_{ts}|$

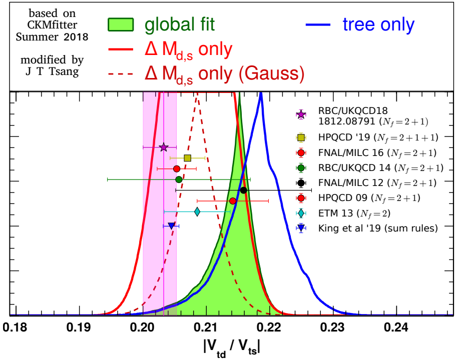
$$\xi^2 \equiv \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_B^2 \hat{B}_B} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}}$$

Determination of V_{ts}/V_{td} [arXiv:1812.08791]





- $N_f = 2+1$, DWF for all flavours.
- 3 lattice spacings inc. 2 $m_\pi^{\rm phys}$
- benign extrapolation $m_h \to m_b^{\rm phys}$

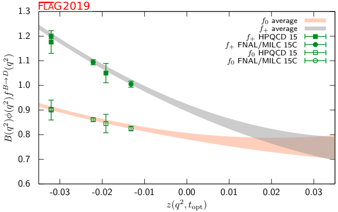


$\Rightarrow$ Update with $m_h \sim m_b^{\rm phys}$ soon

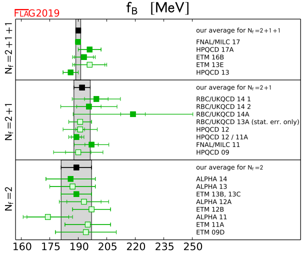
$\Rightarrow$ Full operator basis

Semileptonic decays: Overview

Semileptonic form factor $f^{B\to D}$



Leptonic decay constants f_B



(Some) tree-level decays $[+\ell\nu]$

Pseudoscalar to pseudoscalar

- $b\to u$: $(B\to\pi,\ B_s\to K)$
- $b\to c$: $(B\to D,\ B_s\to D_s)$

Pseudoscalar to vector

- $b\to c$: $(B\to D^*)$

(Some) loop-level decays $[+\ell^+\ell^-]$

Pseudoscalar to pseudoscalar

- "$b\to d$": $(B\to\pi)$
- "$b\to s$": $(B\to K)$

Pseudoscalar to vector

- "$b\to s$": $(B\to K^*)$

Many processes but comparably few results

Semileptonic decays: form factors, PS vs V

Note: Pseudoscalars (PS) are QCD-stable, Vectors (V) are QCD-unstable

✓ Pseudoscalar to pseudoscalar at tree-level

- 2 form factors: f_+ and f_0

(✓) Pseudoscalar to pseudoscalar at loop-level ("rare decays")

- 3 form factors: f_+, f_0 and f_T
- Fewer results than at tree-level

(✗) Pseudoscalar to vector at tree-level

- 4 form factors: V, A_0, A_1, A_2
- $1 \to 2$ transitions (e.g. $D^* \to D\pi$) understood on the lattice, but more involved and technical
- In current studies V are treated as QCD-stable .

✗ Pseudoscalar to vector at loop-level ("rare decays")

- 7 form factors: V, A_0, A_1, A_2, T_1, T_2, T_3
- Single unquenched result for $B \to K^*\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$ treating V as stable [PRD**89** 094501 (2014)]

Semileptonic decays: q^2-coverage

For generic semileptonic $P \to D$ decay we are interested in $f(q^2)$ for

$$q^2 = (E_P - E_D)^2 - (\vec{p}_P - \vec{p}_D)^2 \in [0, (m_P - m_D)^2]$$

Finite volume $\Rightarrow$ discrete momenta: $\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$.

$q^2(|\vec{p}|))$ [in GeV^2]

process	$q^2_{\max}$	$\lvert\vec{p}\rvert = 0.5$	$\lvert\vec{p}\rvert = 1.0$
$D \to \pi$	3.0	1.6	-0.3
$B \to \pi$	26.5	22.4	17.2
$B \to D$	11.6	10.9	9.0

$|\vec{p}|$ s.t. $q^2(|\vec{p}|) = 0$ [in GeV]

$q^2 = 0$	$D \to \pi$	$B \to \pi$	$B \to D$
$\lvert p\rvert$	0.9	2.6	2.3

- Work in B rest-frame
- Cut-off effects grow as $(ap)^n$
- Noise grows as $|\vec{p}|$ increases
- Covering q^2 becomes harder as
  - $m_P - m_D$ becomes larger
  - m_P becomes heavier

$\Rightarrow$ Lattice is most precise near $q^2_{\max}$, experiment for $q^2 \ll q^2_{\max}$.

$\Rightarrow$ **Not possible to cover full kinematic range at physical masses!**

Extrapolating over the full kinematic range: z-expansion

- Data typically limited to $q^2 \in [q^2_{\mathrm{min,sim}}, q^2_{\mathrm{max}}]$.
- Want form factors over **full** range $[0, q^2_{\mathrm{max}}]$.
- Map $q^2 \in [0, q^2_{\mathrm{max}}]$ to $z \in [z_{\mathrm{min}}, z_{\mathrm{max}}]$ with $|z| < 1$ and branch cut t_+.

$$z(q^2; t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- Form factor is a polynomial in z after poles have been removed

BGL: Boyd, Grinstein, Lebed [PRL **74** 4603]:

$$f_X(q^2) = \left(\prod_{\mathrm{poles}} \frac{1}{B_X(q^2)}\right) \frac{1}{\phi_X(q^2, t_0)} \sum_{n \geq 0} a_n(t_0) z^n$$

BCL: Bourrely, Lellouch, Caprini [PRD **82** 099902]:

$$f^{BCL}(q^2) = \left(\prod_{\mathrm{poles}} \frac{1}{1 - q^2/m^2_{\mathrm{pole}}}\right) \sum_{k \geq 0} b_k(t_0) z^k$$

Two methods to extrapolate to $q^2 = 0$

- Lattice data at $a \neq 0$, discrete $\vec{p}$ and typically $m_q \neq m_q^{\rm phys}$ and $q^2 \neq 0$.
- Lattice best near $q^2_{\rm max}$, experiment best for $q^2 \ll q^2_{\rm max}$

"Two-step"

1. **"Lattice to continuum"**
   - $a \to 0$, $m_q \to m_q^{\rm phys}$
   - continuous $E(|\vec{p}|)$ or q^2
   - Assemble full error budget
   - Choose representative $q^2_{\rm ref}$. Provide form factors $(q^2_{\rm ref})$ including **all** correlations
2. **"Continuum analysis"**
   Carry out model independent z-expansion over full range

"Modified z-expansion"

$a_n \to a_n(a, m_q)$ in the z-expansion and simultaneously do lattice and kinematic extrapolations.

energy. Because the modified z-expansion is not derived from an underlying effective field theory, there are several potential concerns with this approach that have yet to be studied. The most significant is that there is no theoretical derivation relating the coefficients of the modified z-expansion to those of the physical coefficients measured in experiment; it therefore introduces an unquantified model dependence in the form-factor shape. As a result, the applicability of uni-

But FLAG advises caution [FLAG2019]

Would be good to have a side-by-side comparison

$B \to \pi \ell \nu$ - literature

Published (within recent years)

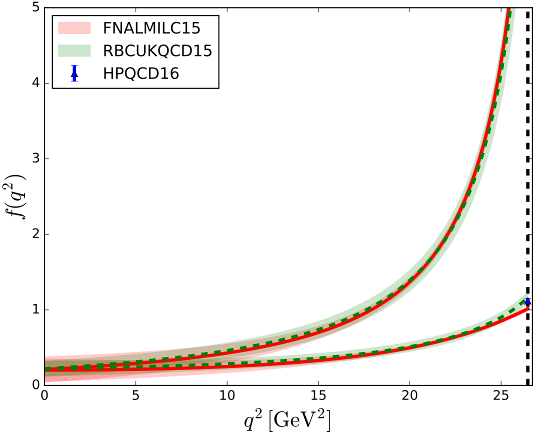
- RBC/UKQCD'15 [PRD **91** 074510]
  $N_f = 2+1$ Domain Wall Fermions (light) + RHQ bottom. $f_0(q^2)$ and $f_+(q^2)$ for $q^2 \in [19\,\mathrm{GeV}^2, q^2_{\mathrm{max}}]$ followed by BCL z-expansion
- FNAL/MILC'15 [PRD **92** 014024]
  $N_f = 2+1$ asqtad (light) + Fermilab (bottom). $f_0(q^2)$ and $f_+(q^2)$ for $q^2 \in [19.8\,\mathrm{GeV}^2, q^2_{\mathrm{max}}]$ followed by BCL z-expansion
- HPQCD'16 [PRD **93** 034502]
  $N_f = 2+1+1$, HISQ (light) + NRQCD (bottom). Only $f_0(q^2_{\mathrm{max}})$

Works in progress

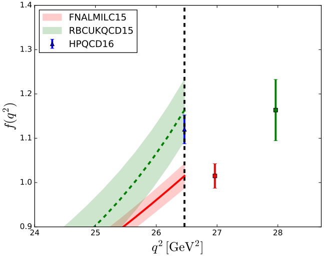
- RBC/UKQCD update with 3^{rd} lattice spacing **soon** [talk by R. Hill @ APLAT'20]
- JLQCD with $N_f = 2+1$, DWF (light and bottom) [PoS LATTICE2019 (2019) 143]
- FNAL/MILC with $N_f = 2+1+1$, HISQ (light) and Fermilab (bottom) [PoS LATTICE2019 (2019) 236]

$B \to \pi \ell \nu$ - results

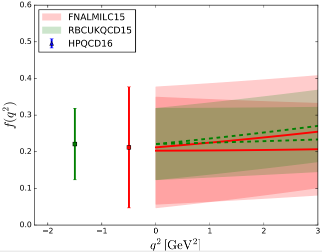
$f_+(q^2)$ (upper bands), $f_0(q^2)$ (lower bands)



$f_0(q^2_{\rm max})$



$f_0(q^2 = 0) = f_+(q^2 = 0)$



Combining with experimental data yields

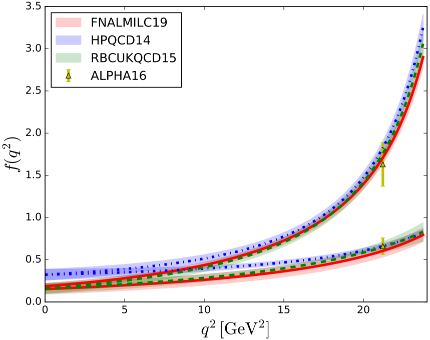
$\Rightarrow |V_{ub}| = 3.74(14) \times 10^{-3}$ [FLAG Review 2019]

$B_s \to K\ell\nu$ - literature

Published

- HPQCD'14 [PRD **90** 054506 and re-analised in HPQCD'18 PRD **98** 114509]
  $N_f = 2+1$ asqtad (light) + NRQCD (bottom). Modified BCL z-expansion, (single fit accounts for extrapolations: $a \to 0$; $m_l \to m_l^{\rm phys}$; kinematic range). Data in $q^2 \in [17\,\mathrm{GeV}^2, q^2_{\rm max}]$.
- RBC/UKQCD'15 [PRD **91** 074510]
  $N_f = 2+1$ Domain Wall Fermions (light) + RHQ bottom. $f_0(q^2)$ and $f_+(q^2)$ for $q^2 \in [17.6\,\mathrm{GeV}^2, q^2_{\rm max}]$ followed by BCL z-expansion
- ALPHA'16 [PLB **757** 473]
  $N_f = 2$ Wilson (light) + static (bottom). f_0, f_+ at $q^2 = 21.22\,\mathrm{GeV}^2$, no chiral extrapolation
- FNAL/MILC'19 [PRD **100** 034501]
  $N_f = 2+1$ asqtat (light) + Fermilab (bottom). $f_0(q^2)$ and $f_+(q^2)$ for $q^2 \in [17\,\mathrm{GeV}^2, q^2_{\rm max}]$ followed by BCL z-expansion

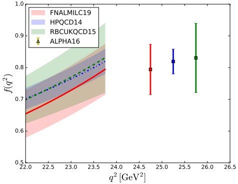
$B_s \to K\ell\nu$ - results



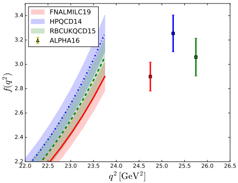
In progress:

- RBC/UKQCD'20 (SOON): Error at $q^2 \sim 21\,\mathrm{GeV}^2$
  f_0: 6.7% $\to$ 4.0%
  f_+: 5.5% $\to$ 4.0%
  [JTT talk at APLAT'20]
- FNAL/MILC with $N_f = 2+1+1$, HISQ (light) and Fermilab (bottom) [PoS LATTICE2019 (2019) 236]

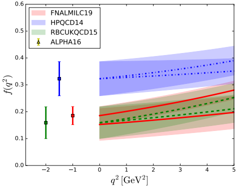
$f_0(q^2_{\mathrm{max}})$



$f_+(q^2_{\mathrm{max}})$

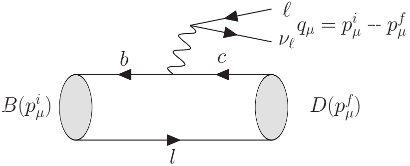


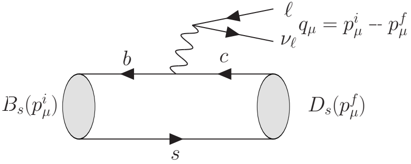
$f_{+/0}(q^2 = 0)$



Remark on $B \to D^{(*)}\ell\nu$ vs. $B_s \to D_s^{(*)}\ell\nu$

To determine $|V_{cb}|$ we are interested in a $b \to c$ transitions.





- Only spectator quark differs
  $\Rightarrow$ B_s complimentary to B decays
  $\Rightarrow$ $R(D_s^{(*)})$ good proxy for $R(D^{(*)})$?
- strange quarks are easier to deal with on the lattice:
  $\Rightarrow$ statistically cleaner
  $\Rightarrow$ computationally cheaper
- When $m_\pi^{\mathrm{sim}} \neq m_\pi^{\mathrm{phys}}$
  $\Rightarrow$ chiral extrapolation only sea-quark effects:

$\Rightarrow$ $B_s \to D_s^{(*)}$ ideal testing ground for $B \to D^{(*)}$

$B \to D\ell\nu$ - literature

Additional difficulty since charm is neither heavy nor light

Published $B \to D$

- FNAL/MILC'15 [PRD **92** 034506]
  $N_f = 2+1$ asqtad (light) + Fermilab (charm and bottom). $f_0(q^2)$ and $f_+(q^2)$ for $q^2 \in [8.5\,\mathrm{GeV}^2, q^2_{\mathrm{max}}]$ followed by BGL z-expansion
- HPQCD'15 [PRD **92** 054510]
  $N_f = 2+1$ asqtad (sea). HISQ (light+charm) + NRQCD (bottom). Modified BCL z-expansion, (single fit accounts for extrapolations: $a \to 0$; $m_l \to m_l^{\mathrm{phys}}$; kinematic range). Data in $q^2 \in [9.5\,\mathrm{GeV}^2, q^2_{\mathrm{max}}]$.

Works in progress

- RBC/UKQCD $N_f = 2+1$, DWF (l+c), RHQ (bottom) [PoS Lattice2019 (2019) 184]
- JLQCD with $N_f = 2+1$, DWF (all quarks) [PoS LATTICE2019 (2019) 139]

$B_s \to D_s \ell \nu$ - literature

Published $B_s \to D_s$

- FNAL/MILC'12 [PRD **92** 034506] $N_f = 2+1$ asqtad (light) + Fermilab (charm and bottom). $f_0(q^2)$ and $f_+(q^2)$ for $q^2 \in [8.5\,\mathrm{GeV}^2, q^2_{\mathrm{max}}]$ followed by BGL z-expansion
- HPQCD'17 [PRD **95** 114506] $N_f = 2+1$ asqtad (sea). HISQ (strange+charm) + NRQCD (bottom). Modified BCL z-expansion, (single fit accounts for extrapolations: $a \to 0$; $m_l \to m_l^{\mathrm{phys}}$; kinematic range). Data in $q^2 \in [9.5\,\mathrm{GeV}^2, q^2_{\mathrm{max}}]$.
- HPQCD'20 [PRD **101** 074513] $N_f = 2+1+1$ HISQ. HISQ (l,s,c,b). Modified z-expansion. Extrapolation from $m_b < m_b^{\mathrm{phys}}$. Data over full q^2

Works in progress

- RBC/UKQCD'20 $N_f = 2+1$, DWF (s+c), RHQ (bottom) publication in preparation. Expect $\sim 4\%$ uncertainty on form factors.

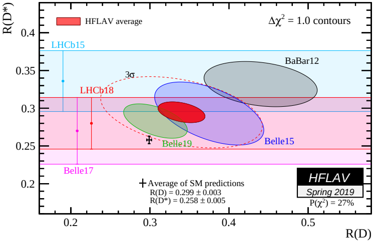
Lepton flavour universality tests

Recall

$$R^{\tau/\mu}(D_{(s)}) = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \frac{d\Gamma}{dq^2}(B_{(s)} \to D_{(s)}\tau\nu_\tau)}{\int_{m_\mu^2}^{q^2_{\max}} dq^2 \frac{d\Gamma}{dq^2}(B_{(s)} \to D_{(s)}\mu\nu_\mu)}$$

where e.g. for $B \to D\ell\nu$

$$\frac{d\Gamma}{dq^2} = \frac{\eta_{EW}\,|V_{ub}|^2\, G_F^2\, |\vec{p}|}{24\pi^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2\left[\left(1+\frac{m_\ell^2}{2q^2}\right)|\vec{p}|^2\,|f_+(q^2)|^2+\frac{3(m_B^2-m_D^2)^2}{8q^2m_B^2}m_\ell^2|f_0(q^2)|^2\right]$$



$R(D)$	0.299(11)	FNAL/MILC'15
$R(D)$	0.300(8)	HPQCD'15
$R(D_s)$	0.301(6)	HPQCD'17
$R(D_s)$	0.2993(46)	HPQCD'20

- $R(D_s)$ is a good proxy for $R(D)$
- RBC/UKQCD'20 in preparation

No result for $B_{(s)} \to D^*_{(s)}$ yet away from $q^2_{\max}$ but **many ongoing efforts**!

Improving lepton flavour universality tests

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \underbrace{\frac{\eta_{EW}\,|V_{qb}|^2\,G_F^2\,|\vec{p}|}{24\pi^3}}_{\Phi(q^2)} \underbrace{\left(1-\frac{m_\ell^2}{q^2}\right)^2 \left[\left(1+\frac{m_\ell^2}{2q^2}\right)\right.}_{\omega_\ell} \underbrace{|\vec{p}|^2\,|f_+(q^2)|^2}_{F_V^2} + \left.\underbrace{\frac{3(m_B^2-m_P^2)^2}{8q^2m_B^2}m_\ell^2|f_0(q^2)|^2}_{(F_S^\ell)^2/(1+m_\ell^2/2q^2)}\right]$$

can be rewritten so that only ω_ℓ and F_S^ℓ depend on the lepton mass, i.e.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \Phi(q^2)\omega_\ell\left[F_V^2 + (F_S^\ell)^2\right]$$

Define $R_{\mathrm{opt}}^{\tau/\mu}(P;q^2_{\mathrm{min}})$ as (changes in red) [Flynn, Soni et al., motivated by Isidori, Sumensari'20]

$$R_{\mathrm{opt}}^{\tau/\mu}(D) \equiv \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \mathrm{d}q^2\,\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(B\to D\tau\nu_\tau)}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \mathrm{d}q^2\left[\frac{\omega_\tau(q^2)}{\omega_\mu(q^2)}\right]\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(B\to D\mu\nu_\mu)}$$

Noting that $(F_S^\ell)^2 \propto m_\ell^2 \sim 0$ for $\ell = \mu, e$ the SM prediction becomes

$$R^{\tau/\mu}(D) = \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \mathrm{d}q^2\Phi(q^2)\omega_\tau(q^2)\left[F_V^2+(F_S^\tau)^2\right]}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \mathrm{d}q^2\Phi(q^2)\omega_\tau(q^2)F_V^2} = 1 + \frac{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \mathrm{d}q^2\Phi(q^2)\omega_\tau(q^2)(F_S^\tau)^2}{\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \mathrm{d}q^2\Phi(q^2)\omega_\tau(q^2)F_V^2}$$

$\Rightarrow$ **Experiment and theory might profit from cancellations!**

Conclusions and Outlook

Experiment

- Two complimentary experiments with access to many decay channels

Status

- Effective actions for b
- $m_\pi > m_\pi^{\mathrm{phys}}$
- q^2 range requires modelling
- PS to PS: form factors under control (tree and loop)
- PS to V (tree): vector treated as QCD-stable, mostly q^2_{max}
- $|V_{ts}/V_{td}| = 0.2033(4)_{\mathrm{e}}(^{+16}_{-30})_{\mathrm{t}}$

Prospects

- Fully relativistic treatment of b
- $m_\pi = m_\pi^{\mathrm{phys}}$, $m_b \to m_b^{\mathrm{phys}}$
- covering q^2-range, test z-expansions
- PS to V (tree): q^2-range
- treatment of vector: known framework but challenging
- $|V_{ts}|, |V_{td}| \sim 1\%$

Lepton Flavour universality tests [experiment and theory]

- New insights from optimised R-ratio?

ADDITIONAL SLIDES

FLAG - Flavour Lattice Averaging Group

In any (research) community it is hard to assess results from the outside.

The Flavour Lattice Averaging Group (FLAG) aims to

- Summarise lattice results
- Assess their quality
- Provide averages of different results


FLAG 2019 Flavour Lattice Averaging Group

Figures for download
Quark masses
V_{ud} and V_{us}
Low-energy constants
Kaon mixing
D-meson decay constants and form factors
B-meson decay constants, mixing parameters, and form factors
The strong coupling α_s
Nucleon matrix elements

Navigation
RecentChanges
FindPage
HelpContents

FLAG Review 2019

The 2019 edition of the FLAG review can be downloaded here. The version dated March 4, 2020 is the one accepted for publication in EPJC. The EPJC version can be obtained from here.

The separate sections can be downloaded as separate pdf-files following the links in the table of contents below, or via the menu in the sidebar. Clicking on the FLAG logo in the upper left corner links back to this main page.

The latest figures can be downloaded in eps, pdf and png format, together with a bib-file containing the bibtex-entries for the calculations which contribute to the FLAG averages and estimates. The downloads are available via the menu in the sidebar.

In the notes we compile detailed information on the simulations used to calculate the quantities discussed in the review. Here we provide the complete tables, in contrast to the paper version of the review which contains this information only for results that have appeared since FLAG 16.

The original complete 2015/2016 review is still accessible from EPJC. The 2016/2017 updates are available from here. The 2013/2014 review is accessible here or from EPJC.

Contents

FLAG-webpage

$\Rightarrow$ This is a very useful and handy tool, BUT...

...**please cite the original papers** and not just FLAG.

...be aware that more recent results might not yet be included.