

Multicomponent dark matter, neutrinos and high scale validity

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Based on the works:

- (i) arXiv :2009.01262,
- (ii) JCAP 04 (2020) 013,
- (iii) PHYSICAL REVIEW D 100, 055027 (2019)

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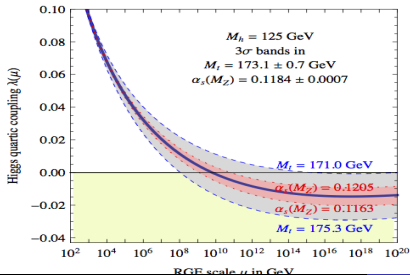
Introduction

A. Here we address two of the most important aspects of present day particle physics and cosmology:

- ▶ **Dark matter** → requires beyond the Standard Model fields [e.g. **Scalar / fermion / boson**].
- ▶ **Neutrino mass** → [most popular one: type-I seesaw requires additional SM singlet RH neutrinos.]

B. These BSM fields: affects the EW vacuum stability at high scale.

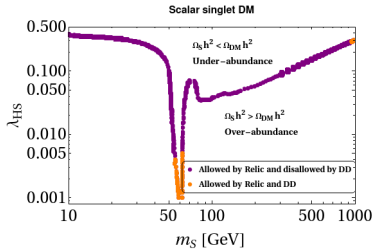
$$\beta_{\lambda}^{\text{SM}} = 24\lambda^2 + \frac{3}{4}g_1^2g_2^2 - 3g_1^2\lambda + \frac{3}{8}g_1^4 - 9g_2^2\lambda + \frac{9}{8}g_2^4 + 12y_t^2\lambda - 6y_t^4.$$



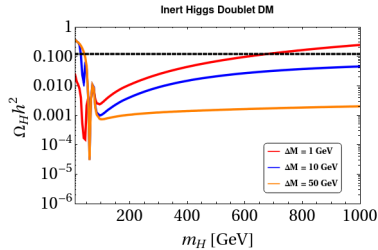
I. an additional scalar can alter the situation towards stability.

II. On the other hand, additional fermion having coupling with the SM Higgs can make it worse..

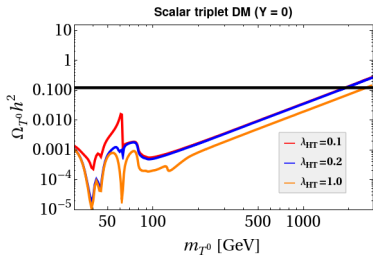
Some standard WIMP DM models



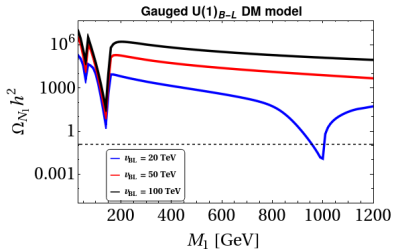
(a)



(b)



(c)



(d)

Lack of precise information of DM quantum numbers

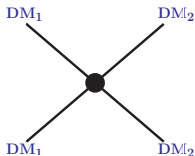


Proposal

Multicomponent DM

Introduction of multicomponent DM:

- Opens up the new **DM-DM interaction**.



- DM-DM interaction influences the relic, however do not contribute to DD. Hence an evade stringent constraints coming from the direct search experiments.

Boltzmann Equation:

$$\begin{aligned}\frac{dy_1}{dx} &= \frac{-1}{x^2} \left[\langle \sigma_{\nu_{11} \rightarrow X X} \rangle (y_1^2 - (y_1^{EQ})^2) + \langle \sigma_{\nu_{11} \rightarrow 22} \rangle \left(y_1^2 - \frac{(y_1^{EQ})^2}{(y_2^{EQ})^2} y_2^2 \right) \Theta(m_1 - m_2) \right. \\ &\quad \left. - \langle \sigma_{\nu_{22} \rightarrow 11} \rangle \left(y_2^2 - \frac{(y_2^{EQ})^2}{(y_1^{EQ})^2} y_1^2 \right) \Theta(m_2 - m_1) \right], \\ \frac{dy_2}{dx} &= \frac{-1}{x^2} \left[\langle \sigma_{\nu_{22} \rightarrow X X} \rangle (y_2^2 - (y_2^{EQ})^2) + \langle \sigma_{\nu_{22} \rightarrow 11} \rangle \left(y_2^2 - \frac{(y_2^{EQ})^2}{(y_1^{EQ})^2} y_1^2 \right) \Theta(m_2 - m_1) \right. \\ &\quad \left. - \langle \sigma_{\nu_{11} \rightarrow 22} \rangle \left(y_1^2 - \frac{(y_1^{EQ})^2}{(y_2^{EQ})^2} y_2^2 \right) \Theta(m_1 - m_2) \right].\end{aligned}$$

Here, $y_i = 0.264 M_{\text{Pl}} \sqrt{g_*} \mu Y_i$ with $Y_i = \frac{n_i}{s}$, $x = \frac{\mu}{T}$, $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Relic density:

$$\Omega_i h^2 = \frac{854.45 \times 10^{-13}}{\sqrt{g_*}} \frac{m_i}{\mu} y_i \left(\frac{\mu}{m_i} x_\infty \right), \quad \Omega_{\text{Tot}} h^2 = \Omega_1 h^2 + \Omega_2 h^2$$

Direct detection: The *effective* SI-DD cross sections:

$$\sigma_{i,\text{eff}}^{\text{SI}} = \frac{\Omega_i}{\Omega_{\text{Tot}}} \sigma_i^{\text{SI}}$$

The questions one can ask is, can multicomponent DM

- Provide a solution for a scalar singlet model which is allowed by DD in the sub-TeV range?
- Provide a DM candidate in the $m_W - 500$ GeV range for a Inert Higgs doublet (IHD) DM scenario?
- Provide a DM candidate below 1.8 TeV for a scalar triplet ($Y = 0$) DM scenario?
- Provide a DM candidate in the region apart from the resonance regions in a gauged $U(1)_{B-L}$ scenario?

Some possibilities of a multicomponent DM framework:

- 2 Scalar singlet [[JCAP 04 \(2017\)043](#)]
- Scalar singlet + Inert doublet [[JHEP 03 \(2020\) 090](#)]
- **Scalar singlet + Scalar Triplet** [[arXiv:2009.01262](#)]
- **Two inert doublets** [[Phys.Rev.D 100 \(2019\) 5, 055027](#)]
- **Inert doublet in a gauged $U(1)_{B-L}$ model** [[JCAP 04 \(2020\) 013](#)]

Our Proposal

Consider a **multicomponent scenario** with a focus on **sub-TeV range** of **scalar singlet DM** and a **below 1.8 TeV** range of **scalar triplet DM** ($Y = 0$).

Proposal : a hybrid with scalar singlet + scalar triplet!

DM: Scalar singlet and neutral component of the scalar triplet .
[DM-DM conversion would be important]

Stability of the Higgs vacuum: Higgs portal couplings of scalar singlet and scalar triplet \rightarrow can make the EW vacuum stable.

Extension of the SM by: $Z_2 \times Z'_2$

Particle	$SU(2)$	$U(1)_Y$	Z_2	Z'_2
H	2	$\frac{1}{2}$	+	+
T	3	0	-	+
S	1	0	+	-

$$V_H = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2,$$

$$V_T = \frac{M_T^2}{2} \text{tr}[T^2] + \frac{\lambda_T}{4!} (\text{tr}[T^2])^2,$$

$$V_S = \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4!} S^4,$$

$$V_{\text{int}} = \frac{\lambda_{HT}}{2} (H^\dagger H) \text{tr}[T^2] + \frac{\lambda_{HS}}{2} (H^\dagger H) S^2 + \frac{\kappa}{4} \text{tr}[T^2] S^2$$

The scalar fields are then parametrised as

$$H = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + h + iz) \end{pmatrix}, \quad T = \begin{pmatrix} \frac{1}{\sqrt{2}}T^0 & -T^+ \\ -T^- & -\frac{1}{\sqrt{2}}T^0 \end{pmatrix}, \quad S.$$

After the EWSB, the masses of the scalar particles are given as

$$\begin{aligned} m_h^2 &= 2\lambda_H v^2 \\ m_{T^0, T^\pm}^2 &= M_T^2 + \frac{\lambda_{HT}}{2} v^2 \\ m_S^2 &= M_S^2 + \frac{\lambda_{HS}}{2} v^2. \end{aligned}$$

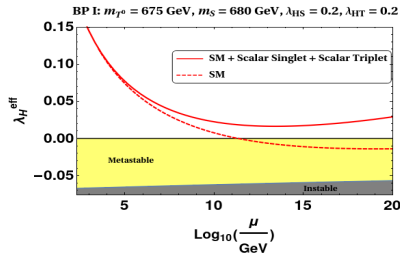
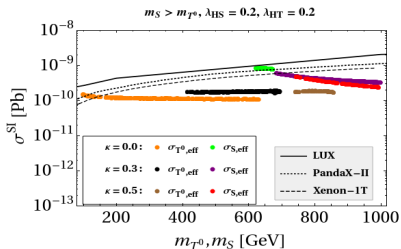
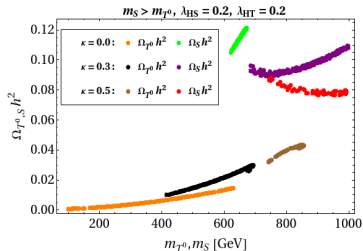
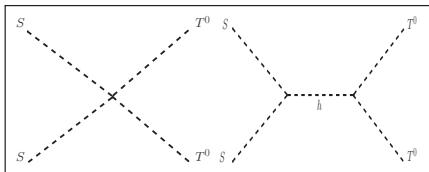
A small mass difference of **166 MeV** between the charged and the neutral scalars can be generated at 1-loop

$$\Delta m = (m_{T^\pm} - m_{T^0})_{1-loop} = \frac{\alpha m_{T^0}}{4\pi} \left[f\left(\frac{M_W}{m_{T^0}}\right) - c_W^2 f\left(\frac{M_Z}{m_{T^0}}\right) \right].$$

This makes the T^0 a viable DM candidate.

DM Phenomenology and vacuum stability

DM-DM conversion ($m_S > m_T$):



$$\beta_{\lambda_H} = \beta_{\lambda_H}^{SM} + \frac{3}{2} \lambda_{HT}^2 + \frac{1}{2} \lambda_{HS}^2$$



The next question we ask is

Can multicomponent provide DM in the mass range $m_W - 500$ GeV of IHD?

↓ Yes¹!!

Proposal : a hybrid with 2 IHDs!

DM: Lightest neutral component of both the IHDs .
[DM-DM conversion would be important]

Neutrino mass: Together with the RHNs, ν - masses can be generated at 1-loop.

¹A similar analysis with one singlet and one IHD can be found in JHEP 03 (2020) 090

Extension of the SM: $SU(2)_L \times U(1)_Y \times \mathbb{Z}_2 \times \mathbb{Z}'_2$

Field	$SU(3)_c \times SU(2)_L \times U(1)_Y$	\mathbb{Z}_2	\mathbb{Z}'_2
η_1	$(1, 2, \frac{1}{2})$	-	+
η_2	$(1, 2, \frac{1}{2})$	+	-
N_1	$(1, 1, 0)$	-	+
N_2	$(1, 1, 0)$	+	-

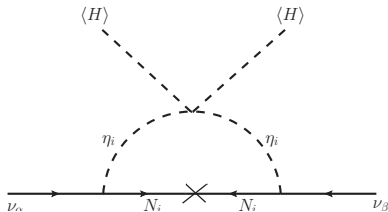
Lagrangian

$$-\mathcal{L}^{\text{new}} = Y_{\alpha 1} \bar{L}_\alpha \tilde{\eta}_1 N_1 + Y_{\alpha 2} \bar{L}_\alpha \tilde{\eta}_2 N_2 + \frac{1}{2} M_1 \bar{N}_1^c N_1 + \frac{1}{2} M_2 \bar{N}_2^c N_2 + h.c.,$$

Scalar potential

$$\begin{aligned}
 V_{\text{int}} = & \lambda_3 (H^\dagger H) (\eta_1^\dagger \eta_1) + \lambda_4 (H^\dagger \eta_1) (\eta_1^\dagger H) + \frac{\lambda_5}{2} \left[(H^\dagger \eta_1)^2 + (\eta_1^\dagger H)^2 \right] \\
 & + \tilde{\lambda}_3 (H^\dagger H) (\eta_2^\dagger \eta_2) + \tilde{\lambda}_4 (H^\dagger \eta_2) (\eta_2^\dagger H) + \frac{\tilde{\lambda}_5}{2} \left[(H^\dagger \eta_2)^2 + (\eta_2^\dagger H)^2 \right] \\
 & + \lambda'_3 (\eta_1^\dagger \eta_1) (\eta_2^\dagger \eta_2) + \lambda'_4 (\eta_1^\dagger \eta_2) (\eta_2^\dagger \eta_1) + \frac{\lambda'_5}{2} \left[(\eta_1^\dagger \eta_2)^2 + (\eta_2^\dagger \eta_1)^2 \right].
 \end{aligned}$$

$Y_{\alpha i} \bar{L}_{\alpha} \tilde{\eta}_i N_i$: ν -mass generation at **1-loop**



$$(m^{\nu})_{\alpha\beta} = \sum_{i=1,2} Y_{\alpha i} Y_{\beta i} \frac{M_i}{32\pi^2} \left[\frac{m_{H_i}^2}{m_{H_i}^2 - M_i^2} \ln \frac{m_{H_i}^2}{M_i^2} - \frac{m_{A_i}^2}{m_{A_i}^2 - M_i^2} \ln \frac{m_{A_i}^2}{M_i^2} \right],$$

With $Y = U_{\text{PMNS}} \sqrt{m_{\nu}^{\text{diag}}} R^{\dagger} \sqrt{\Lambda^{-1}}$ [CI parameterization²]
where

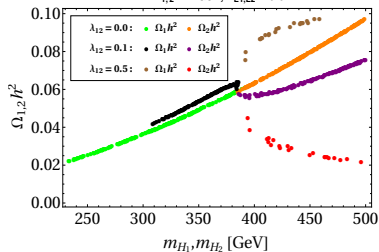
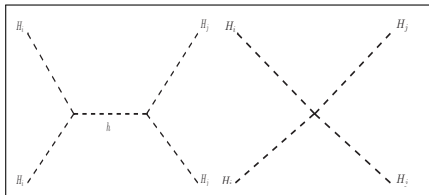
$$m_{\nu}^{\text{diag}} = \text{dia}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \quad R = \begin{pmatrix} 0 & \cos z & \sin z \\ 0 & -\sin z & \cos z \end{pmatrix} \text{ and } R R^{\text{T}} = 1.$$

²Nucl.Phys. B618 (2001) 171-204

DM Phenomenology

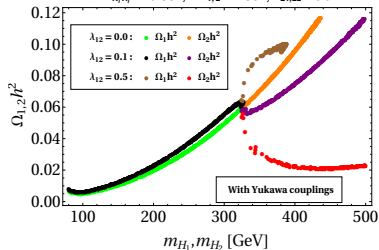
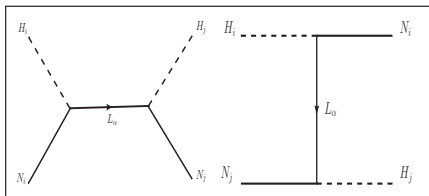
DM-DM conversion ($m_{H_2} > m_{H_1}$, $\lambda'_3, \lambda'_4, \lambda'_5 = \lambda_{12}$ and without $Y_{\alpha i}$)

$\Delta M_{1,2} = 1 \text{ GeV}$, $\lambda_{L1,L2} = 0.01$



DM-DM conversion (with $Y_{\alpha i}$ and $M_i - m_{H_i} = 10 \text{ GeV}$)

$\Delta M_{N_i H_i} = 10 \text{ GeV}$, $\Delta M_{1,2} = 1 \text{ GeV}$, $\lambda_{L1,L2} = 0.01$



And finally,

Can multicomponent provide DM candidate in a gauged $U(1)_{B-L}$ scenario apart from the region near resonances ?

↓ **Yes, but one needs to pay a price!!**

Proposal : a hybrid with Gauged $U(1)_{B-L} + \text{IHD!}$

DM: Lightest neutral component of the IHD and the lightest RHN .
[DM-DM conversion would be important]

Neutrino mass: RHNs together with the IHD, ν - masses can be generated at 1-loop.

High scale validity: The fate of EW vacuum will now depend on the choice of DM conversion coupling.

- Extension of the SM by
 - Gauged $U(1)_{B-L}$
 - Two discrete symmetries: $\mathbb{Z}_2 \times \mathbb{Z}'_2$
- Charges of the particle under different symmetry groups

Field	$SU(2)_L \times U(1)_Y$	$U(1)_{B-L}$	\mathbb{Z}_2	\mathbb{Z}'_2
ϕ_2	$(2, \frac{1}{2})$	0	-	-
N_1	$(1, 0)$	-1	-	+
N_2, N_3	$(1, 0)$	-1	-	-
S	$(1, 0)$	2	+	+

$$\mathcal{L}_{new} = i\bar{N}\not{\partial}N + |D_\mu\phi_1|^2 + |D_\mu S|^2 + \frac{1}{4}(Z_{BL})_{\mu\nu}(Z_{BL})^{\mu\nu} - \zeta_{i\alpha}\bar{L}_i\tilde{\phi}_2 N_\alpha - y_{\alpha\beta}\bar{N}_\alpha^c N_\beta^c S - y_{11}\bar{N}_1^c N_1^c S - V(\phi_1, S)$$

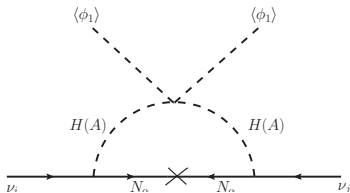
where $D_\mu = \partial_\mu - ig\frac{\tau}{2}W_\mu - ig'\frac{B}{2} - ig_{BL}Q(Z_{BL})_\mu$

$$\begin{aligned}
 V(\phi_1, \phi_2, S) = & -\mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 - \mu_S^2 |S|^2 \\
 & + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\
 & + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{\lambda_5}{2} [(\phi_1^\dagger \phi_2)^2 + (\phi_2^\dagger \phi_1)^2] \\
 & + \lambda_6 (\phi_1^\dagger \phi_1) |S|^2 + \lambda_7 (\phi_2^\dagger \phi_2) |S|^2 + \lambda_8 |S|^4
 \end{aligned}$$

After EWSB, $\langle \phi_1 \rangle = v$, $\langle S \rangle = v_{BL}$, $\langle \phi_2 \rangle = 0$ and ϕ_h mixes with ϕ_s

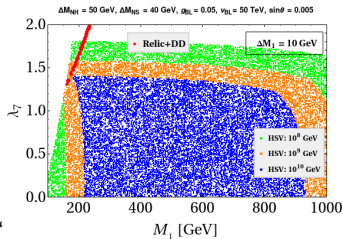
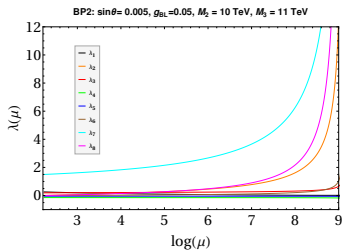
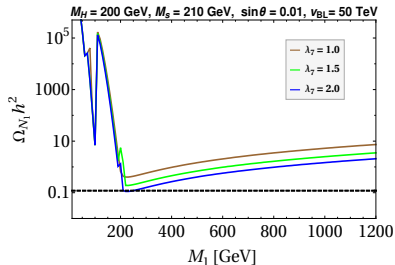
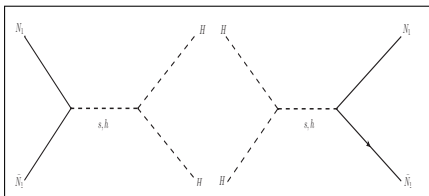
$$\begin{pmatrix} \phi_h \\ \phi_s \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}$$

Neutrino mass: $\zeta_{i\alpha} \bar{L}_i \phi_2^\dagger N_\alpha$, with $\alpha = 2, 3$



DM Phenomenology and High scale validity

DM-DM conversion :



$$\beta_{\lambda_7} = 6\lambda_2\lambda_7 + 4\lambda_3\lambda_6 + 2\lambda_4\lambda_6 + 4\lambda_7^2 + 8\lambda_7\lambda_8 + 4\lambda_7 \text{Tr}[y^\dagger y] - 24\lambda_7 g_{BL}^2$$

- Multicomponent DM models can evade ever tightening bound on the direct detection (DD) rates while enlarging relic density allowed parameter space.
- The proposed scenarios opens up an attractive possibility of **DM-DM conversion**, a phenomenon that goes on to become the main theme of these studies.
- Conversion processes can lead to the desired relic density for:
 - Scalar singlet plus scalar triplet scenario in the sub-TeV mass regime.
 - IHD DM scenario in the mass range $m_W - 500$ GeV.
 - Gauged $U(1)_{B-L}$ model apart from the near resonance regions.
- The multicomponent scenarios discussed above can generate neutrino mass and at the same time can also modify the fate of EW vacuum.
- Multicomponent DM studies can also open up an interesting collider prospects (lighter DM masses are allowed).

Backup Slides

$$m_{H_1}^2 = \mu_1^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v^2,$$

$$m_{A_1}^2 = \mu_1^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v^2,$$

$$m_{\eta_1^+}^2 = \mu_1^2 + \frac{1}{2}\lambda_3 v^2,$$

$$m_{H_2}^2 = \mu_2^2 + \frac{1}{2}(\tilde{\lambda}_3 + \tilde{\lambda}_4 + \tilde{\lambda}_5)v^2,$$

$$m_{A_2}^2 = \mu_2^2 + \frac{1}{2}(\tilde{\lambda}_3 + \tilde{\lambda}_4 - \tilde{\lambda}_5)v^2,$$

$$m_{\eta_2^+}^2 = \mu_2^2 + \frac{1}{2}\tilde{\lambda}_3 v^2.$$

Annihilation channels

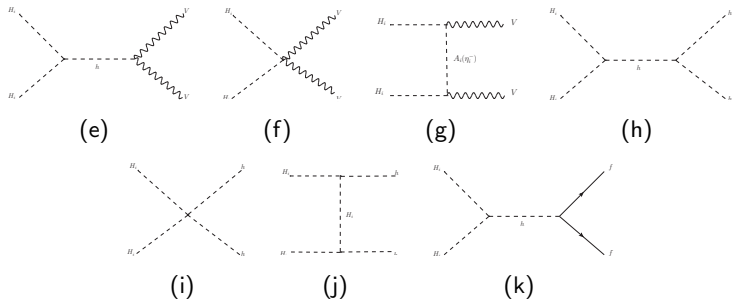


Figure: Annihilation channels

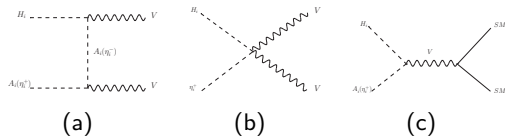


Figure: Coannihilation Channels

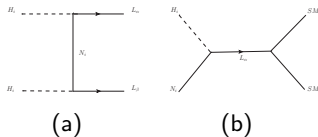
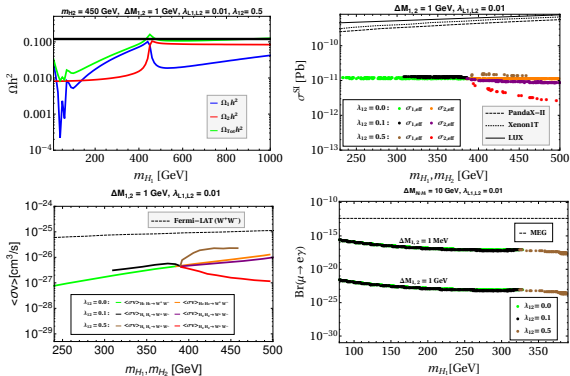
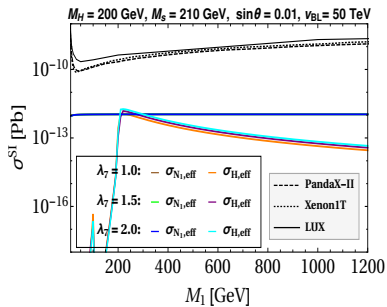
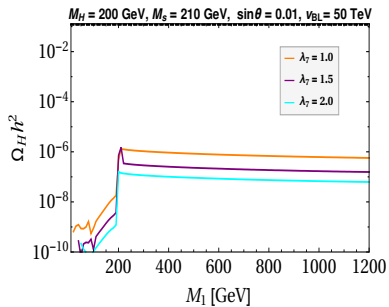


Figure: (Co)annihilation channels in presence of singlet neutral fermions

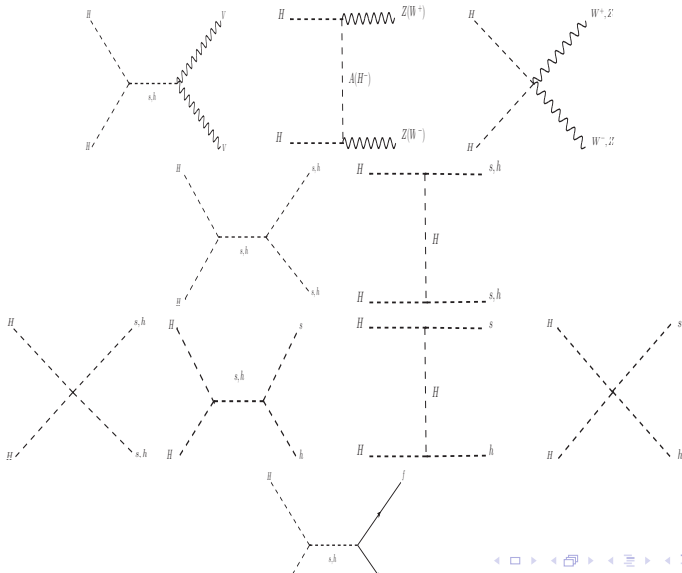


BP	m_{H_1} [GeV]	m_{H_2} [GeV]	$\Omega_1 h^2$	$\Omega_2 h^2$
Without Yukawa interactions	250	492	0.026	0.095
With Yukawa interactions	250	380	0.033	0.085

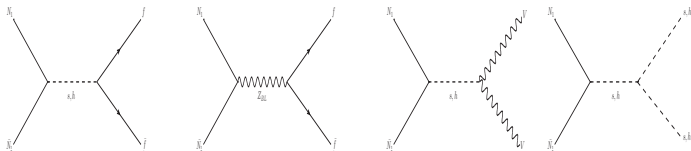
Table: In producing the above values, we considered: $\lambda_{L_1} = \lambda_{L_2} = 0.01$ and $\lambda_{12} = 0$



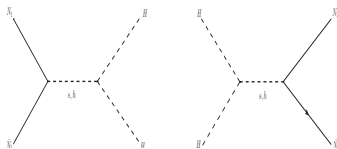
Annihilation and co-annihilation processes for H



Annihilation processes for N_1



$N_1 - H$ conversion processes



The **masses of the IHD component** after the symmetry breaking:

$$\begin{aligned}M_H^2 &= \mu_2^2 + \frac{1}{2}\lambda_L v^2 + \frac{1}{2}\lambda_7 v_{BL}^2, \\M_A^2 &= \mu_2^2 + \frac{1}{2}\lambda_L v^2 + \frac{1}{2}\lambda_7 v_{BL}^2, \\M_{H^+}^2 &= \mu_2^2 + \frac{1}{2}\lambda_3 v^2 + \frac{1}{2}\lambda_7 v_{BL}^2.\end{aligned}$$

where $\lambda_L = \lambda_3 + \lambda_4 + \lambda_5$.

Masses of the 3 RHNs will be given as:

$$M_N = \sqrt{2} v_{BL} \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & y_{23} \\ 0 & y_{23} & y_{33} \end{pmatrix}.$$

We take $y_{23} = 0$ for simplicity for the rest of the analysis, in which case M_N is diagonal with entries $M_i = \sqrt{2} y_{ii} v_{BL}$.

Choice of parameters

- We have the following free parameters in our set-up:

$$M_1, M_2, M_3, M_H, M_A, M_{H^+}, M_S, \theta, \lambda_L, \lambda_7, g_{BL}, v_{BL}$$

- Dependent parameters in the set-up:

$$\mu_2^2 = M_H^2 - \frac{1}{2}\lambda_L v^2 - \frac{1}{2}\lambda_7 v_{BL}^2,$$

$$\lambda_1 = \frac{(M_h^2 c_\theta^2 + M_S^2 s_\theta^2)}{v^2},$$

$$\lambda_3 = \lambda_L + \frac{2(M_{H^+}^2 - M_H^2)}{v^2},$$

$$\lambda_4 = \frac{M_H^2 + M_A^2 - 2M_{H^+}^2}{v^2},$$

$$\lambda_5 = \frac{(M_H^2 - M_A^2)}{v^2},$$

$$\lambda_6 = \frac{(M_S^2 - M_h^2)s_\theta c_\theta}{v v_{BL}},$$

$$\lambda_8 = \frac{(M_h^2 s_\theta^2 + M_S^2 c_\theta^2)}{2v_{BL}^2}$$

$$y_{ii} = \frac{M_{ii}}{\sqrt{2}v_{BL}}.$$

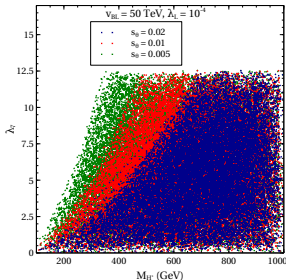


Figure: Parameter space in the $\lambda_7 - M_{H^+}$ plane allowed by the $\mu\gamma\gamma$ constraint

Scalar interactions:

$$\lambda_{HHh} = (\lambda_3 + \lambda_4 + \lambda_5)v c_\theta - \lambda_7 v_{BL} s_\theta, \quad (4a)$$

$$\lambda_{H H s} = (\lambda_3 + \lambda_4 + \lambda_5)v s_\theta + \lambda_7 v_{BL} c_\theta, \quad (4b)$$

$$\lambda_{AAh} = (\lambda_3 + \lambda_4 - \lambda_5)v c_\theta - \lambda_7 v_{BL} s_\theta, \quad (4c)$$

$$\lambda_{AA s} = (\lambda_3 + \lambda_4 - \lambda_5)v s_\theta + \lambda_7 v_{BL} c_\theta, \quad (4d)$$

$$\lambda_{H^+ H^- h} = \lambda_3 v c_\theta - \lambda_7 v_{BL} s_\theta, \quad (4e)$$

$$\lambda_{H^+ H^- s} = \lambda_3 v s_\theta + \lambda_7 v_{BL} c_\theta. \quad (4f)$$

Yukawa interactions:

$$y_h N_1 N_1 = -\frac{1}{\sqrt{2}} y_{11} s_\theta, \quad (5a)$$

$$y_s N_1 N_1 = \frac{1}{\sqrt{2}} y_{11} c_\theta, \quad (5b)$$

$$y_{hff} = \frac{M_f}{v} c_\theta, \quad (5c)$$

$$y_{sff} = \frac{M_f}{v} s_\theta \text{ where } f \text{ is a SM fermion.} \quad (5d)$$

Gauge interactions:

$$g_{hVV} = \frac{2M_V^2}{v} c_\theta, \quad (6a)$$

$$g_{sVV} = \frac{2M_V^2}{v} s_\theta \text{ where } V = W^+, Z \quad (6b)$$

$$g_{hZ_{BL}Z_{BL}} = -\frac{2M_V^2}{v_{BL}} s_\theta, \quad (6c)$$

$$g_{sZ_{BL}Z_{BL}} = \frac{2M_V^2}{v_{BL}} c_\theta. \quad (6d)$$

• LHC diphoton signal strength:

- ▶ Measured Higgs signal strength at LHC gives constraint on $\sin\theta \leq 0.36^3$.
- ▶ The presence of H^+ will alter the decay width of $h \rightarrow \gamma\gamma$ through **one loop**⁴

$$\begin{aligned}\mathcal{M}_{h \rightarrow \gamma\gamma} &= \frac{4}{3} c_\theta A_f \left(\frac{M_h^2}{4M_t^2} \right) + c_\theta A_V \left(\frac{M_h^2}{4M_W^2} \right) \\ &\quad + \frac{\lambda_{hH^+H^-} v}{2M_{H^+}^2} A_S \left(\frac{M_h^2}{4M_{H^+}^2} \right), \\ \Gamma_{h \rightarrow \gamma\gamma} &= \frac{G_F \alpha^2 M_h^3}{128\sqrt{2}\pi^3} |\mathcal{M}_{h \rightarrow \gamma\gamma}|^2.\end{aligned}$$

where A_f , A_V and A_S are the loop functions. [ref]
The latest $\mu_{\gamma\gamma}$ values from 13 TeV LHC read [ref]

$$\begin{aligned}\mu_{\gamma\gamma} &= 0.99_{-0.14}^{+0.14} \text{ (ATLAS),} \\ &= 1.18_{-0.14}^{+0.17} \text{ (CMS).}\end{aligned}$$

³Eur.Phys.J. C76 (2016) no.5, 268

⁴Phys.Rev. D85 (2012) 095021