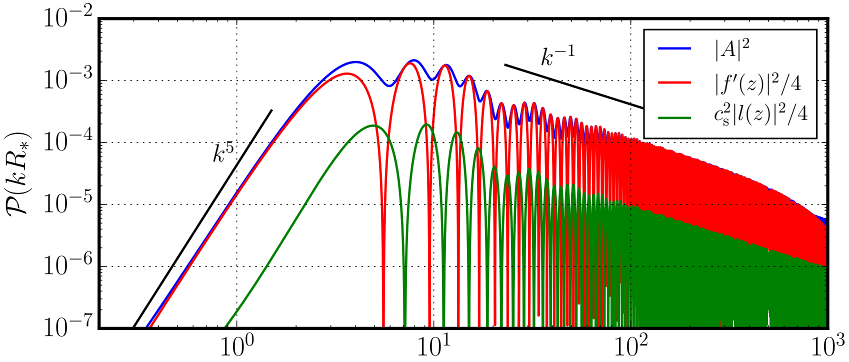
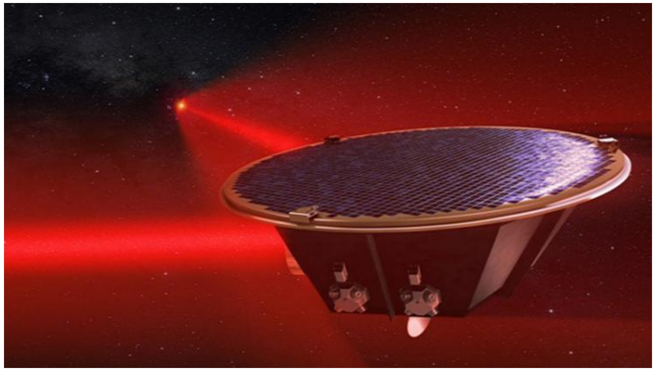




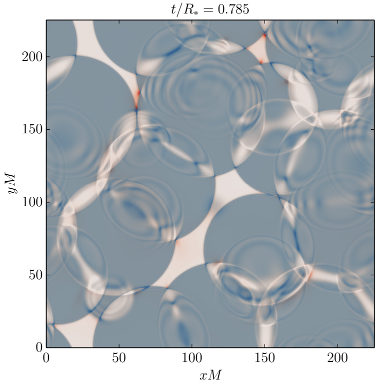
Gravitational waves from phase transitions

Mark Hindmarsh

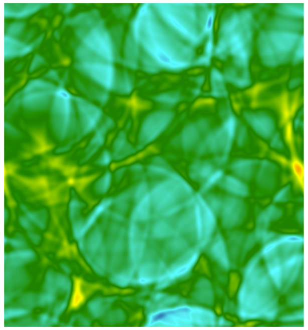
Helsinki Institute of Physics & Dept of Physics, University of Helsinki

and

Dept of Physics and Astronomy, University of Sussex



Anomalies 2020
11. syyskuuta 2020

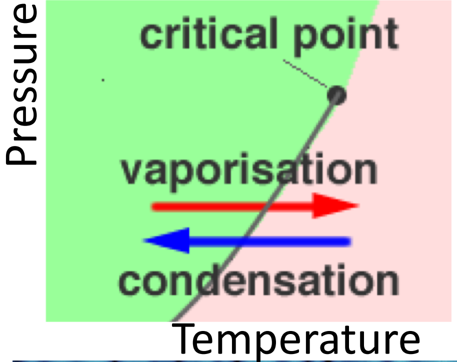


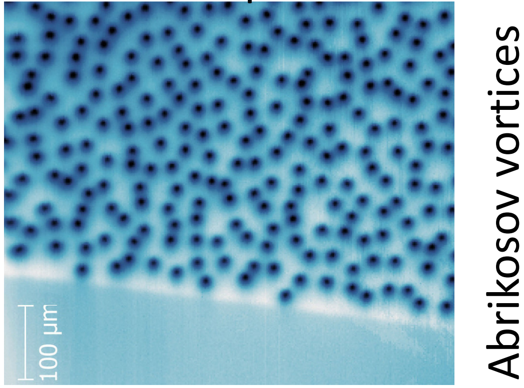
Phase transitions in the early Universe

- At very high temperatures and pressures, the state of matter in the Universe changes
  - $T_c \sim 100$ MeV (1 ms) QCD
  - $T_c \sim 100$ GeV (10 ps) Electroweak
  - $T_c >> 100$ GeV new symmetries?
- Departures from equilibrium and homogeneity (shear stress)
  - First order phase transition: relativistic condensation or `fizz'

    Steinhardt (1982)
  - Formation of topological defects

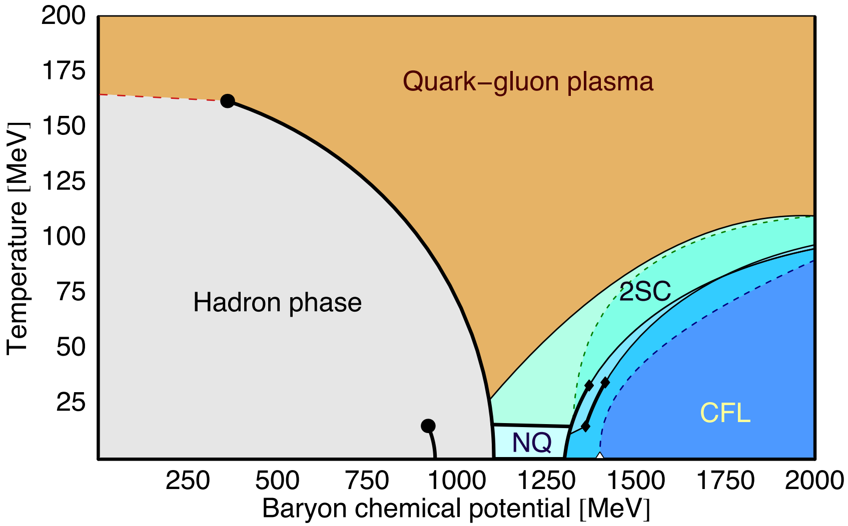
    Kibble (1976)

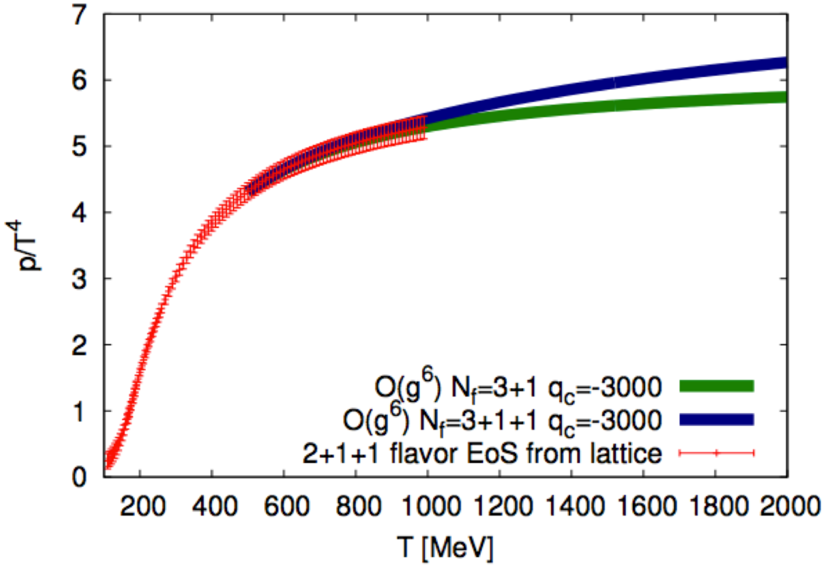




QCD phases

- QCD: rich phase diagram
- Universe: $n_B/n_\gamma \approx 6.1 \times 10^{-10}$
- Behaviour at low chemical potential well-established by lattice QCD Borsanyi et al (2016)
- Transition from QGP to hadronic phase is a cross-over
- Departures from equilibrium very small: no GWs

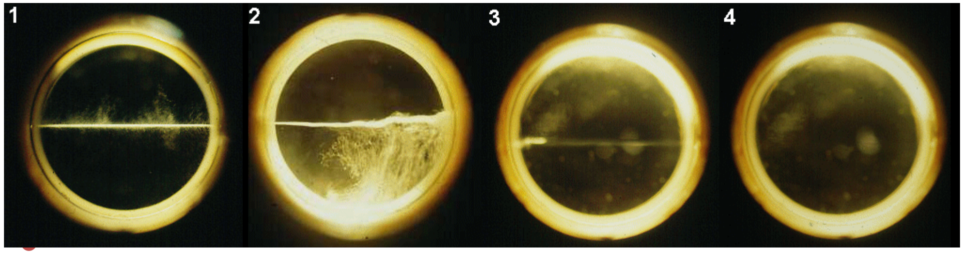




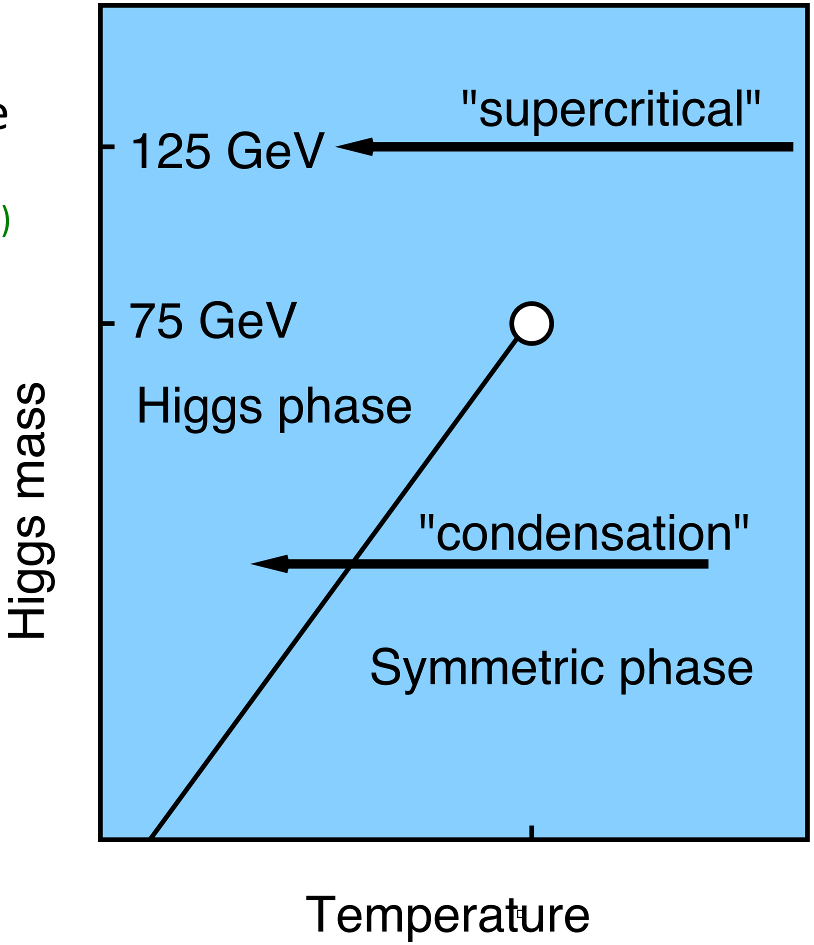
Borsanyi et al 2016

Electroweak transition

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over - a supercritical fluid

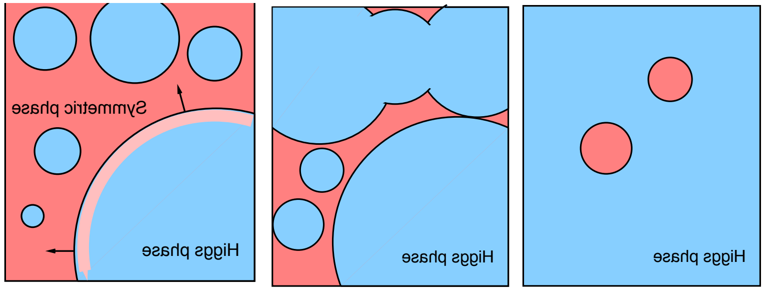


- Search for 1st order transition is a search for physics beyond SM



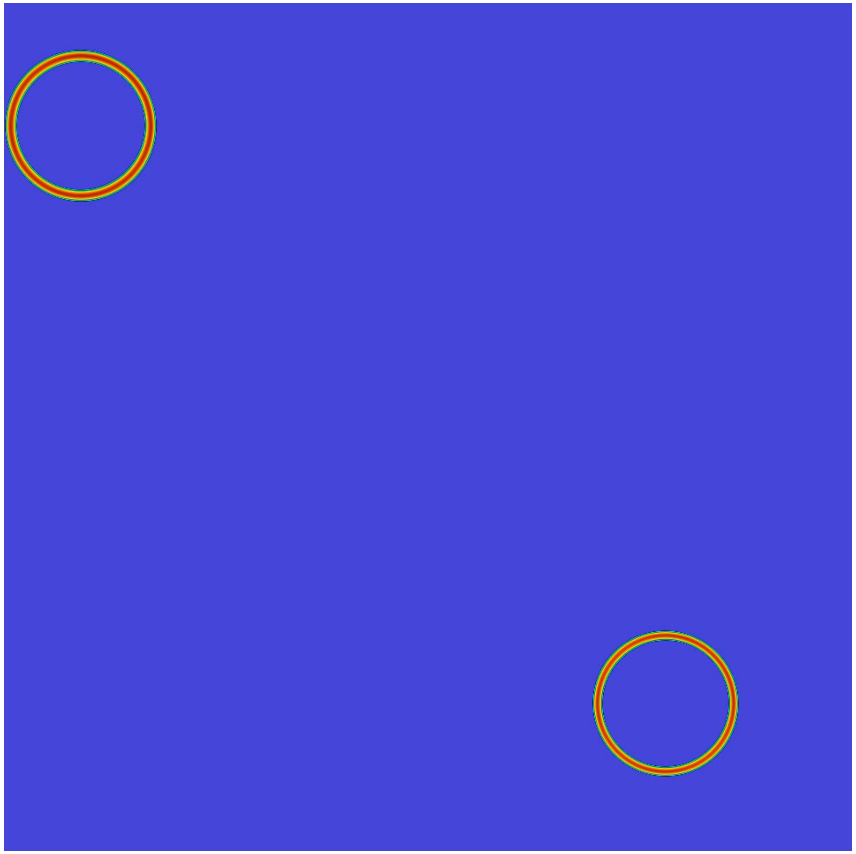
Little bangs in the Big Bang

- 1st order transition Langer 1969 proceeds by nucleation of bubbles of low-*T* phase
- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Universal “fizz”



Steinhardt (1982); Hogan (1983,86); Gyulassy et al (1984); Witten (1984)

Fluid kinetic energy

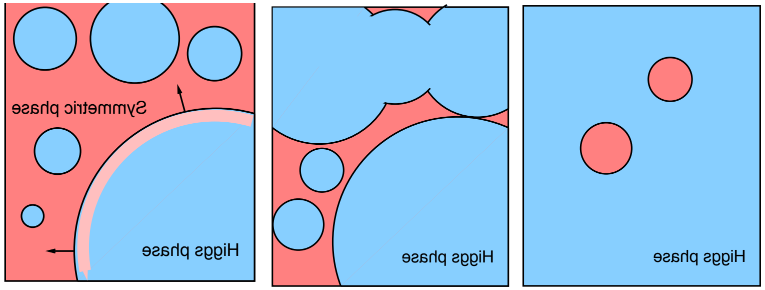


David Weir, Helsinki University

MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

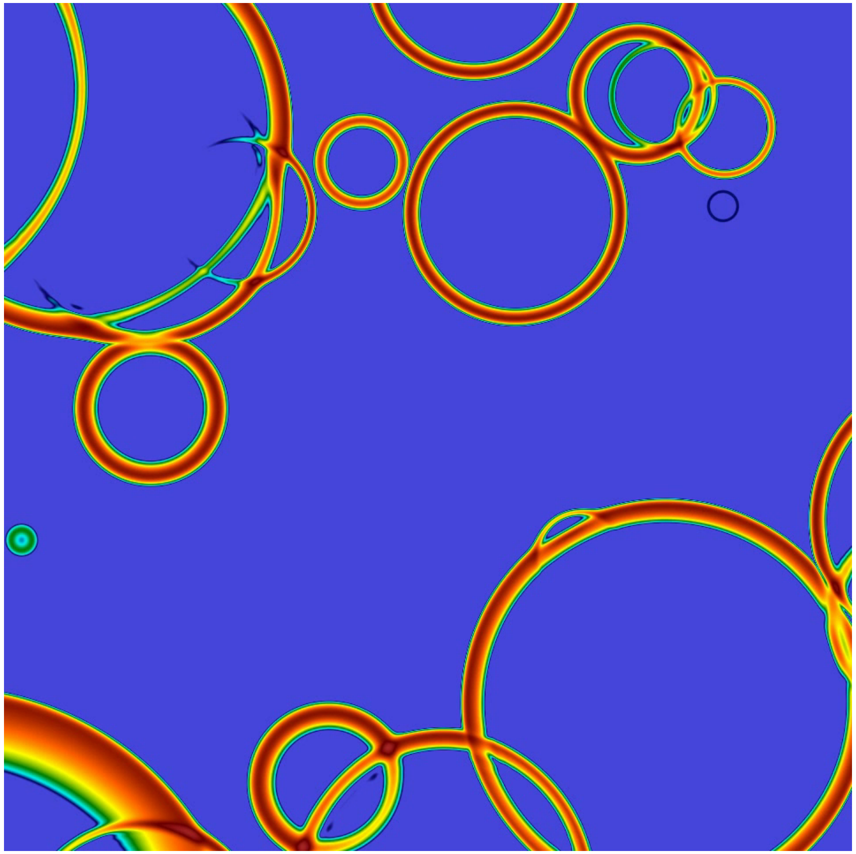
Little bangs in the Big Bang

- 1st order transition Langer 1969 proceeds by nucleation of bubbles of low-*T* phase
- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Universal “fizz”



Steinhardt (1982); Hogan (1983,86); Gyulassy et al (1984); Witten (1984)

Fluid kinetic energy

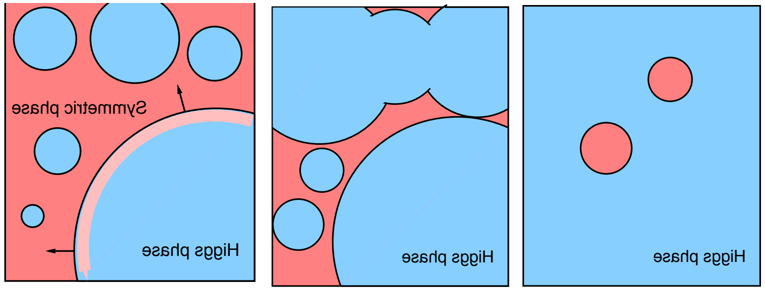


David Weir, Helsinki University

MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

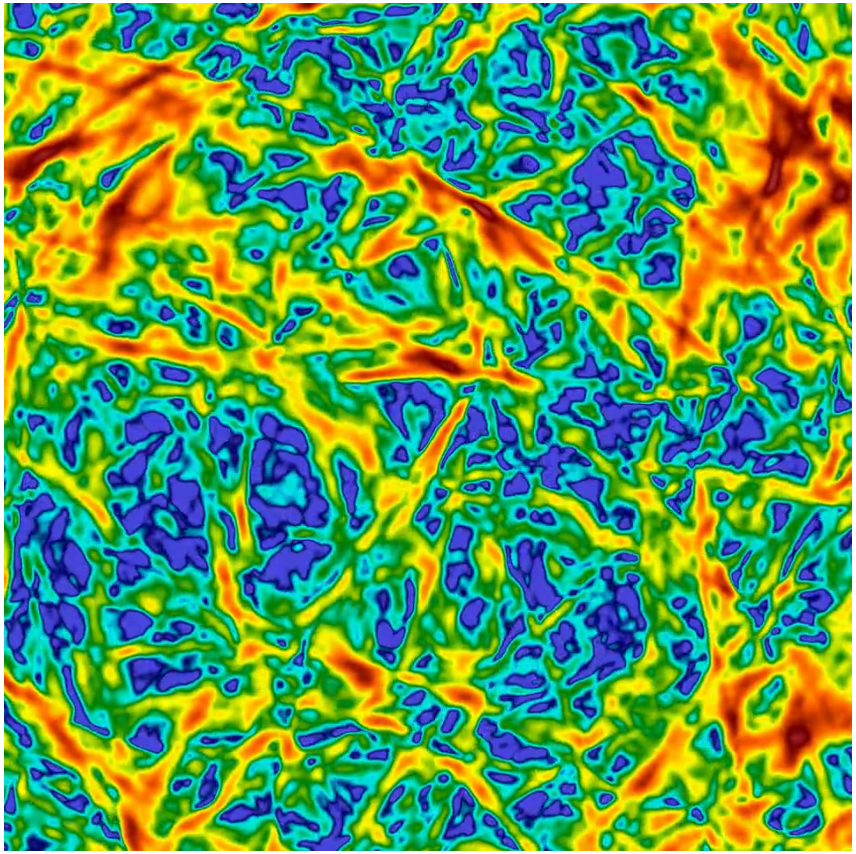
Little bangs in the Big Bang

- 1st order transition Langer 1969 proceeds by nucleation of bubbles of low-*T* phase
- Nucleation rate/volume *p*(*t*) rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Universal “fizz”



Steinhardt (1982); Hogan (1983,86);
Gyulassy et al (1984); Witten (1984)

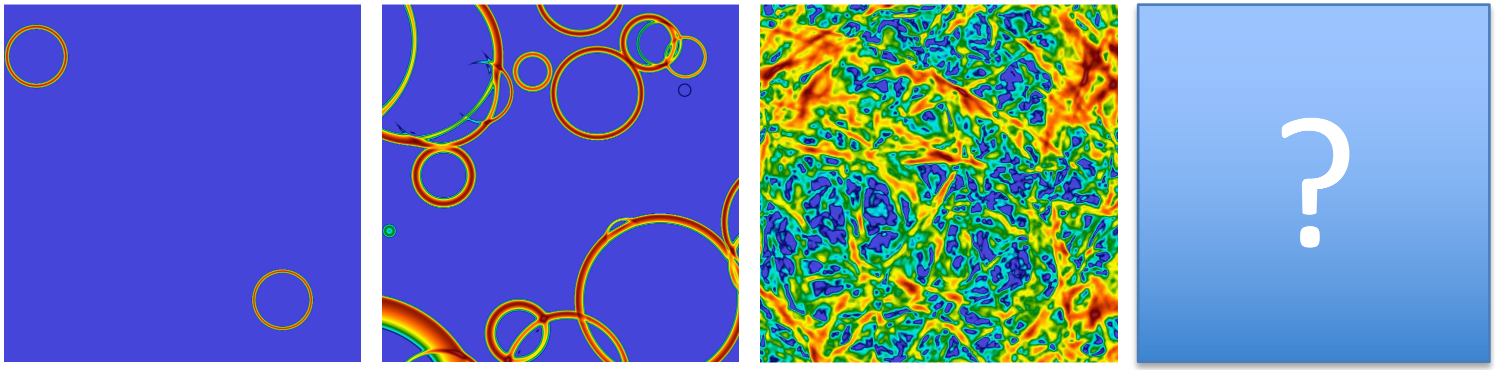
Fluid kinetic energy



David Weir, Helsinki University

MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

Phases of a phase transition



1. Nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\rm nl} \sim L_f/\bar{U}_f$$

$L_{\rm f}$ – fluid flow length scale
$U_{\rm f}$ – RMS fluid velocity

'exponential' nucleation

$$p(t) = p_n e^{\beta(t-t_n)}$$

$$\tau_{\rm co} = \beta^{-1}$$

Guth, Weinberg 1983; Enqvist et al 1992;
Turner, Weinberg, Widrow 1992;

p – nucleation rate/volume
β – transition rate parameter

Dynamics of an early universe phase transition

- Ingredients:

Ignatius et al (1994), Kurki-Suonio, Laine (1996)

  - Higgs field $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$$
    - *η* coupling to fluid (models energy transfer, friction)
  - Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i P + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - *E* energy density, Z_i momentum density, V_i velocity, *W* γ-factor
  - Discretisation

Wilson & Matthews (2003)
Different approach: Giblin, Mertens (2013)

  - Metric perturbation

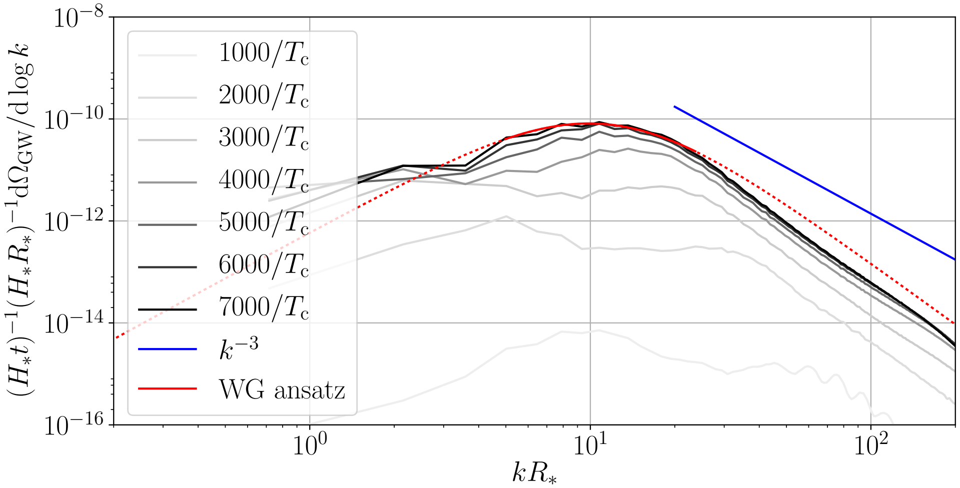
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

Garcia-Bellido, Figueroa, Sastre (2008)

Simulations of phase transitions

- 2015: 1M hrs CSC, Finland
- 2015/6: 17M CPU-hours
  Tier-0 (Hazel Hen, Stuttgart)
- 4200^3 lattice on 24k cores
- Output: GW power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln f} = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$

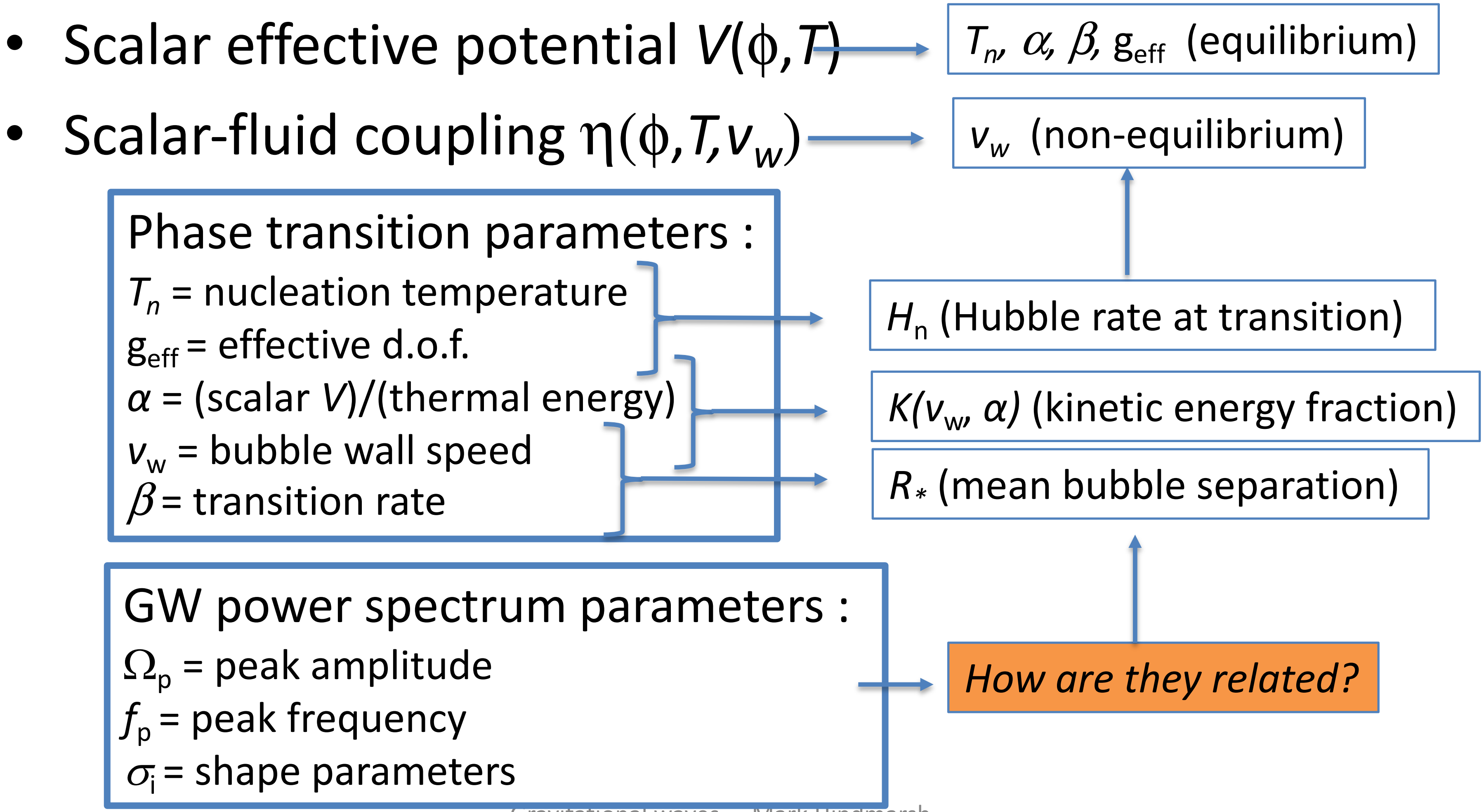


SUOMEN AKATEMIA
FINLANDS AKADEMI
ACADEMY OF FINLAND

- Transition strength: $\alpha = 0.0046$
- Wall speed: $v_{\rm w} = 0.44$
- Peak at $kR_* \sim 10$
- “Domed” peak
- Approx k^{-3} spectrum at high k

Hindmarsh, Huber, Rummukainen, Weir 2017

Connection to fundamental theory



- Scalar effective potential $V(\phi,T)$
- Scalar-fluid coupling $\eta(\phi,T,v_w)$

T_n, α, β, $g_{\rm eff}$ (equilibrium)

v_w (non-equilibrium)

Phase transition parameters :

T_n = nucleation temperature

$g_{\rm eff}$ = effective d.o.f.

α = (scalar V)/(thermal energy)

$v_{\rm w}$ = bubble wall speed

β = transition rate

$H_{\rm n}$ (Hubble rate at transition)

$K(v_{\rm w}, \alpha)$ (kinetic energy fraction)

R_* (mean bubble separation)

GW power spectrum parameters :

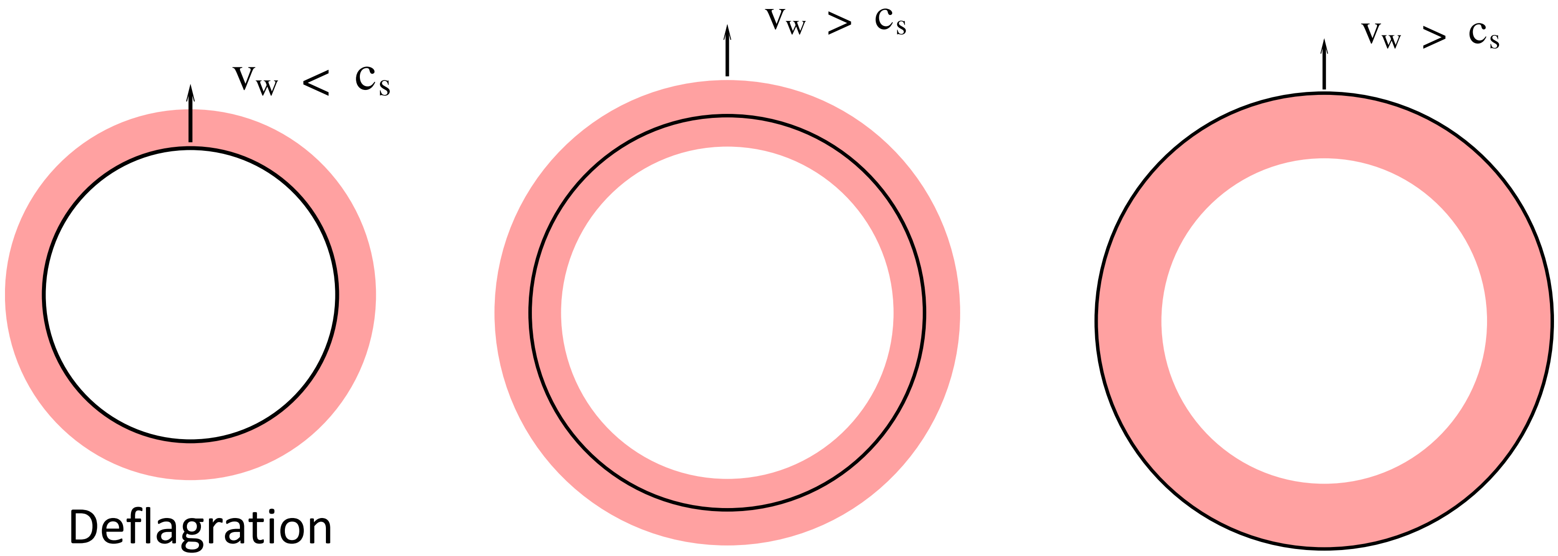
$\Omega_{\rm p}$ = peak amplitude

$f_{\rm p}$ = peak frequency

$\sigma_{\rm i}$ = shape parameters

How are they related?

Relativistic combustion



Deflagration

Supersonic deflagration
(“hybrid”)

Detonation

Landau & Lifshitz
Steinhardt (1984)
Kurki-Suonio, Laine (1991)
Espinosa et al (2010)

- Scalar potential energy (free energy) to kinetic energy, heat energy
- Wall velocity v_w - pressure difference $\Delta V(\phi,T)$. scalar-fluid coupling $\eta(\phi,T)$
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
- Similarity solution $v(r/t)$, $w(r/t)$
- Some cases … runaway ($v_w \to 1$) (weakly coupled near-vacuum transition)

Bodeker Moore 2010, 2017

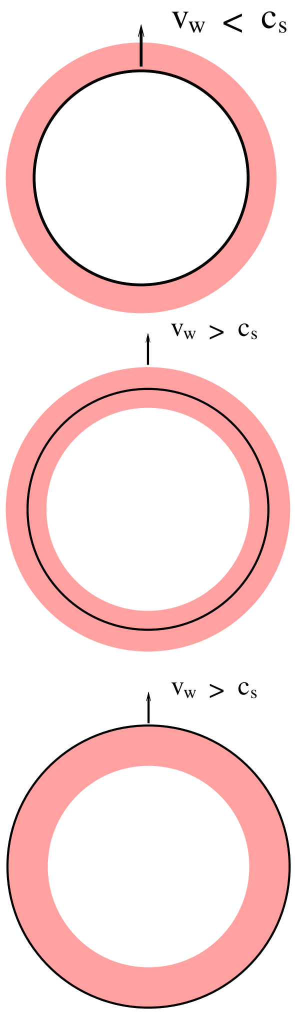
GWs from first order phase transitions

- Parameters of transition:
  - T_n = Temperature at nucleation
  - β = transition rate (= - d log p / dt)
  - v_w = Bubble wall speed
  - α = ("Potential energy")/("Heat energy")
- Derived parameters:
  - R_* = mean bubble separation $= (8\pi)^{1/3} v_w/\beta$
  - K = fluid kinetic energy fraction (depends on α, v_w)

Steinhardt '84
Espinosa et al 2010

- Aim: GW power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln f} = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$



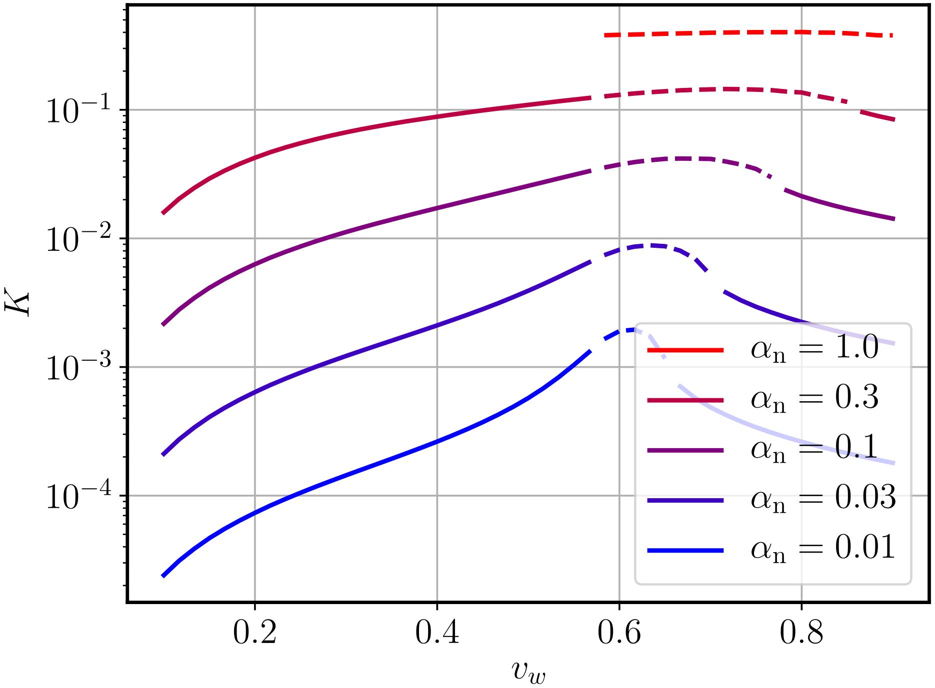
Estimating GW power

- GW energy fraction:
  - τ_v duration of stresses
  - τ_c coherence time
- Numerical simulations:
  - $\tau_c \sim R_*$ (bubble separation)
- Analytical estimate:
  - $\tau_v = H_n^{-1}/(1 + H_n R_*/U_f)$
  - N.B. $K = (4/3)U_f^2$
  - U_f (weighted) RMS velocity
  - Pure acoustic

$$\Omega_{\rm gw} \simeq 10^{-2} \frac{(H_{\rm n} R_*)^2}{H_{\rm n} R_* + \sqrt{K}} K^2$$

$$\Omega_{\rm gw} \sim (H_{\rm n}\tau_v)(H_{\rm n}\tau_c)K^2$$

Estimate *K* (kinetic energy fraction) from self-similar hydro solution



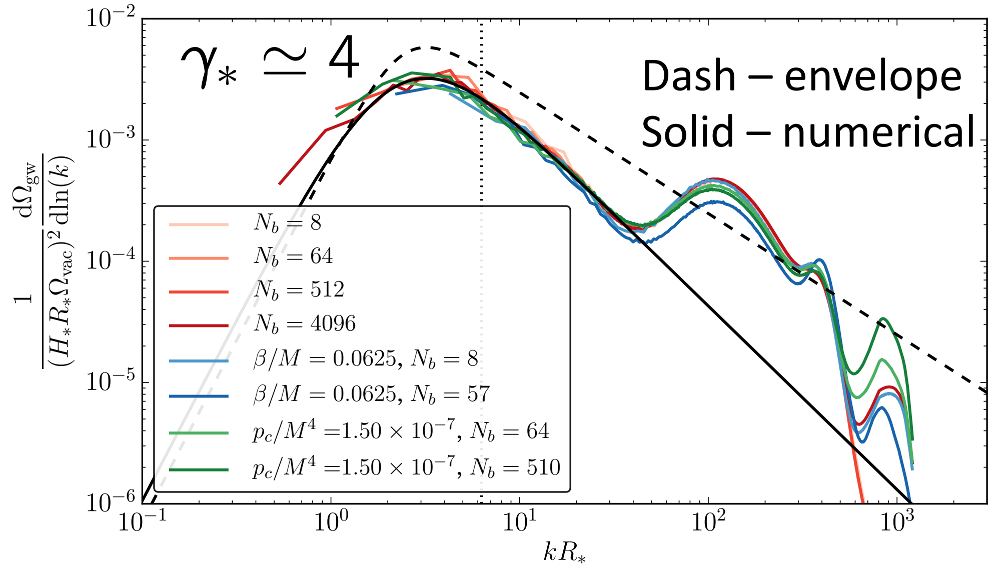
Scalar field only

- History: envelope approximation

  Kamionkowski, Kosowsky, Turner 1994; Huber, Konstandin 2008

- Numerical simulations show differences

  Cutting, MH, Weir 2018



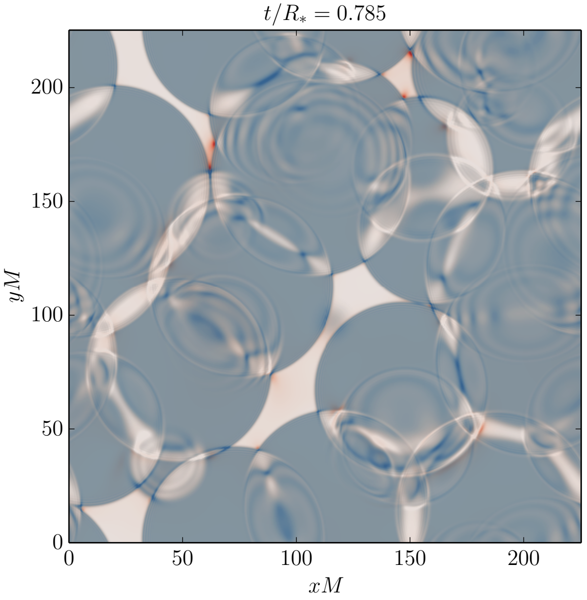
$\gamma_* \simeq 4$

Dash – envelope

Solid – numerical

$$\frac{d\Omega_{\rm gw}^{\rm fit}}{d\ln k} = \Omega_{\rm p}^{\rm fit} \frac{(3+b)^c \tilde{k}^b k^3}{(b\tilde{k}^{(3+b)/c} + 3k^{(3+b)/c})^c}$$

$$\Omega_{\rm p}^{\rm fit} = (3.22 \pm 0.04) \times 10^{-3}\, (H_{\rm n} R_*)^2 \Omega_\phi^2,$$
$$\tilde{k} R_* = 3.20 \pm 0.04,$$
$$b = 1.51 \pm 0.04, \quad c = 2.18 \pm 0.15$$



LISA CWG party line 2019

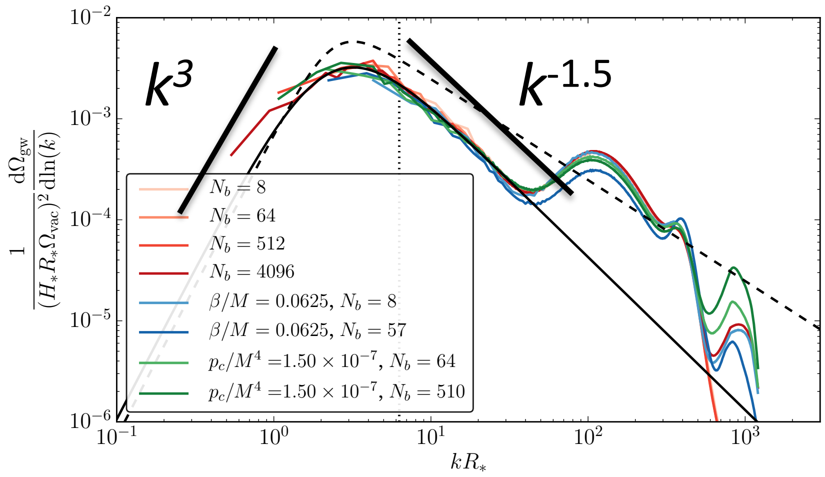
Caprini et al 2019

- Three contributions to total power:
  - Scalar field ϕ
  - Acoustic ac
  - Turbulent tu

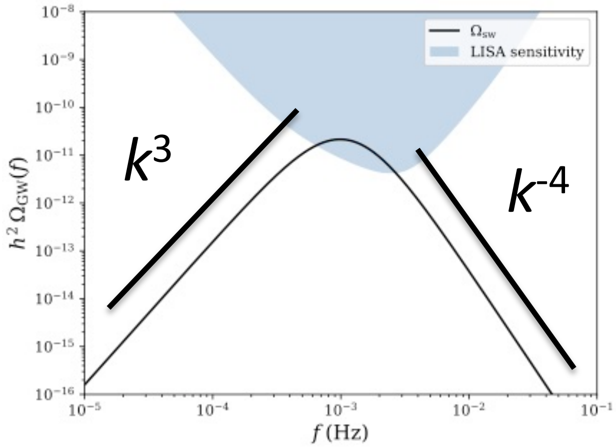
$$\Omega_{\rm gw} = \Omega_{\rm gw}^{\phi} + \Omega_{\rm gw}^{\rm ac} + \Omega_{\rm gw}^{\rm tu}$$

- Scalar field: bubble wall collisions
  - relevant only for runaway walls
- Acoustic production:
  - M.H. et al 2013, 2015, 2017, 2019
- Turbulent production:
  - Uncertain, probably subdominant around peak
  - Conservative estimate: neglect

Runaway



Constant v_w, $\alpha < 0.1$

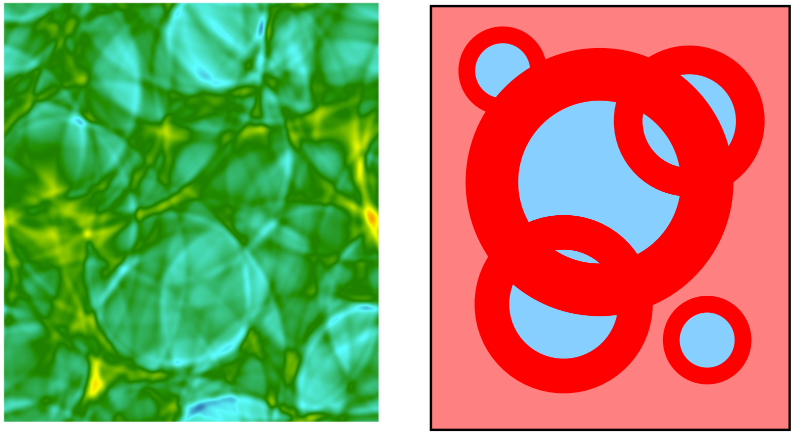


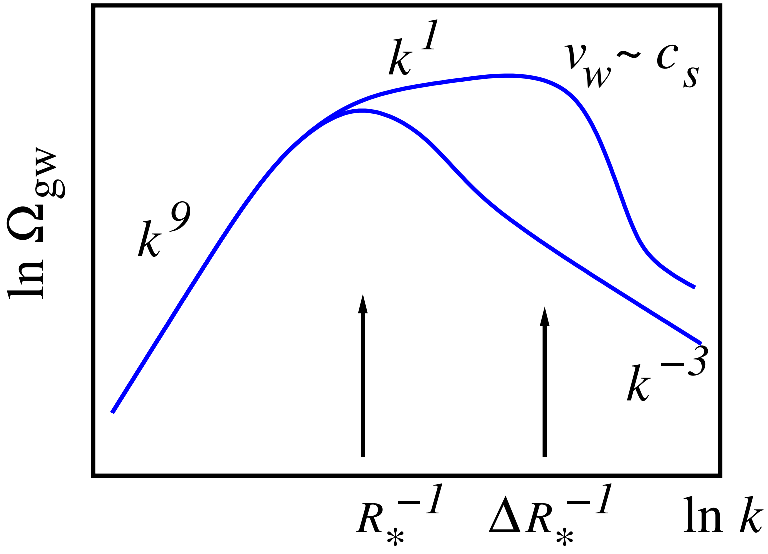
Sound shell model

- Gaussian velocity field from weighted addition of sound shells $\mathbf{v}_\mathbf{q}(t_\mathrm{i})$

  MH 2017, MH, Hijazi (in prep 2019)

- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_* \frac{|v_w - c_s|}{v_w}$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude:
  - Bubble separation
  - (Kinetic energy)^2

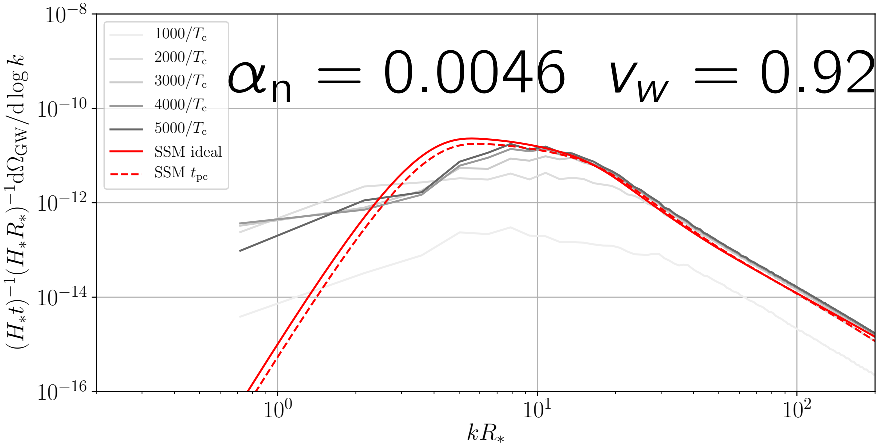


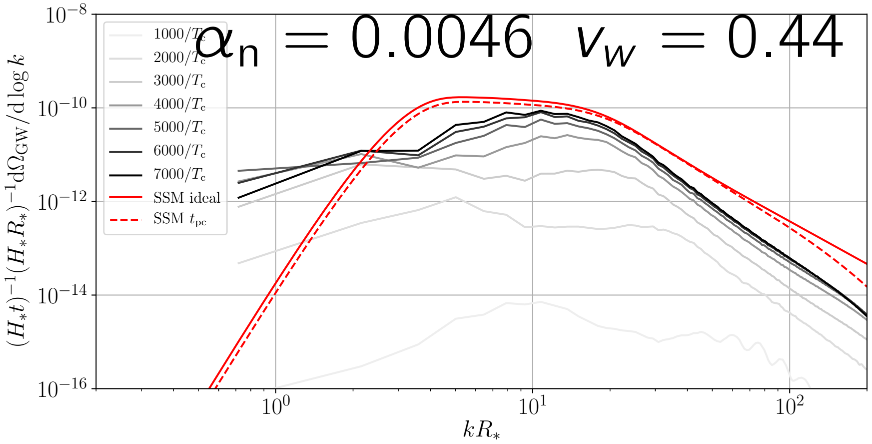


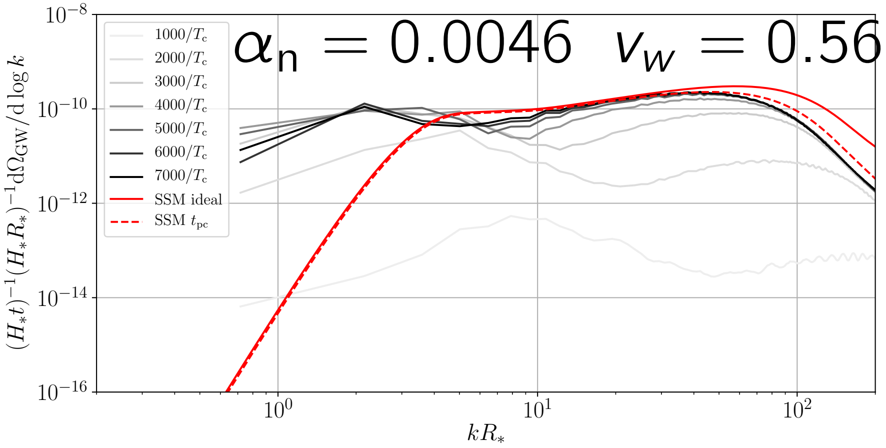
Sound shell model vs. simulations P_{gw}

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

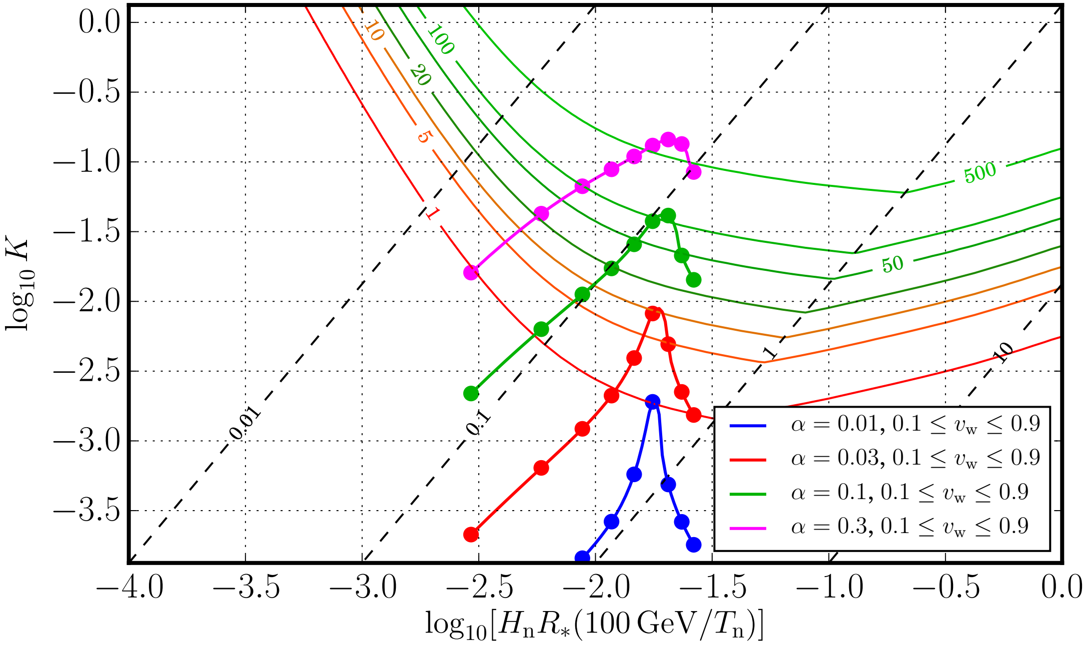
MH et al in prep 2019







Estimated LISA prospects (K,$v_{\rm w}$)



- Estimate signal-to-noise ratio ρ

$$\rho^2 = T_{\rm obs} \int df \left(\frac{\Omega_{\rm gw}(f)}{\Omega_{\rm noise}(f)} \right)^2$$

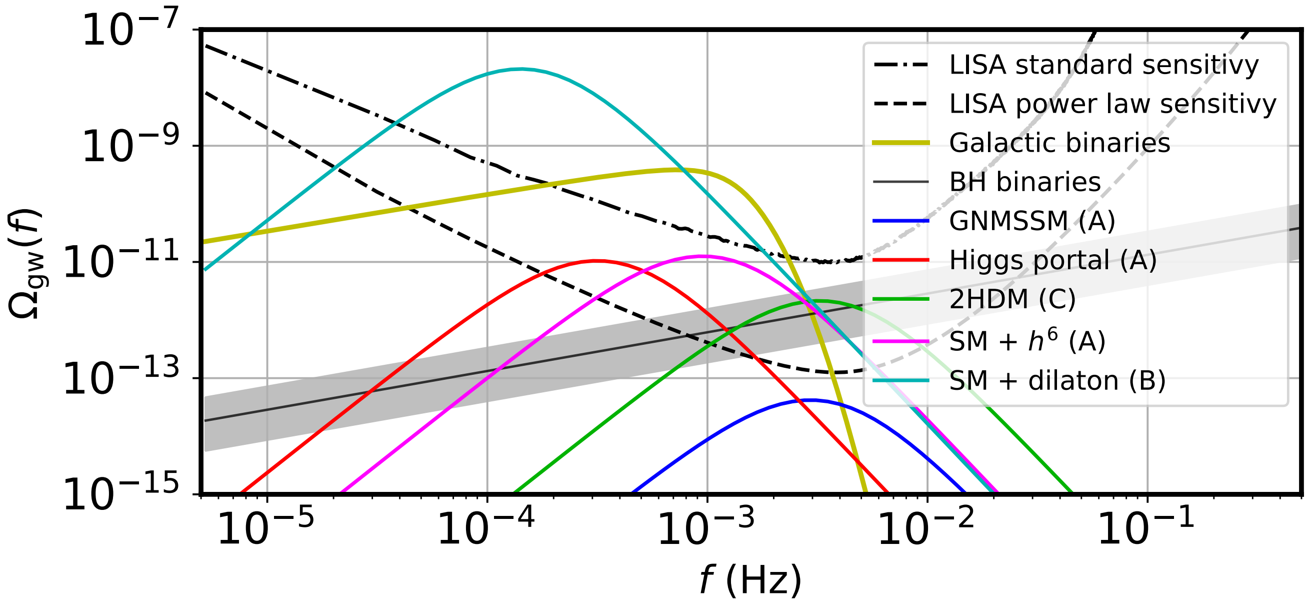
  - Observation time 4 years
  - Neglect foregrounds
    - White Dwarf binaries (annual variation)
    - LIGO BHB precursors (negligible)

- E.g. (favourable cases):
  - $\beta/H_{\rm n} = 100$, $T_{\rm n} = 100$ GeV
- NB $\alpha > 0.1$ highly uncertain

  Cutting, Hindmarsh, Weir
  - But important region for LISA

    Ellis, Lewicki, No (2018)

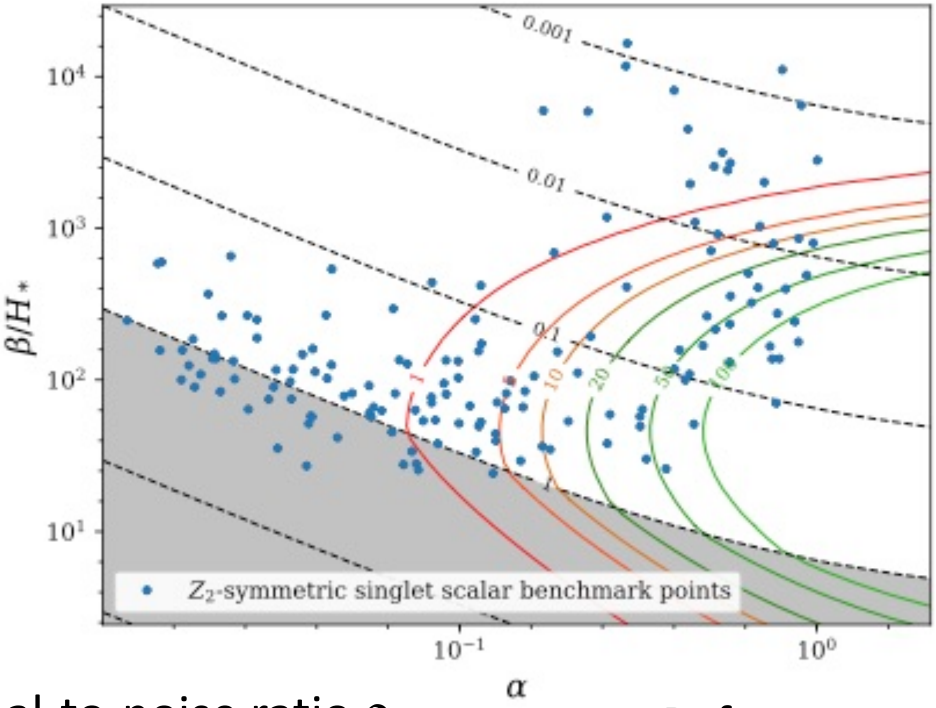
Benchmark models and foregrounds



- White Dwarf binaries
  - Anisotropic, annual variation
- LIGO BHB precursors
  - Below noise, will be well-determined

- Benchmark particle physics models
  - Higgs portal = SM Higgs + scalar
  - 2HDM = 2 Higgs doublet model
  - GNMSSM = general next-to-minimal supersymmetric standard model

Estimated LISA prospects



LISA Cosmology Working Group 2019

- Estimate signal-to-noise ratio ρ

$$\rho^2 = T_{\rm obs} \int df \left(\frac{\Omega_{\rm gw}(f)}{\Omega_{\rm noise}(f)} \right)^2$$

  - Observation time 4 years
  - No foregrounds

- Reference wall speed:
  - $V_w = 0.95$
- NB α > 0.1 highly uncertain

  Cutting, Hindmarsh, Weir
  - But important region for LISA

    Ellis, Lewicki, No (2018)

Summary

- Good understanding of GWs from near-linear flows.
  - $\alpha \leq 0.1$, $v_\mathrm{w} > 0.4$
- Dominant source is sound
- Total power estimate:

$$\Omega_{\mathrm{gw},0} \simeq F_{\mathrm{gw},0} \frac{(H_\mathrm{n} R_*)^2}{H_\mathrm{n} R_* + \sqrt{K}} K^2 \tilde{\Omega}_\mathrm{gw}$$

Standard cosmology:

$$F_{\mathrm{gw},0} = 3.6 \times 10^{-5} \left(\frac{100}{g_\mathrm{eff}}\right)^{\frac{1}{3}}$$

Numerical simulations:

$$\tilde{\Omega}_\mathrm{gw} = \mathrm{O}(10^{-2})$$

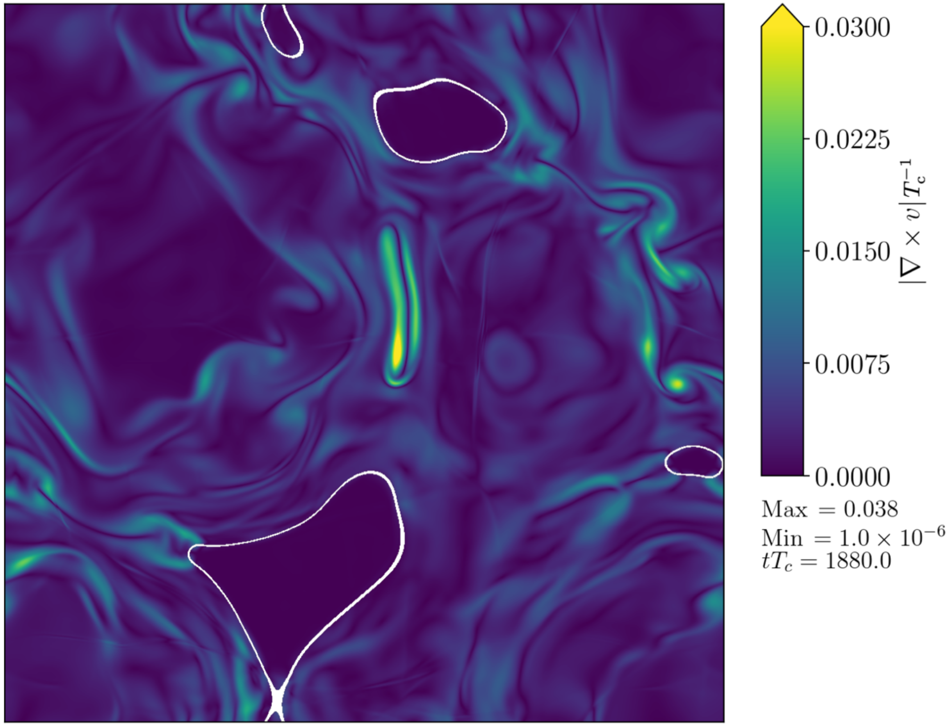
- Naïve extrapolation:
  an upper bound on GWs from PTs: $\Omega_{\mathrm{gw},0} \lesssim 10^{-7}$

Future challenges

- Stronger transitions lead to non-linear evolution, dynamics not understood
  - Longitudinal/compression modes
    - Kinetic energy suppression
    - Shocks, wave turbulence
  - Transverse/rotational modes
    - Vorticity generation
    - Turbulence

Vorticity, strong transition

Cutting, MH, Weir 2019

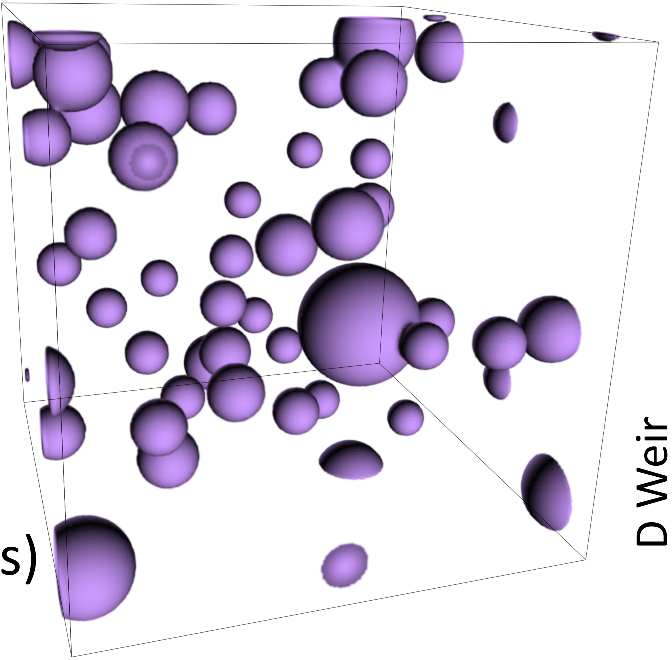


  - Turbulence less efficient at producing GWs?

Roper pol et al 2019

Conclusions

- GWs probe physics at very high energy
  - LISA will probe physics of Higgs transition from 2034
  - Measure/constrain phase transition parameters
- Towards accurate calculations of GW power spectrum from parameters
  - Some understanding of acoustic production, probably dominant
  - Non-linear evolution (turbulence, shocks) not well understood
- Applies to 1st order PTs at all scales



D Weir

$f_{p.0} \simeq 26(H_\mathrm{n} R_*)^{-1}(T_\mathrm{n}/100\,\mathrm{GeV})\,\mu\mathrm{Hz}$ $\quad h_c \lesssim 3 \times 10^{-19}\,(10^2\,\mathrm{GeV}/T_\mathrm{n})$

Back-up slides

D Weir

GWs from phase transitions

$$\Omega_{\rm GW} \sim \frac{1}{G\rho}\left\langle \left|\dot{h}_{ij}(t)\right|^2 \right\rangle$$

- Gravitational waves generated by shear stress fluctuations

$$\dot{h}_{ij} \sim G \int dt' \cos[k(t-t')] T^{TT}_{ij}(k,t')$$

- Shear stress ~ kinetic energy

$$T_{ij} \sim \rho U_i U_j$$

- Kinetic energy from potential energy
  - *K(α,v$_w$)* = fluid kinetic energy fraction

$$\dot{h} \sim \tau(G\rho)K$$

- Timescales τ_v and τ_c
  - τ_v duration of stresses from fluid velocity
  - τ_c coherence time of stress fluctuations

$$\Omega_{\rm gw} \sim \frac{\tau_v \tau_c}{G\rho}(G\rho)^2 K^2$$

$$\Omega_{\rm gw} \sim (H_n\tau_v)(H_n\tau_c)K^2$$

$$\Omega_{\rm gw,0} \sim \Omega_{\rm rad,0}(H_n\tau_v)(H_*\tau_c)K^2$$

GWs from turbulent flows

- Modelling

Green: Gogoberidze, Kahniashvili, Kosowsky 2007
Black: Caprini, Durrer, Servant 2008

Blue: Niksa, Schlederer, Sigl 2018

  - Kraichnan sweeping model: velocity autocorrelation time

$$\tau_k \sim 1/k\bar{v}_\perp$$

  - Pure rotational flow: high *k* GW power spectrum $k^{-5/3}$
  - Mixed acoustic-turbulent $k^{-8/3}$

- MHD simulation

Roper pol et al 2019