Charged current B anomalies with right-handed neutrino

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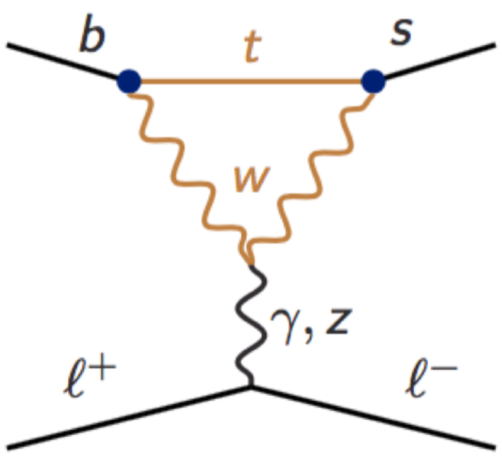
Outline

- Introduction
- Theoretical framework
- New physics effects
- Summary

Introduction

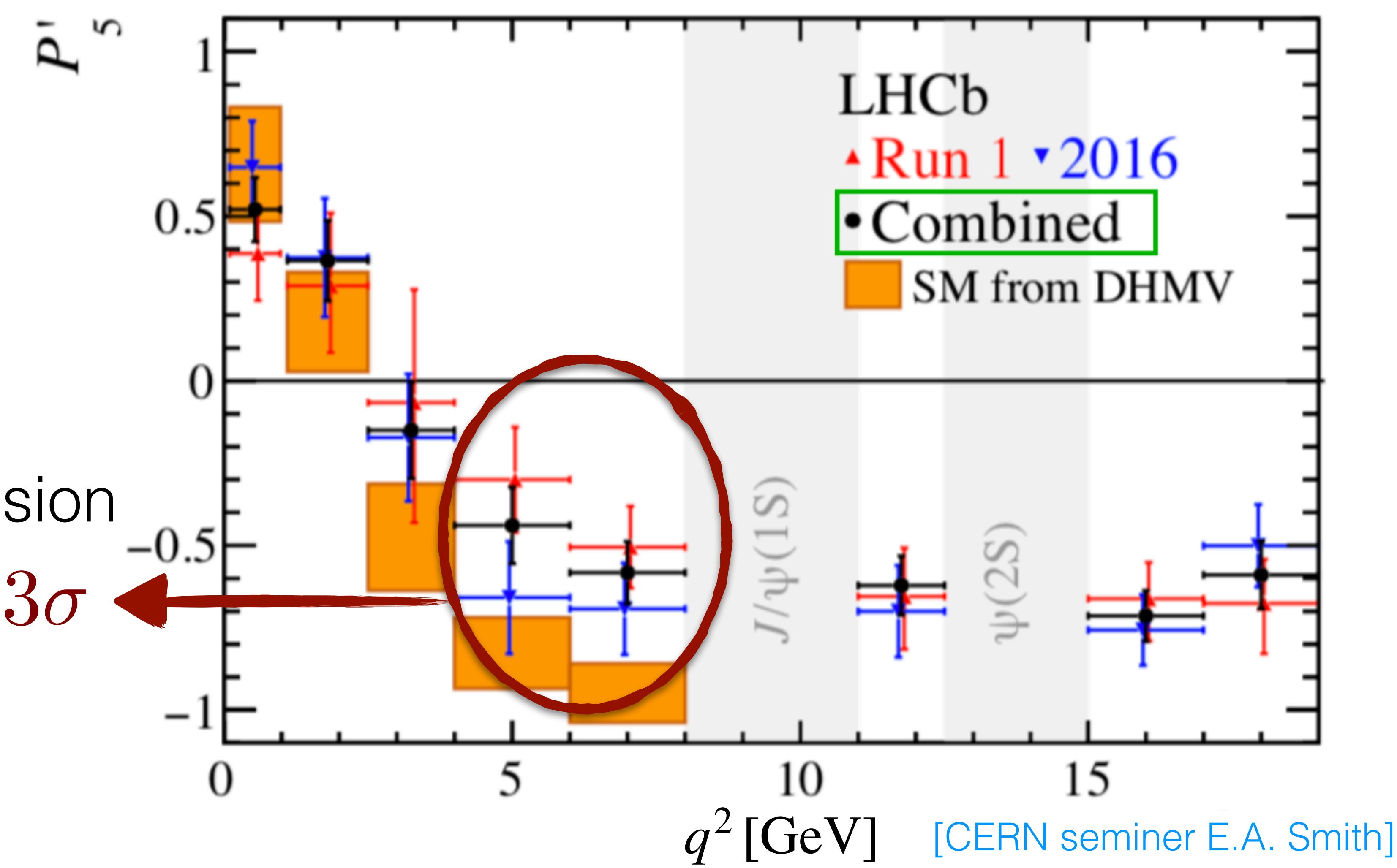
- Angular distribution of multi-body semileptonic decay is powerful tool to access observables in B-physics

- E.g., long-standing discrepancy in $B \to K^*(\to K\pi)\mu\mu$



Local tension

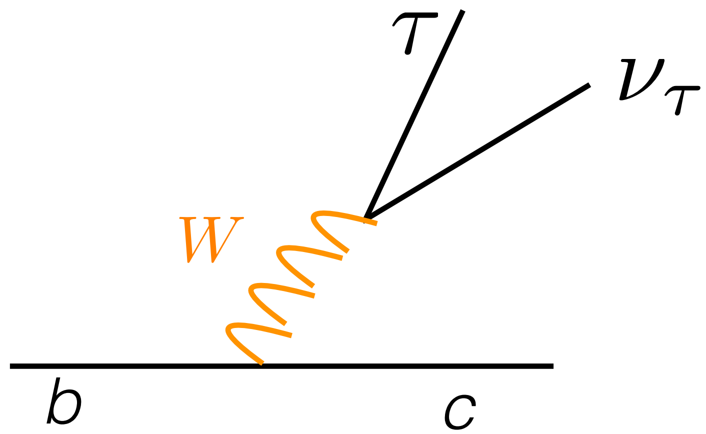
$\sim 2 - 3\sigma$



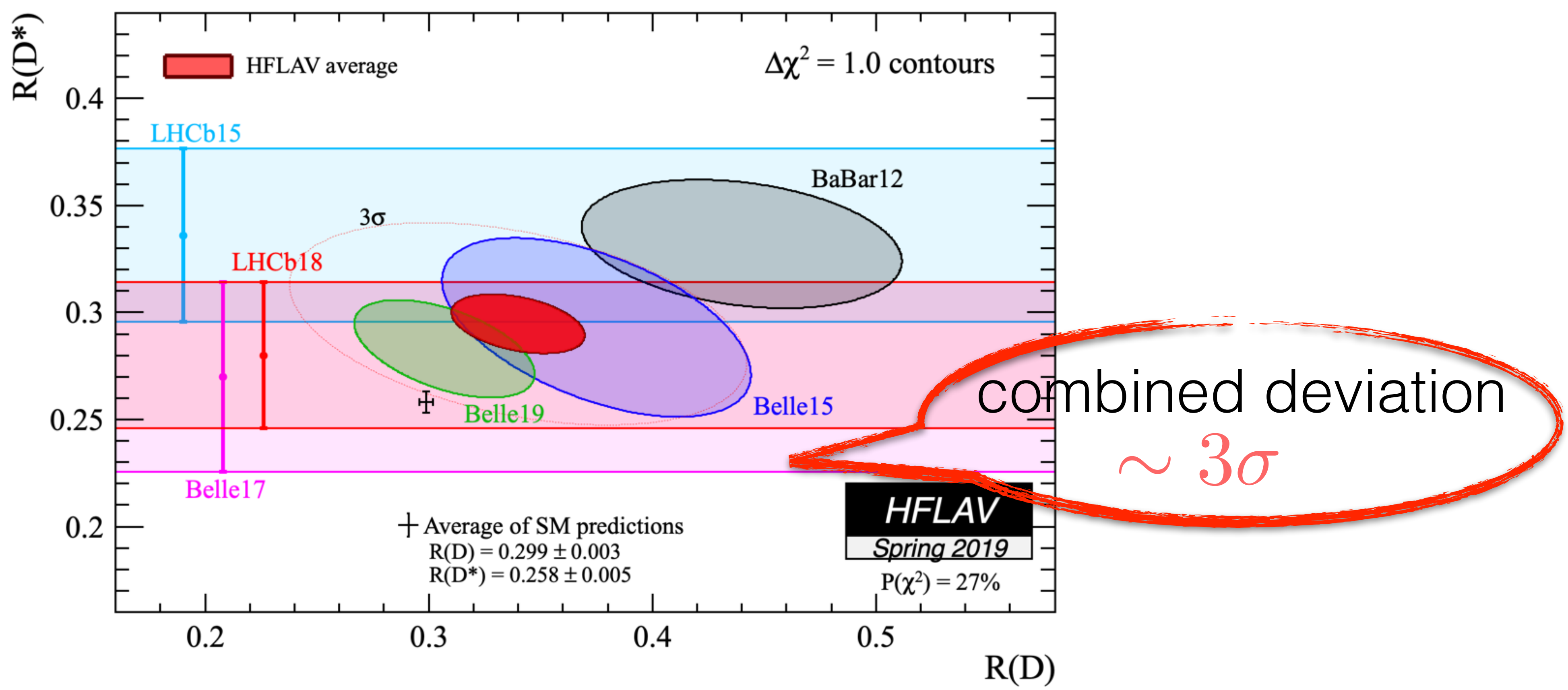
[CERN seminer E.A. Smith]

Motivation

- Exciting discrepancies observed in charged current B decays also



$$R(D^{(*)}) \equiv \frac{\mathrm{BR}(B \to D^{(*)}\tau\nu)}{\mathrm{BR}(B \to D^{(*)}\ell\nu)}\,, \quad \ell \in \{e, \mu\}$$



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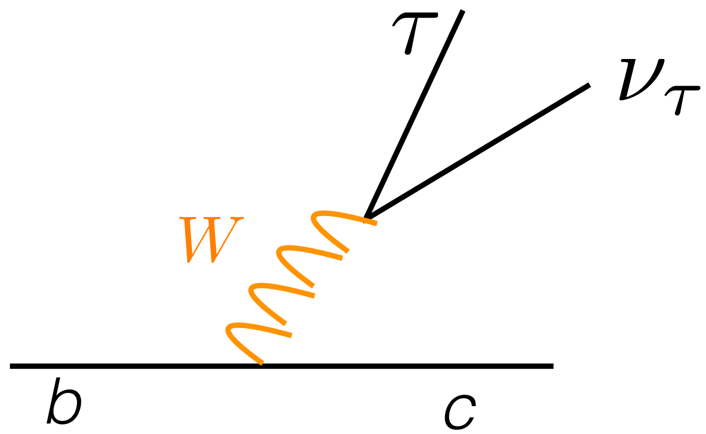
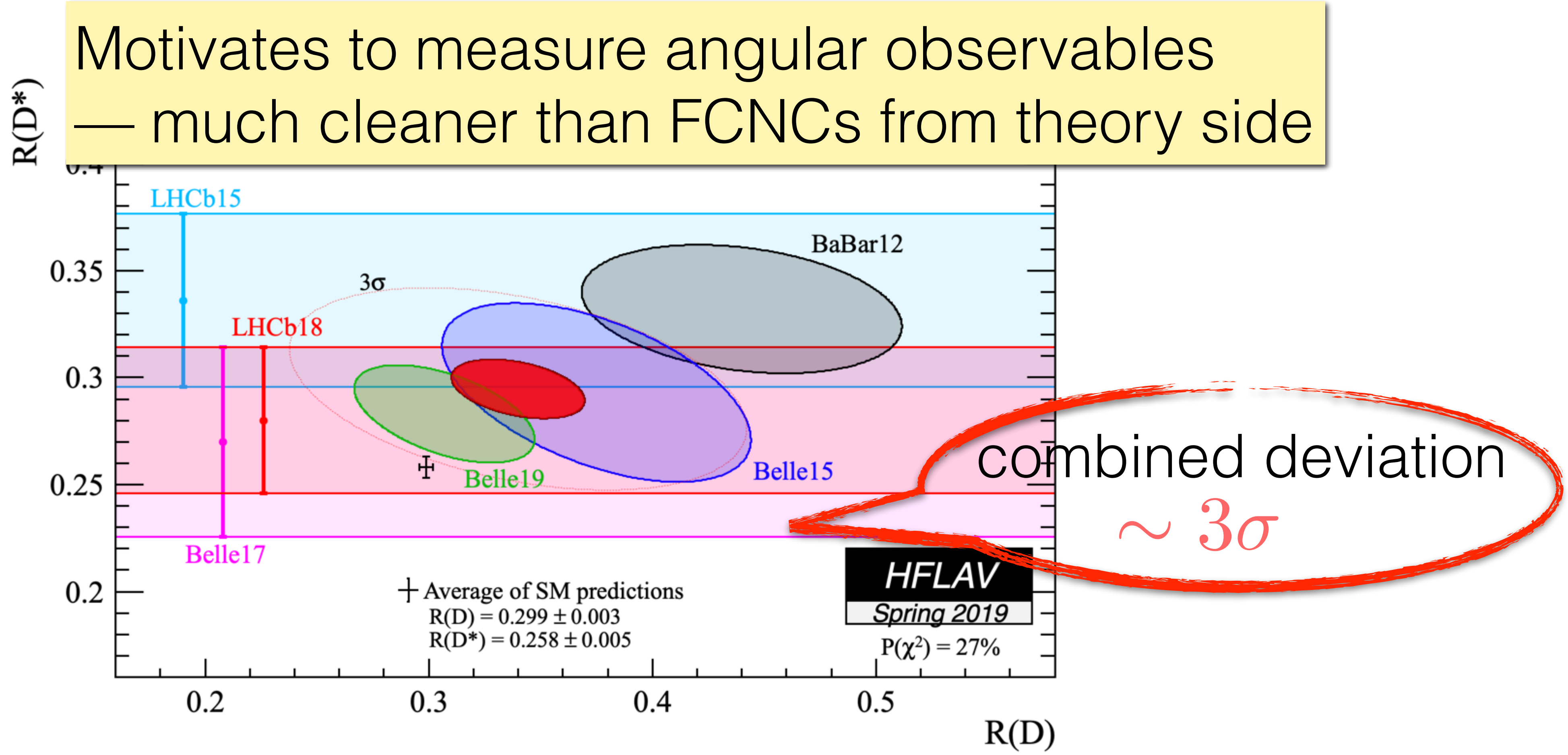


$$R(D^{(*)}) \equiv \frac{\mathrm{BR}(B \to D^{(*)}\tau\nu)}{\mathrm{BR}(B \to D^{(*)}\ell\nu)}\,, \quad \ell \in \{e, \mu\}$$

Motivates to measure angular observables — much cleaner than FCNCs from theory side



Hamiltonian

- Most general dim-6 BSM Hamiltonian for $b \to c\ell\bar{\nu}$

$$\mathcal{H}_{\rm eff} = \frac{4G_F V_{cb}}{\sqrt{2}} \Big\{ \mathcal{O}^V_{LL} + \sum_{\substack{X=S,V,T \\ M,N=L,R}} C^X_{MN} \mathcal{O}^X_{MN} \Big\}$$

Hamiltonian

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$$\mathcal{O}_{MN}^S \equiv (\bar{c} P_M b) \left(\bar{\ell} P_N \nu\right) ,$$
$$\mathcal{O}_{MN}^V \equiv (\bar{c} \gamma^\mu P_M b) \left(\bar{\ell} \gamma_\mu P_N \nu\right) ,$$
$$\mathcal{O}_{MN}^T \equiv (\bar{c} \sigma^{\mu\nu} P_M b) \left(\bar{\ell} \sigma_{\mu\nu} P_N \nu\right) .$$

→ Sandwiched between mesons form factors: non-perturbative

Hamiltonian

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Wilson coefficients:
perturbatively calculable

All $C^X_{MN} = 0$ in the SM

Simple dynamics

$$\mathcal{O}^S_{MN} \equiv (\bar{c} P_M b)\left(\bar{\ell} P_N \nu\right),$$
$$\mathcal{O}^V_{MN} \equiv (\bar{c}\gamma^\mu P_M b)\left(\bar{\ell}\gamma_\mu P_N \nu\right),$$
$$\mathcal{O}^T_{MN} \equiv (\bar{c}\sigma^{\mu\nu} P_M b)\left(\bar{\ell}\sigma_{\mu\nu} P_N \nu\right).$$

Sandwiched between mesons
form factors: non-perturbative

BSM physics induce new Wilson coefficients

Distribution

- Simpler for *D*: $$\frac{d\Gamma(B \to D\ell\bar{\nu})}{dq^2\, d\cos\theta_l} = J_0 + J_1\, \cos\theta_l + J_2\, \cos^2\theta_l$$

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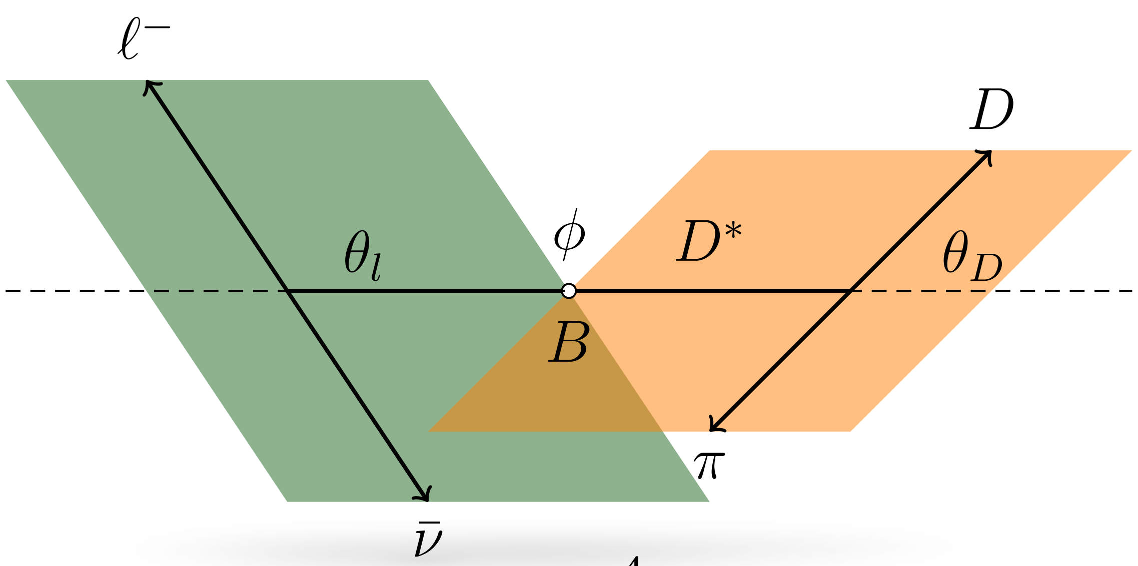
- $D^* \to D\pi$ induces two more angles:

[RM, Peñuelas, Murgui, Pich; *2004.06726*]



$$\frac{d^4\Gamma(B\to D^*(\to D\pi)\ell\bar{\nu})}{dq^2\, d\cos\theta_l\, d\cos\theta_D\, d\phi} =$$

$$\begin{aligned}
&\frac{9}{32\pi}\Big[I_1^c\, \cos^2\theta_D + I_1^s\, \sin^2\theta_D \\
&+ \left(I_2^c\, \cos^2\theta_D + I_2^s\, \sin^2\theta_D\right)\cos 2\theta_l \\
&+ I_3 \sin^2\theta_D \sin^2\theta_l \cos 2\phi \\
&+ I_4 \sin 2\theta_D \sin 2\theta_l \cos\phi \\
&+ I_5 \sin 2\theta_D \sin\theta_l \cos\phi \\
&+ \left(I_6^s \sin^2\theta_D + I_6^c \cos^2\theta_D\right)\cos\theta_l \\
&+ I_7\, \sin 2\theta_D \sin\theta_l \sin\phi \\
&+ I_8\, \sin 2\theta_D \sin 2\theta_l \sin\phi \\
&+ I_9\, \sin^2\theta_D \sin^2\theta_l \sin 2\phi\Big]
\end{aligned}$$

J, *I*'s ∝ NP + FF ⇨ measurable

Observables

- Helicity fractions

$$\frac{d^2\Gamma_{D^*}}{dq^2\, d\cos\theta_D} = \frac{3}{4}\left[F_T^{D^*}\sin^2\theta_D + 2\,F_L^{D^*}\cos^2\theta_D\right]\Gamma_f^{D^*} \qquad \Gamma_f^{D^{(*)}} \equiv d\Gamma^{D^{(*)}}/dq^2$$

- ϕ distribution:

$$\frac{d^2\Gamma}{dq^2\, d\phi} = \frac{1}{2\pi}\left[1 + A_3\cos 2\phi + A_9\sin 2\phi\right]\Gamma_f^{D^*}$$

- Lepton polarisation:

$$\mathcal{P}_\ell^{D^{(*)}} = \frac{d\Gamma^{D^{(*)}}_{\lambda_\ell=1/2}/dq^2 - d\Gamma^{D^{(*)}}_{\lambda_\ell=-1/2}/dq^2}{d\Gamma^{D^{(*)}}/dq^2}$$

Observables

- *CP* averaged asymmetries

$$A_{FB}^{D^{(*)}} = \frac{1}{\Gamma_f^{D^{(*)}}}\left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_l \, \frac{d^2(\Gamma^{D^{(*)}} - \bar{\Gamma}^{D^{(*)}})}{dq^2 d\cos\theta_l}$$

$$A_4 = \frac{1}{\Gamma_f^{D^*}}\left[\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2}\right] d\phi \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_D \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} + \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$$A_5 = \frac{1}{\Gamma_f^{D^*}}\left[\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2}\right] d\phi \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_D \int_{-1}^1 d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} - \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$$A_7 = \frac{1}{\Gamma_f^{D^*}}\left[\int_0^{\pi} - \int_{\pi}^{2\pi}\right] d\phi \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_D \int_{-1}^1 d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} + \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$$A_8 = \frac{1}{\Gamma_f^{D^*}}\left[\int_0^{\pi} - \int_{\pi}^{2\pi}\right] d\phi \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_D \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} - \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$A_{3,4,5}, A_{FB} \propto$ Real part of the amplitude

$A_{7,8,9} \propto$ Imaginary part → **Null tests of SM**

Status with LHN

Obs	Expt	Deviation	
R_D	[BaBar' *12*,' *13* Belle' *15*,' *19*]	1.4σ	3σ (combined R_D, R_{D^*})
R_{D^*}	[BaBar' *12*,' *13* Belle' *15*,' *17*,' *19* LHCb ' *15*,' *18*]	2.5σ	
$P_\tau^{D^*}$	[Belle' *16*, ' *17*]	$-$	
$F_L^{D^*}$	[Belle' *19*]	1.7σ	
$d\Gamma^{D^{(*)}}/dq^2$	[BaBar' *13* Belle' *15*]	$-$	

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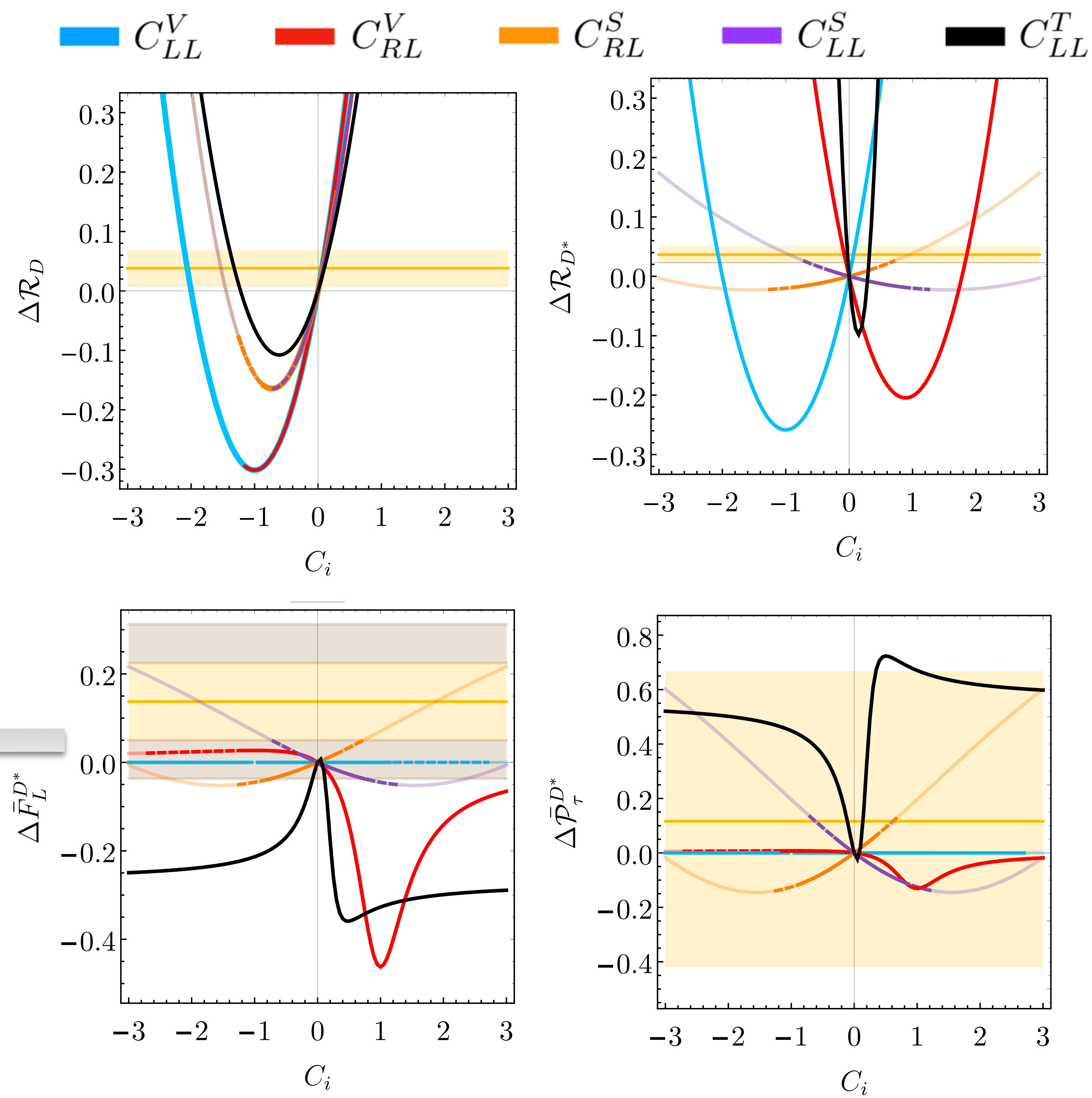
$P_\tau^{D^*}$	[Belle'16,'17]	–
$F_L^{D^*}$	[Belle'19]	1.7σ
$d\Gamma^{D^{(*)}}/dq^2$	[BaBar'13 Belle'15]	–

R_D and R_{D^*} combined: 3σ

Not compatible@1σ

$\mathscr{B}(B_c \to \tau\bar{\nu}) \leq 10 - 30\,\%$

forbids large scalar



Status with RHN

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R_D	[BaBar'*12*,'*13* Belle'*15*,'*19*]	1.4σ } 3σ
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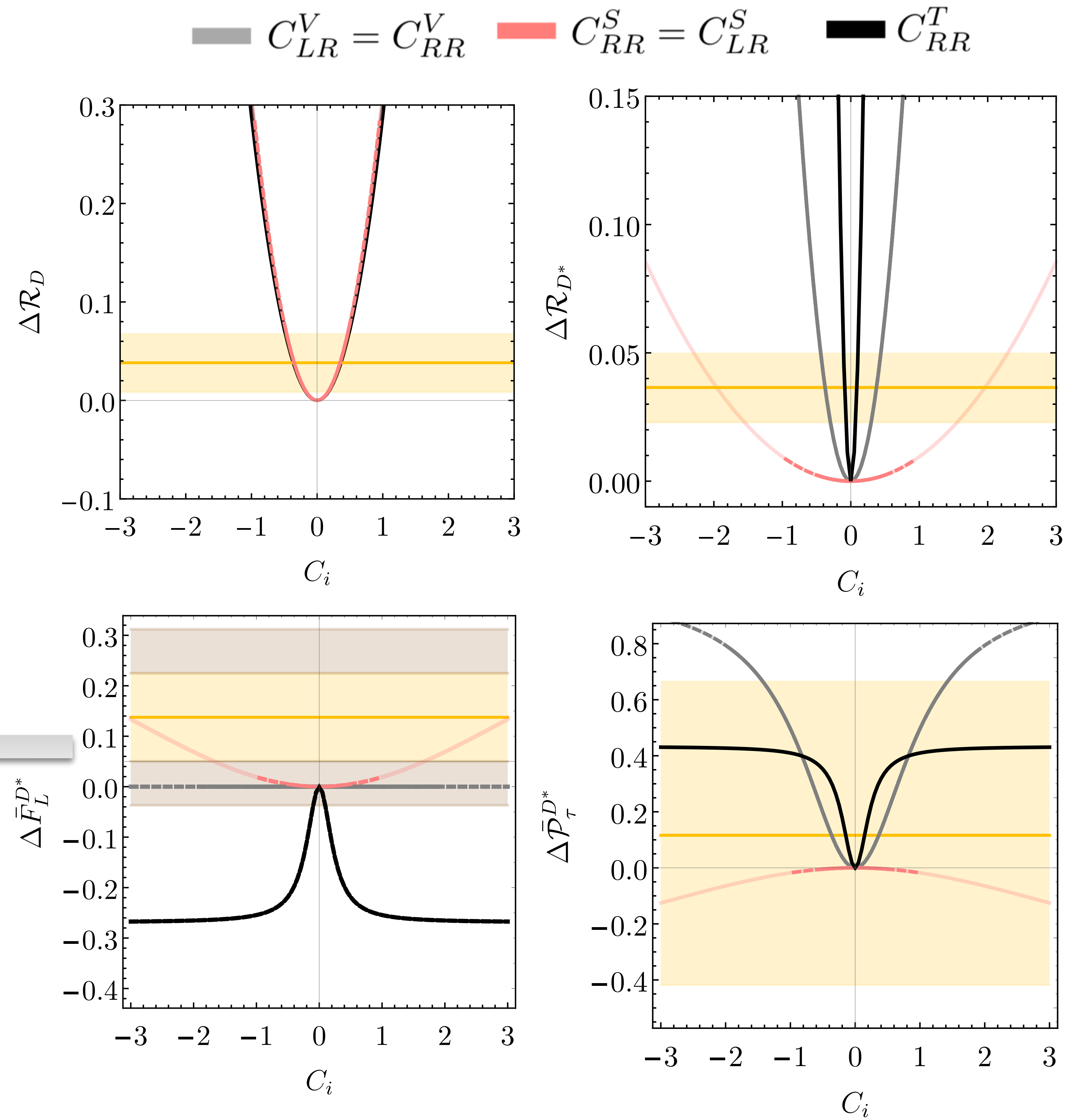
$P_\tau^{D^*}$	[Belle'*16*,'*17*]	–
$F_L^{D^*}$	[Belle'*19*]	1.7σ
$d\Gamma^{D^{(*)}}/dq^2$	[BaBar'*13* Belle'*15*]	–



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R_{D^*}	[BaBar'12,'13 Belle'15,'17,'19 LHCb '15,'18]	2.5σ
$P_\tau^{D^*}$	[Belle' [illegible]	
$F_L^{D^*}$	[Belle'19]	1.7σ
$d\Gamma^{D^{(*)}}/dq^2$	[BaBar'13 Belle'15]	–

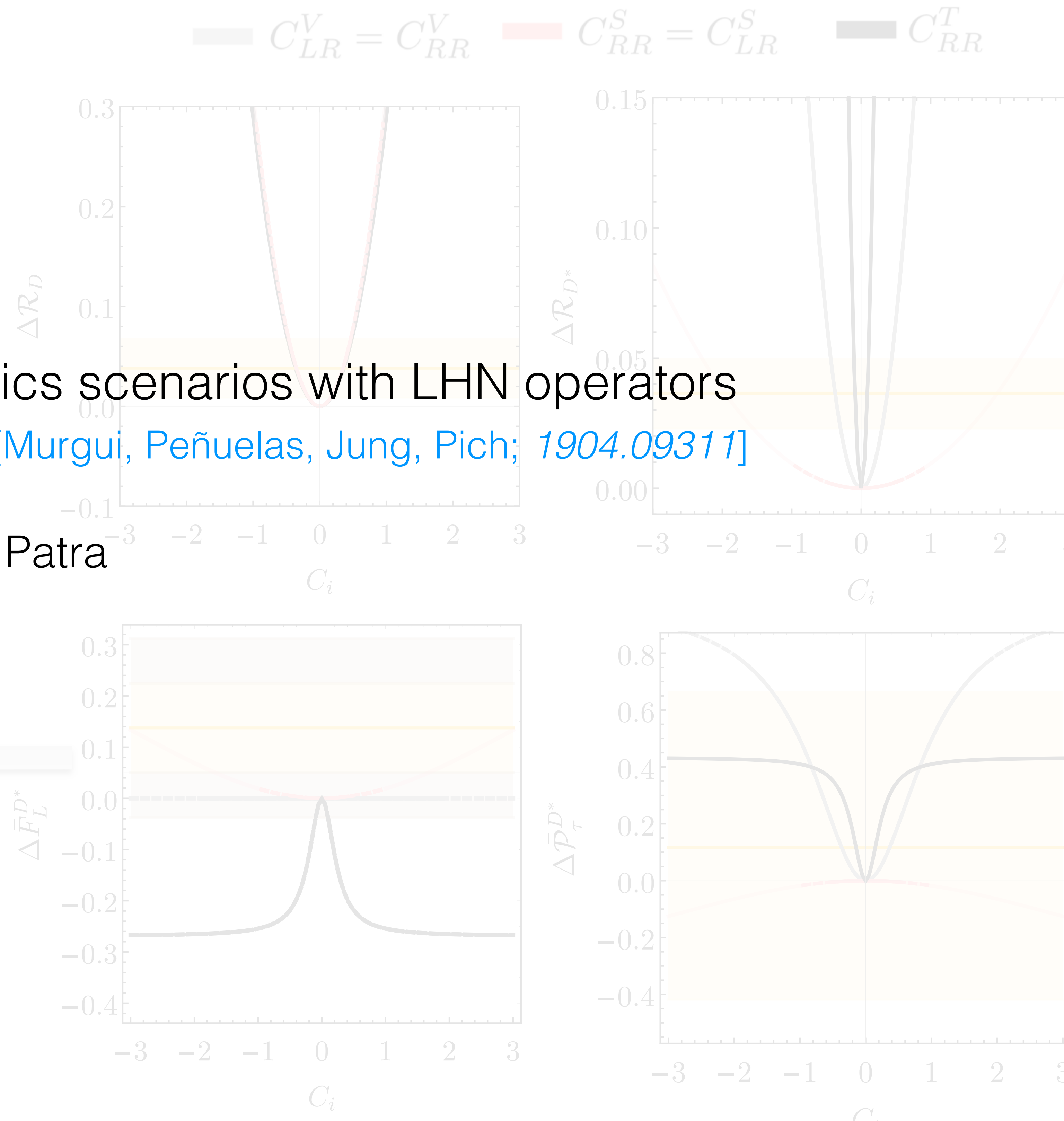
Details of new physics scenarios with LHN operators can be found in [Murgui, Peñuelas, Jung, Pich; *1904.09311*]

— see also talk by S. Patra

Not compatible@1σ

$\mathscr{B}(B_c \to \tau\bar{\nu}) \leq 10 - 30\,\%$

forbid large scalar

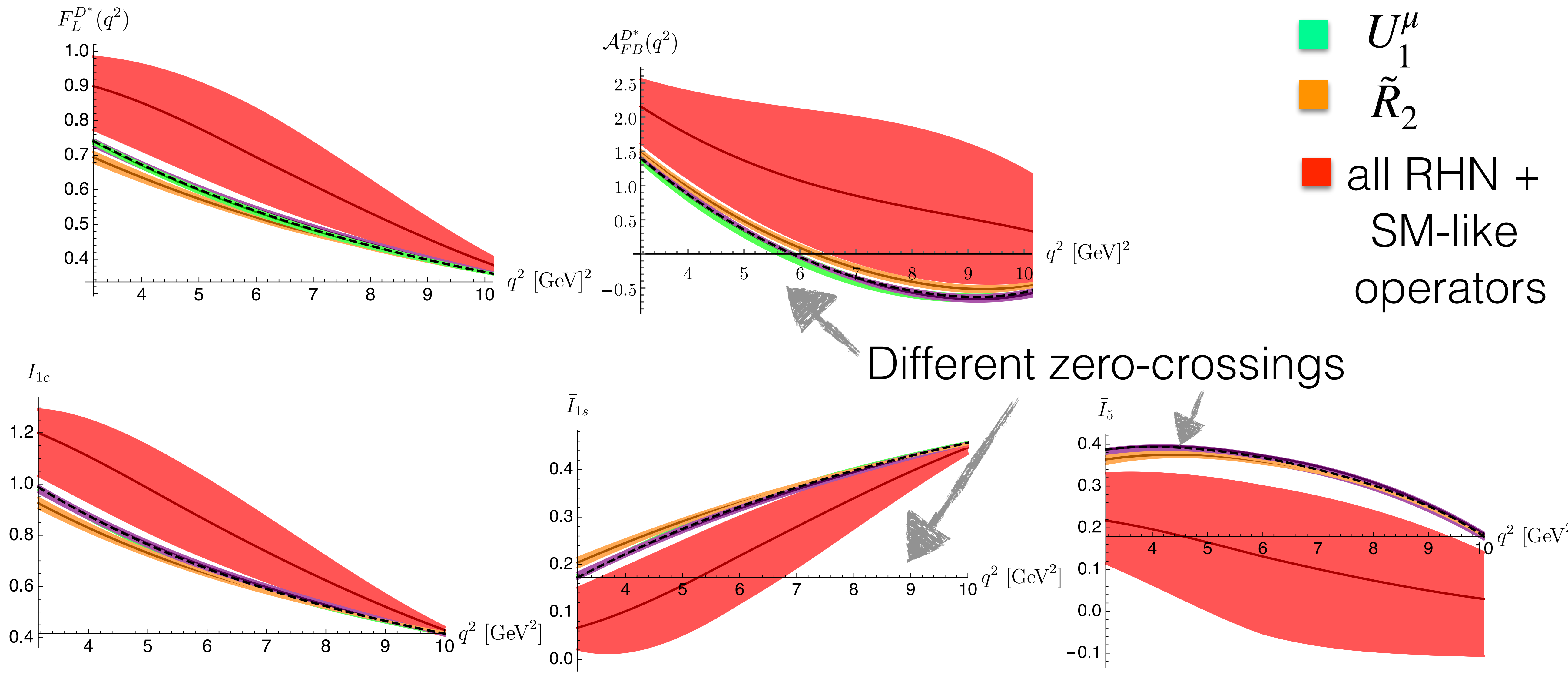


New Physics

- Fit to all measured observables in $B \to D^{(*)}\ell\bar{\nu}$ including differential BR in q^2 in EFT approach motivated by UV mediators

Mediators	Operators	Pull	R_D	R_{D*}	F_L^{D*}	P_τ^{D*}
	$\mathcal{O}_{LL}^{V}, \mathcal{O}_{LR}^{S,V,T}, \mathcal{O}_{RR}^{S,V,T}$	2.4	✓	✓	✓	✓
	$\mathcal{O}_{LR}^{S,V,T}, \mathcal{O}_{RR}^{S,V,T}$	2.5	✓	✓	✗	✓
$S_1(\bar{3}, 1, 1/3)$	$\mathcal{O}_{RR}^{S,V,T}, \mathcal{O}_{LL}^{S,V,T}$	3.3	✓	✓	✗	✓
$\tilde{R}_2(3, 2, 1/6)$	$\mathcal{O}_{RR}^{S,T}$	2.9	✓	✓	✗	✓
$U_1^\mu(3, 1, 2/3)$	$\mathcal{O}_{RR}^{V}, \mathcal{O}_{LR}^{S}, \mathcal{O}_{LL}^{V}, \mathcal{O}_{RL}^{S}$	2.6	✓	✓	✗	✓
$\tilde{V}_2^\mu(3, 2, -1/6)$	$\mathcal{O}_{LR}^{S}$	1.9	✓	✗	✗	✓
$V_\mu(1, 1, -1)$	$\mathcal{O}_{RR}^{V}$	3.7	✓	✓	✗	✓
$\phi(1, 2, 1/2)$	$\mathcal{O}_{XY}^{S}$	2.5	✓	✓	✓	✓

New physics



Easily distinguishable in various q^2 region

Crucial to identify NP mediators

Higher spin states

Properties	D^*	D_2^*
Spin	1^-	2^+
Mass (MeV)	2006	2461
Width (MeV)	< 2	47

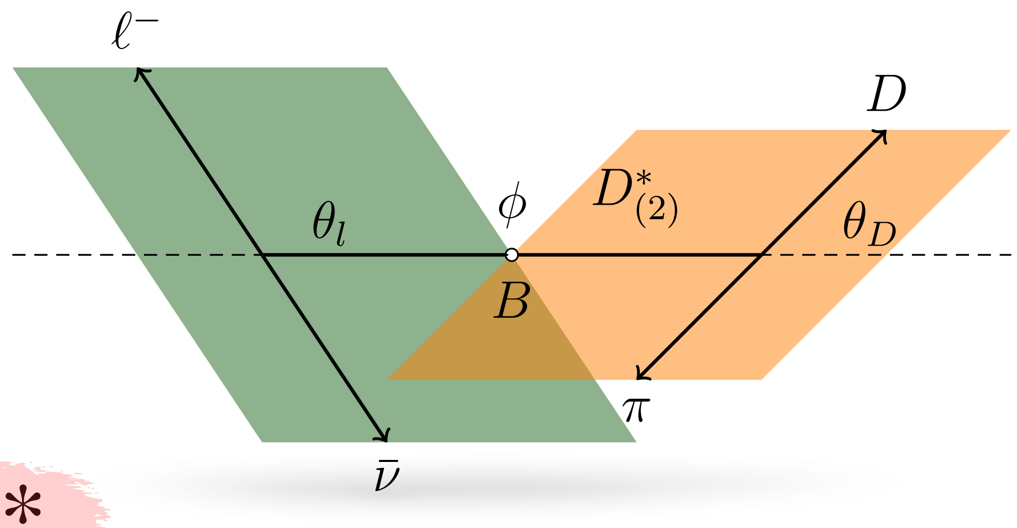
- Tensor mesons $D_2^*(2460)$ provides complementary information

[RM; *1912.03835*]

$\bar{B} \to D_2^*(\to D\pi)\ell\bar{\nu}$ ⇨ important background for $R(D^*)$

$\mathrm{BR} \simeq \mathcal{O}(10^{-3})$ [Belle, BaBar '08]

Higher spin states



$$\frac{d^4\Gamma}{dq^2 d\cos\theta_D d\cos\theta_l d\phi} =$$

D^*	D_2^*
$\frac{9}{32\pi}\Big[I_1^c \cos^2\theta_D + I_1^s \sin^2\theta_D$	$\frac{15}{128\pi}\Big[I_1^c(3\cos^2\theta_D - 1)^2 + 3I_1^s\sin^2 2\theta_D$
$+\left(I_2^c \cos^2\theta_D + I_2^s \sin^2\theta_D\right)\cos 2\theta_l$	$+\left(I_2^c(3\cos^2\theta_D - 1)^2 + 3I_2^s\sin^2 2\theta_D\right)\cos 2\theta_l$
$+I_3\sin^2\theta_D\sin^2\theta_l\cos 2\phi$	$+3I_3\sin^2 2\theta_D\sin^2\theta_l\cos 2\phi$
$+I_4\sin 2\theta_D\sin 2\theta_l\cos\phi$	$+2\sqrt{3}I_4(3\cos^2\theta_D - 1)\sin 2\theta_D\sin 2\theta_l\cos\phi$
$+I_5\sin 2\theta_D\sin\theta_l\cos\phi$	$+2\sqrt{3}I_5(3\cos^2\theta_D - 1)\sin 2\theta_D\sin\theta_l\cos\phi$
$+\left(I_6^s\sin^2\theta_D + I_6^c\cos^2\theta_D\right)\cos\theta_l$	$+\left(3I_6^s\sin^2 2\theta_D + I_6^c(3\cos^2\theta_D - 1)^2\right)\cos\theta_l$
$+I_7\sin 2\theta_D\sin\theta_l\sin\phi$	$+2\sqrt{3}I_7(3\cos^2\theta_D - 1)\sin 2\theta_D\sin\theta_l\sin\phi$
$+I_8\sin 2\theta_D\sin 2\theta_l\sin\phi$	$+2\sqrt{3}I_8(3\cos^2\theta_D - 1)\sin 2\theta_D\sin 2\theta_l\sin\phi$
$+I_9\sin^2\theta_D\sin^2\theta_l\sin 2\phi\Big]$	$+3I_9\sin^2 2\theta_D\sin^2\theta_l\sin 2\phi\Big]$

Higher spin states

- Easily distinguishable via uni-angular distribution in θ_D

$$\frac{d^2\Gamma_{D^*_{(2)}}}{dq^2\, d\cos\theta_D} = \begin{cases} \frac{3}{4}\left[F_T^{D^*}\sin^2\theta_D + 2\,F_L^{D^*}\cos^2\theta_D\right]\Gamma_f^{D^*} \\ \frac{5}{8}\left[F_L^{D_2^*} + 6(F_T^{D_2^*} - F_L^{D_2^*})\cos^2\theta_D + 3(3F_L^{D_2^*} - 2F_T^{D_2^*})\cos^4\theta_D\right]\Gamma_f^{D_2^*} \end{cases}$$

effective for analysis with low statistics

$$\Gamma_f^{D^*_{(2)}} \equiv d\Gamma^{D^*_{(2)}}/dq^2$$

- Difference in inputs: Form factors

Theory	D^*	D_2^*
HQET	CNL [hep-ph/9712417] BGL [hep-ph/9705252]	[1711.03110]
LCSR	[1811.00983]	[1908.00847]
Lattice	[HPQCD, 1711.11013]	

Higher spin states

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$$\Gamma_f^{D^*_{(2)}} \equiv d\Gamma^{D^*_{(2)}}/dq^2$$

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Theory	D^*	D_2^*
HQE[illegible]		[illegible]3110]
LCSR	[1811.00983]	[1908.00847]
Lattice	[HPQCD, 1711.11013]	

Same NP should show up here also

Summary

- Charged current B-anomalies can be addressed with BSM operators with light RHN
- $F_L^{D^*}$ data is not easily achievable in NP scenarios
- 4-body angular distribution provides plethora of observables — important to identify the underlying NP dynamics
- Higher spin states provide complimentary information
— D^* & D_2^* are easily separable from distributions
- Caution for modes with τ due to neutrinos in final state
— experimentally challenging
— further decay of τ modifies the angular distribution

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  — important to identify the underlying NP dynamics
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  — further decay of τ modifies the angular distribution

Backup

Angular coefficients

$$I_1^c = N_F \Big[2\left(1+\frac{m_\tau^2}{q^2}\right)\Big(|\mathcal{A}_0^L|^2 + 4\,|\mathcal{A}_{T0}^L|^2\Big) - \frac{16 m_\tau}{\sqrt{q^2}}\,\mathcal{R}\text{e}[\mathcal{A}_0^L \mathcal{A}_{T0}^{L*}] + \frac{4m_\tau^2}{q^2}\,|A_{tP}^L|^2 + (L\to R)\Big]\,,$$

$$I_1^s = N_F \Big[\frac{1}{2}\left(3+\frac{m_\tau^2}{q^2}\right)\Big(|\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\parallel^L|^2\Big) + 2\left(1+\frac{3m_\tau^2}{q^2}\right)\Big(|\mathcal{A}_{T\perp}^L|^2 + |\mathcal{A}_{T\parallel}^L|^2\Big) - 8\,\frac{m_\tau}{\sqrt{q^2}}\,\mathcal{R}\text{e}[\mathcal{A}_\perp^L \mathcal{A}_{T\perp}^{L\,*} + \mathcal{A}_\parallel^L \mathcal{A}_{T\parallel}^{L\,*}] + (L\to R)\Big]\,,$$

$$I_2^c = -2\,N_F \left(1-\frac{m_\tau^2}{q^2}\right)\Big(|\mathcal{A}_0^L|^2 - 4\,|\mathcal{A}_{T0}^L|^2 + (L\to R)\Big),$$

$$I_2^s = \frac{1}{2}\,N_F \left(1-\frac{m_\tau^2}{q^2}\right)\Big(|\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\parallel^L|^2 - 4\left(|\mathcal{A}_{T\perp}^L|^2 + |\mathcal{A}_{T\parallel}^L|^2\right) + (L\to R)\Big)\,,$$

$$I_3 = N_F \left(1-\frac{m_\tau^2}{q^2}\right)\Big(|\mathcal{A}_\perp^L|^2 - |\mathcal{A}_\parallel^L|^2 - 4\left(|\mathcal{A}_{T\perp}^L|^2 - |\mathcal{A}_{T\parallel}^L|^2\right) + (L\to R)\Big)\,,$$

$$I_4 = \sqrt{2}\,N_F \left(1-\frac{m_\tau^2}{q^2}\right)\mathcal{R}\text{e}[\mathcal{A}_0^L \mathcal{A}_\parallel^{L*} - 4\,\mathcal{A}_{T0}^L \mathcal{A}_{T\parallel}^{L*} + (L\to R)]\,,$$

$$I_5 = 2\sqrt{2}\,N_F \Big[\mathcal{R}\text{e}[(\mathcal{A}_0^L - 2\,\frac{m_\tau}{\sqrt{q^2}}\,\mathcal{A}_{T0}^L)\,(\mathcal{A}_\perp^{L*} - 2\,\frac{m_\tau}{\sqrt{q^2}}\,\mathcal{A}_{T\perp}^{L*}) - (L\to R)] - \frac{m_\tau^2}{q^2}\,\mathcal{R}\text{e}[A_{tP}^{L*}\,(\mathcal{A}_\parallel^L - 2\,\frac{\sqrt{q^2}}{m_\tau}\mathcal{A}_{T\parallel}^L) + (L\to R)]\Big]\,,$$

$$I_6^c = N_F\,\frac{8m_\tau^2}{q^2}\,\mathcal{R}\text{e}[A_{tP}^{L*}\,(\mathcal{A}_0^L - 2\,\frac{\sqrt{q^2}}{m_\tau}\,\mathcal{A}_{T0}^L) + (L\to R)]\,,$$

$$I_6^s = 4\,N_F\,\mathcal{R}\text{e}[(\mathcal{A}_\parallel^L - 2\,\frac{m_\tau}{\sqrt{q^2}}\,\mathcal{A}_{T\parallel}^L)(\mathcal{A}_\perp^{L*} - 2\frac{m_\tau}{\sqrt{q^2}}\mathcal{A}_{T\perp}^{L*}) - (L\to R)]\,,$$

$$I_7 = -2\sqrt{2}\,N_F \Big[\mathcal{I}\text{m}[(\mathcal{A}_0^L - 2\,\frac{m_\tau}{\sqrt{q^2}}\,\mathcal{A}_{T0}^L)\,(\mathcal{A}_\parallel^{L*} - 2\,\frac{m_\tau}{\sqrt{q^2}}\,\mathcal{A}_{T\parallel}^{L*}) - (L\to R)] + \frac{m_\tau^2}{q^2}\,\mathcal{I}\text{m}[A_{tP}^{L*}\,(\mathcal{A}_\perp^L - 2\,\frac{\sqrt{q^2}}{m_\tau}\,\mathcal{A}_{T\perp}^L) + (L\to R)]\Big]\,,$$

$$I_8 = \sqrt{2}\,N_F \left(1-\frac{m_\tau^2}{q^2}\right)\mathcal{I}\text{m}[\mathcal{A}_0^{L*}\mathcal{A}_\perp^L - 4\,\mathcal{A}_{T0}^{L*}\mathcal{A}_{T\perp}^L + (L\to R)]\,,$$

$$I_9 = 2\,N_F \left(1-\frac{m_\tau^2}{q^2}\right)\mathcal{I}\text{m}[\mathcal{A}_\parallel^L \mathcal{A}_\perp^{L*} - 4\,\mathcal{A}_{T\parallel}^L \mathcal{A}_{T\perp}^{L*} + (L\to R)]\,.$$