The anomalous magnetic moment of the muon

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The Muon g-2 Theory Initiative

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Outline

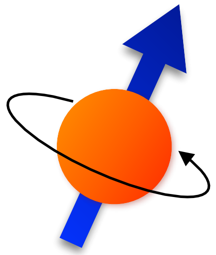
- The Story of the Muon
- Lepton moments
- Muon g-2 Theory Initiative
- Hadronic Vacuum Polarization
  - data driven methods
  - Introduction to lattice QCD
  - lattice QCD+QED calculations
- Hadronic Light-by-Light
  - dispersive methods
  - lattice QCD+QED calculations
- Summary and Outlook

INT workshop slides:
https://indico.fnal.gov/event/21626/

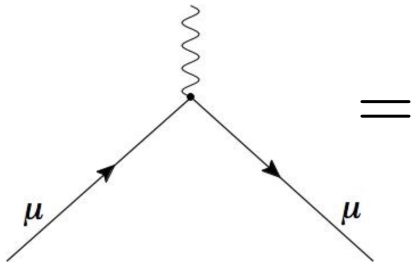
White Paper:
[T. Aoyama et al, arXiv:2006.04822]

Anomalous magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\dfrac{e}{2m}\vec{S}$

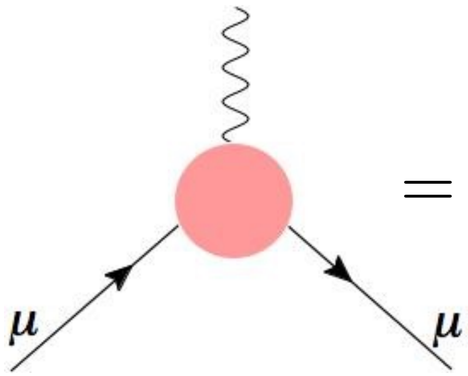


Dirac (leading order): $g = 2$



$$= (-ie)\,\bar{u}(p')\gamma^{\mu}u(p)$$

Quantum effects (loops):



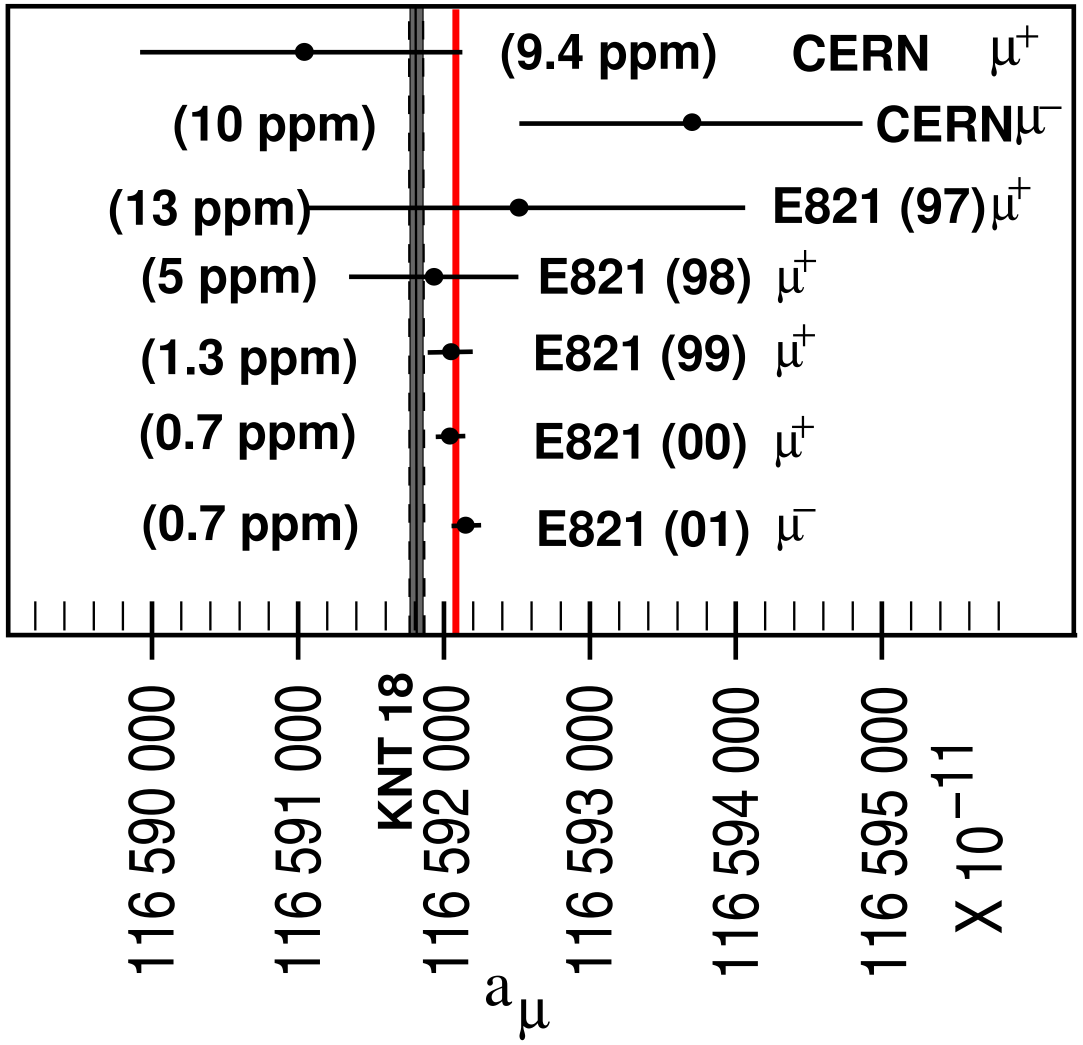
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

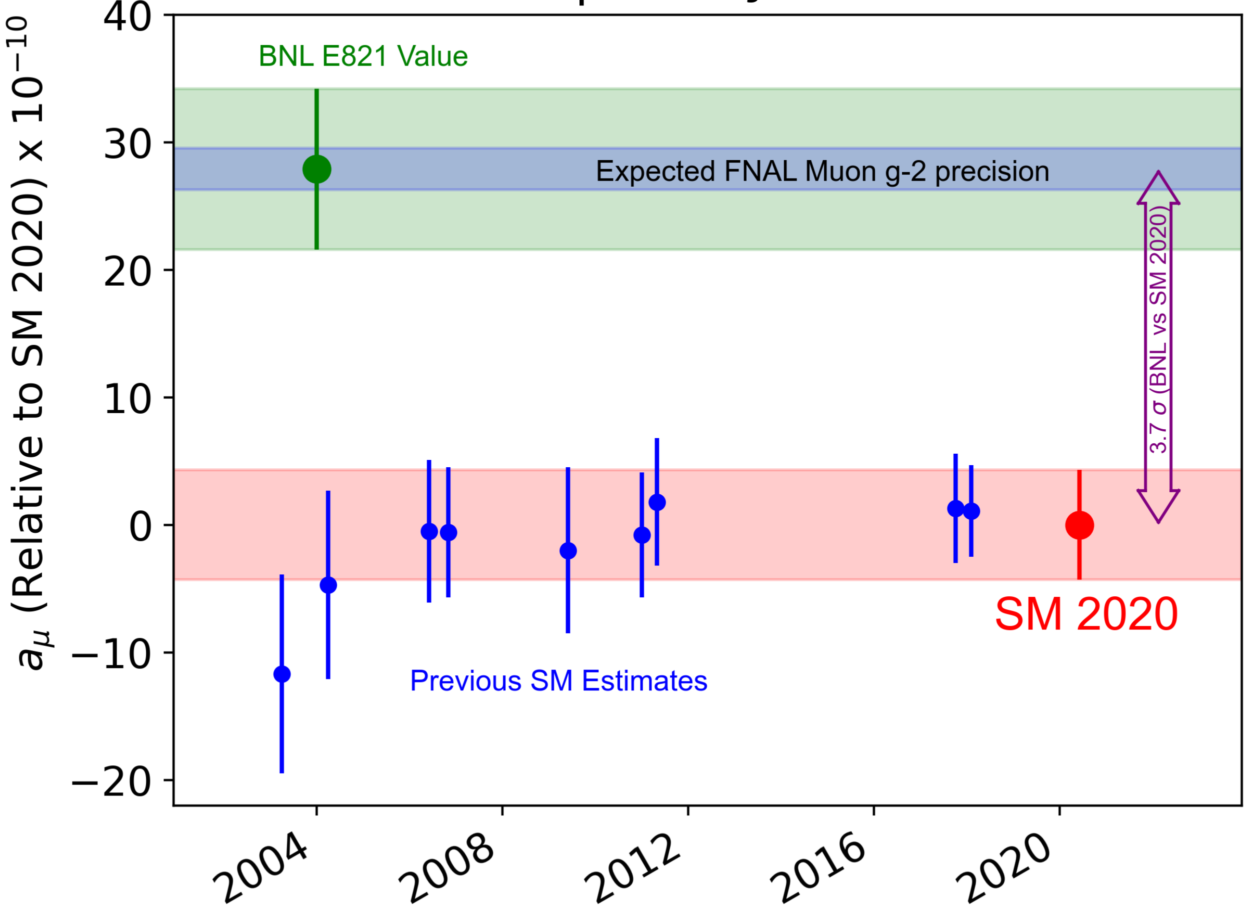
Anomalous magnetic moment: $\boxed{a \equiv \dfrac{g-2}{2} = F_2(0)}$

Muon g-2: experiment vs theory

[L. Roberts, arXiv:1811.06974]

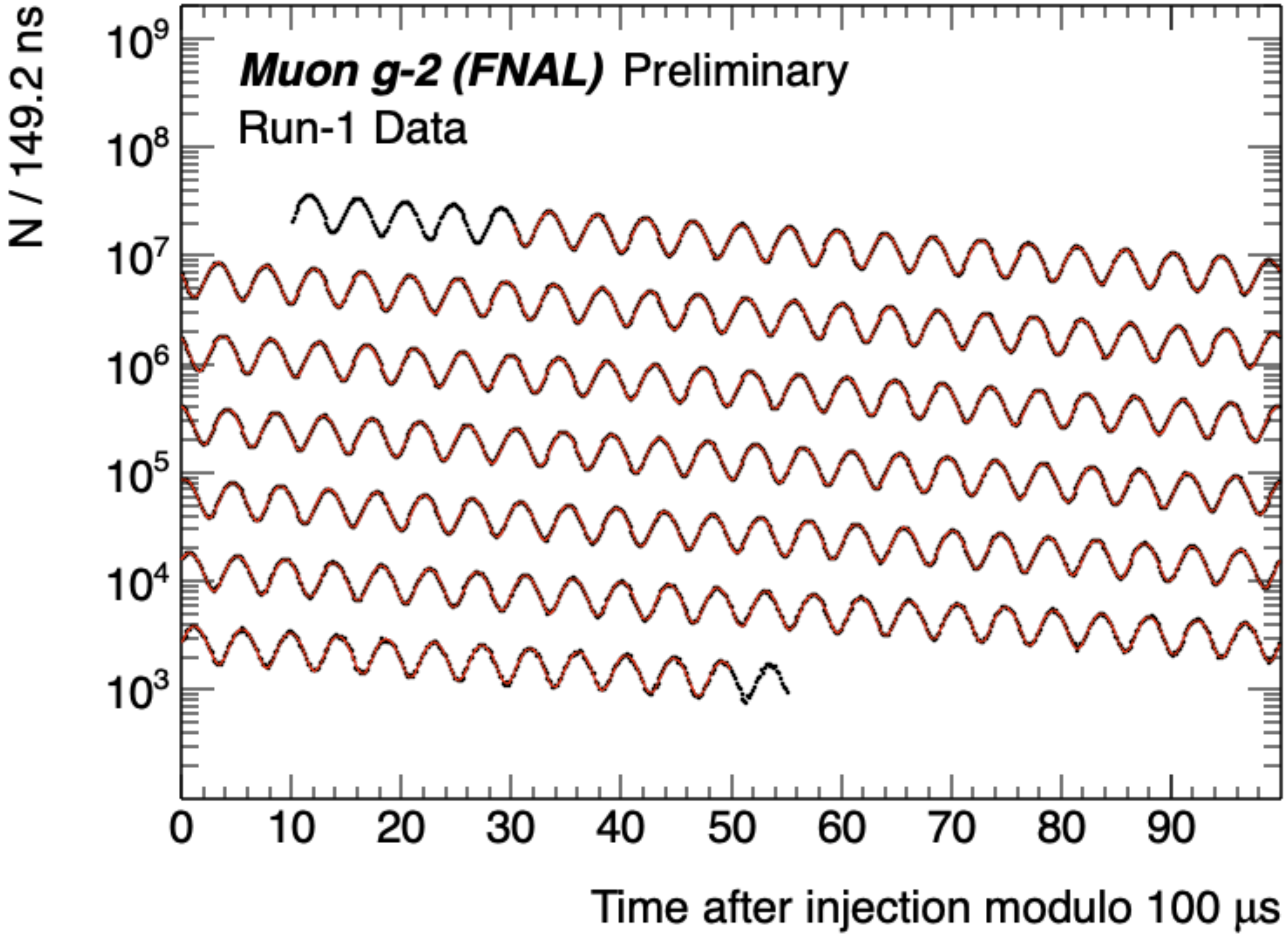


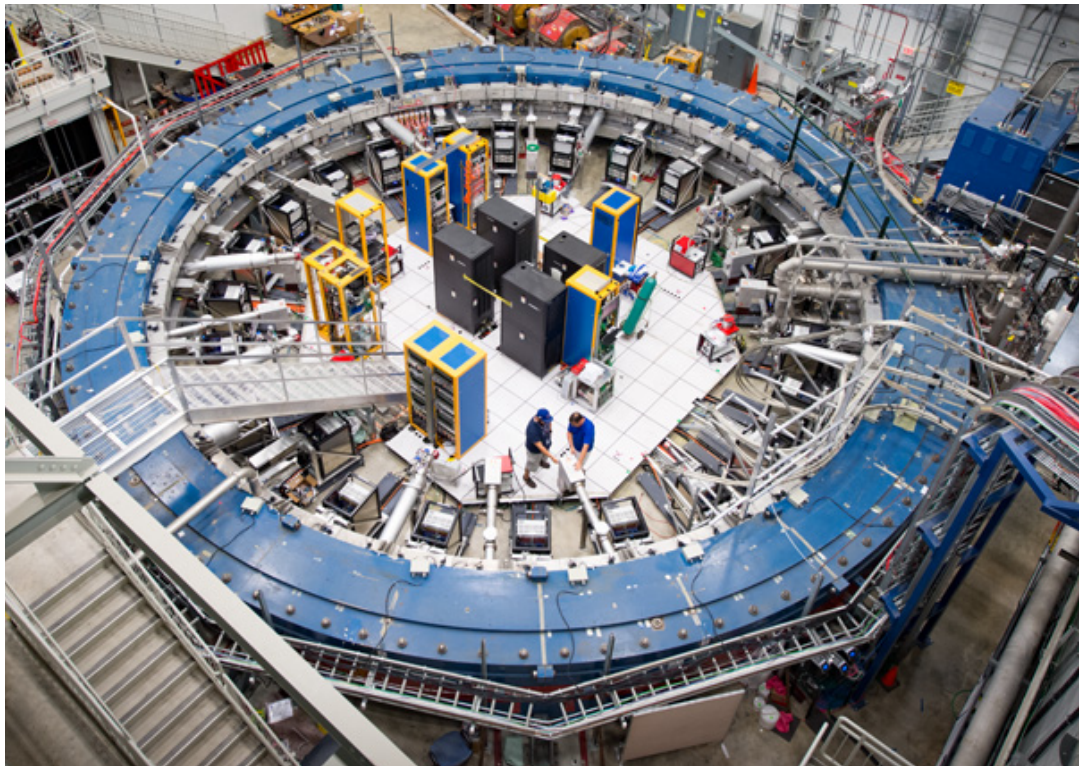
plot by Mark Lancaster



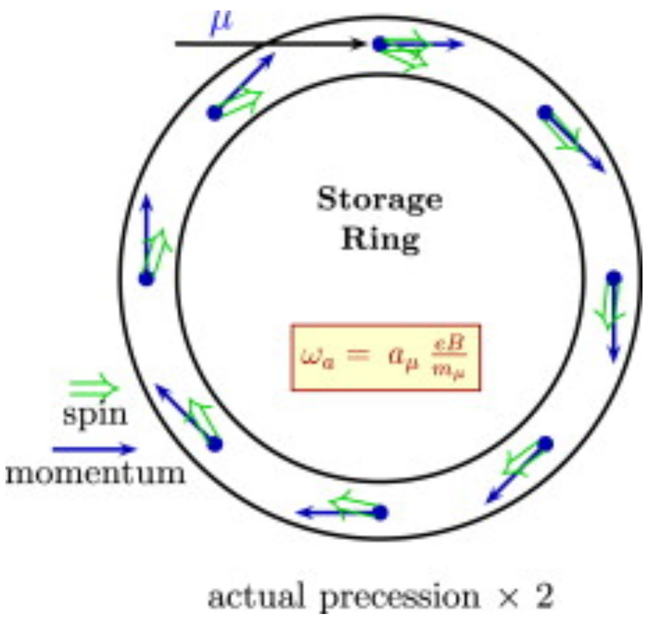
Muon g-2: experiment

David Hertzog for E989 @ INT g-2 workshop





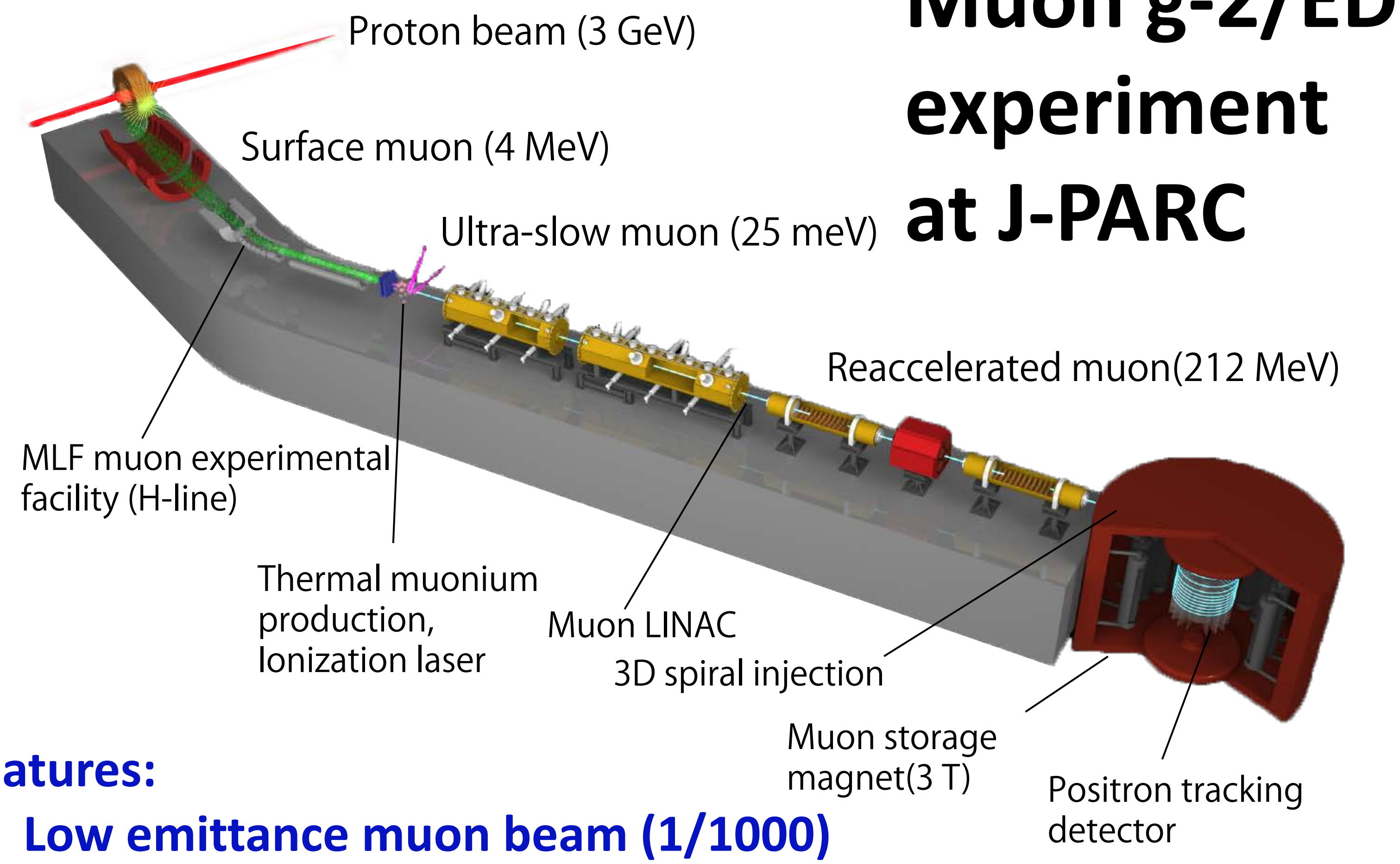
Analysis of data from run 1 is underway.
Expect public release in 2020.



Muon g-2: experiment

T. Mibe for E34 @ INT g-2 workshop

Muon g-2/EDM experiment at J-PARC



Features:

- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**
- **Completely different from BNL/FNAL method**

- 2018:
  Stage II approval by IPNS and IMSS directors.
- March 2019:
  Endorsed by KEK-SAC as a near-term priority
- 2020:
  Funding request
- 2024-2026:
  data taking runs

Lepton g-2: SM contributions

$$\boxed{a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})}$$

Lepton g-2: SM contributions

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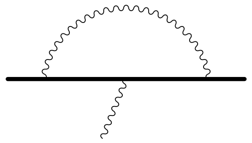
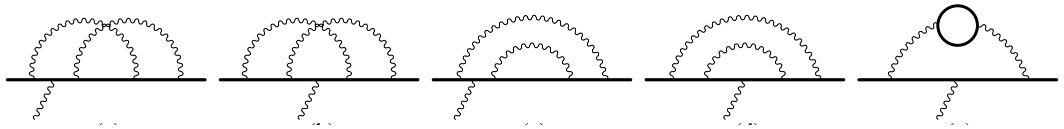
QED

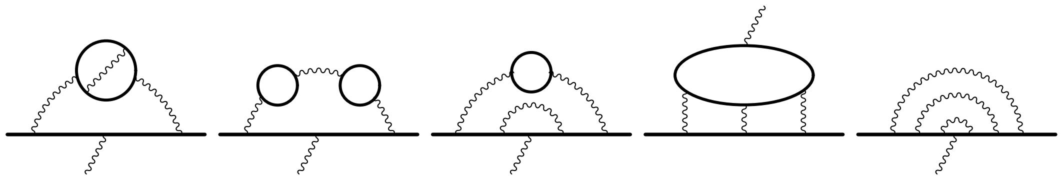
α :



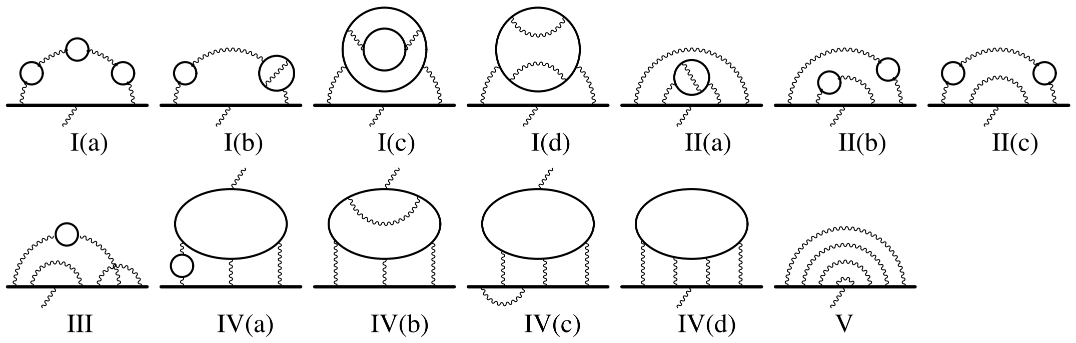
α^2 :



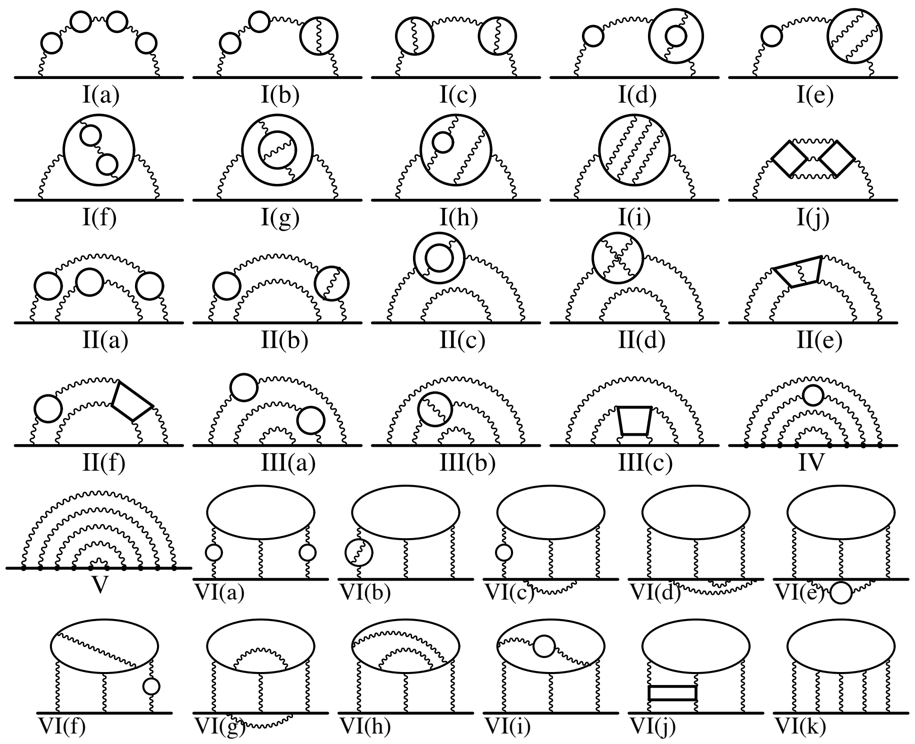
α^3 :



α^4 :



α^5 :

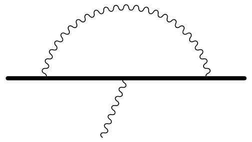


Lepton g-2: SM contributions

$$\boxed{a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})}$$

QED

α :

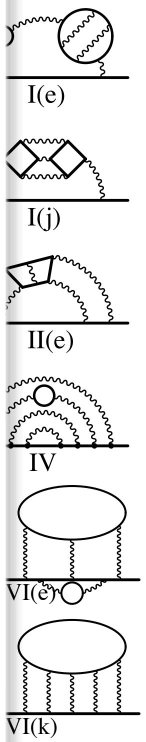


α^5 :

α^2 :

α^3 :

α^4 :



QED corrections start with $\alpha/(2\pi)$.

The complete 5th-order calculation yields:

$$a_\mu^{\mathrm{QED}}(\alpha(\mathrm{Cs})) = 116\,584\,718.931\,(104) \times 10^{-11}$$

[T. Aoyma et al, 2012, 2019, Laporta 2017,…]

uncertainty dominated by $\mathcal{O}(\alpha^6)$ contributions

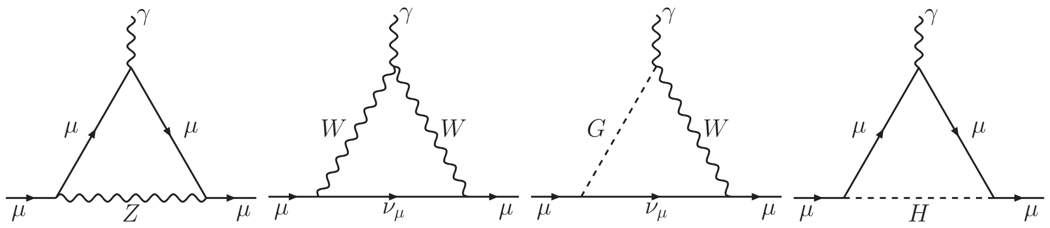
Lepton g-2: SM contributions

$$\boxed{a_\ell = a_\ell(\text{QED}) + a_\ell(\text{EW}) + a_\ell(\text{hadronic})}$$

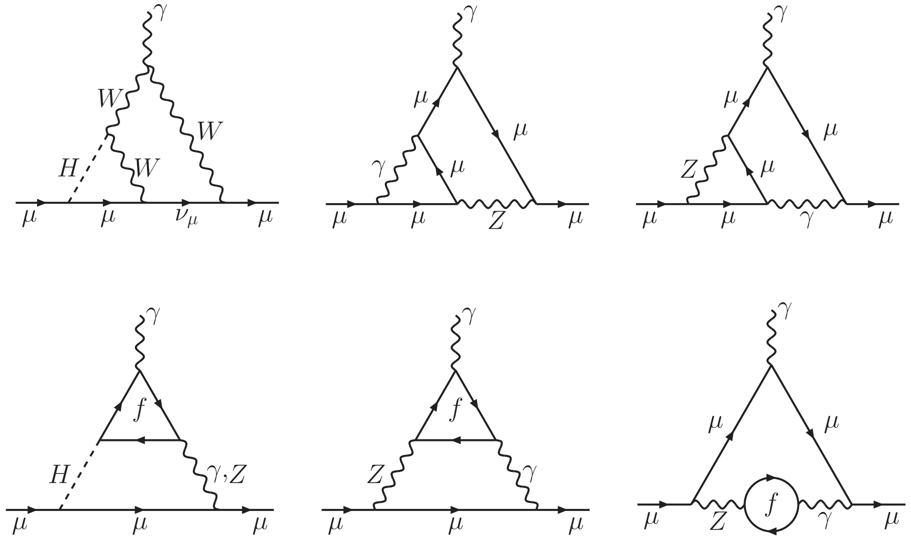
Electroweak

(contributions from W,Z,H,.. bosons)

1-loop



2-loop



Lepton g-2: SM contributions

$$\boxed{a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})}$$

Electroweak

(contributions from W,Z,H,.. bosons)

1-loop

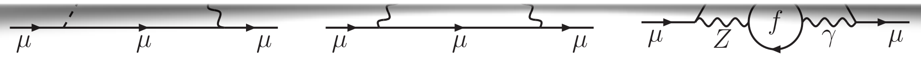
Complete 2nd-order calculation yields:

$$a_\mu^{\mathrm{EW}} = 153.6\,(1.0) \times 10^{-11}$$

[Gnendiger et al, 2013]

uncertainty dominated by hadronic loops.

2-

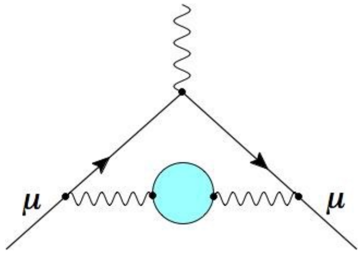


Lepton g-2: SM contributions

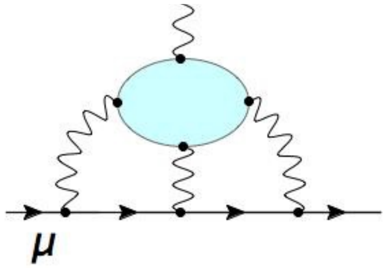
$$\boxed{a_\ell = a_\ell(\mathrm{QED}) + a_\ell(\mathrm{EW}) + a_\ell(\mathrm{hadronic})}$$

leading hadronic

α^2



α^3



✦ The complete hadronic contributions are written as:

$$a_\ell(\mathrm{hadronic}) = a_\ell^{\mathrm{HVP,\,LO}} + a_\ell^{\mathrm{HVP,\,NLO}} + a_\ell^{\mathrm{HVP,\,NNLO}}$$

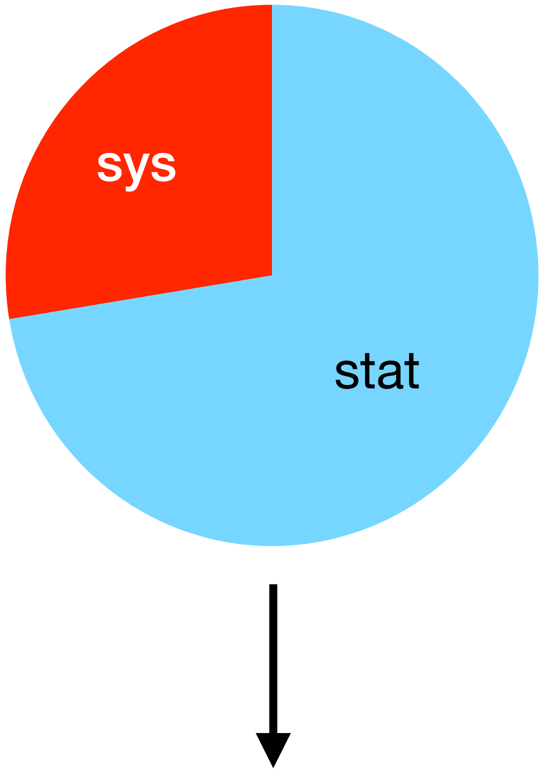
$$+ a_\ell^{\mathrm{HLbL}} + a_\ell^{\mathrm{HLbL,\,NLO}}$$

α^2 α^3 α^4

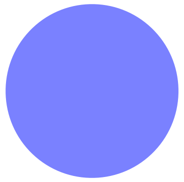
$\sim \quad 10^{-7}$

Muon g-2: experiment vs theory

BNL E821

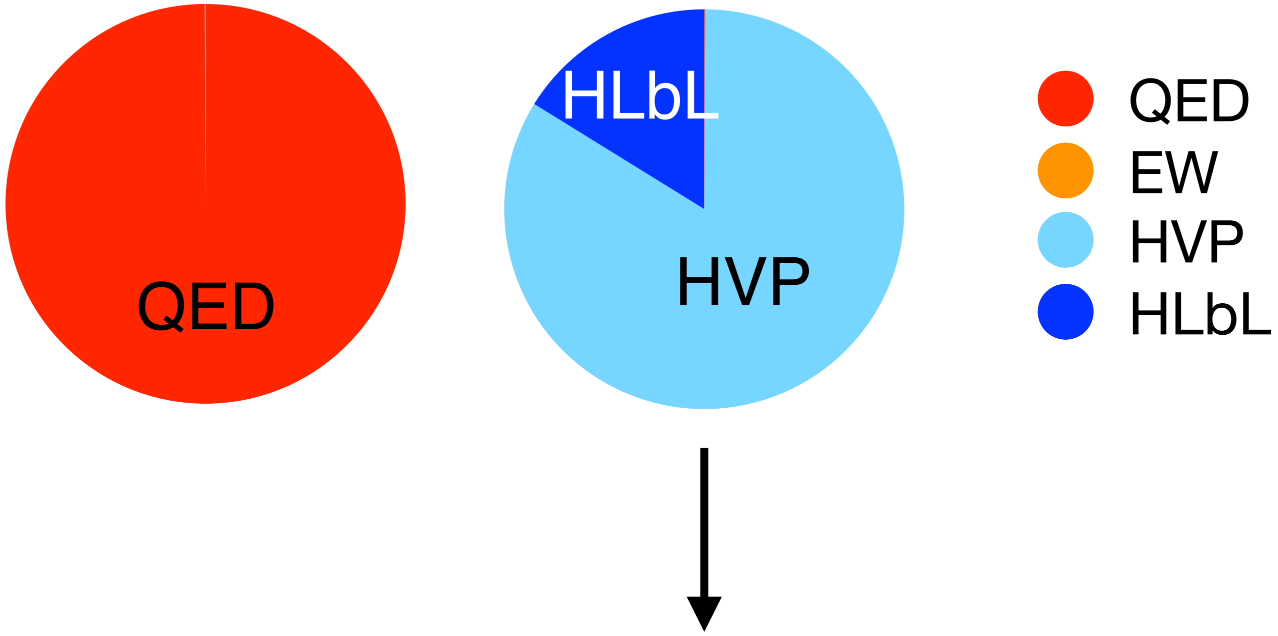


FNAL E989 + J-PARC E34



SM theory

contribution error^2



Muon g-2: experiment vs theory

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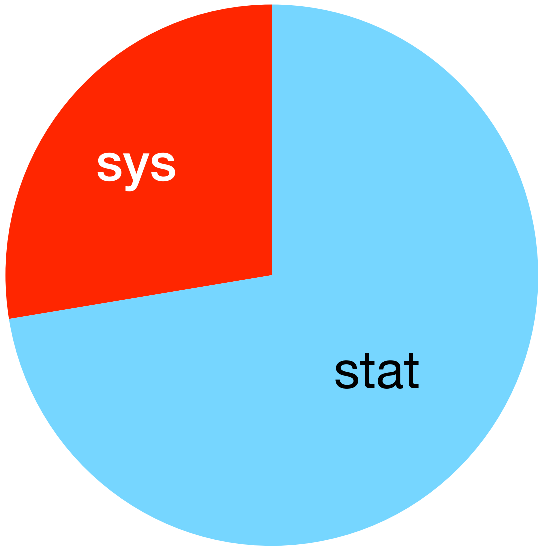

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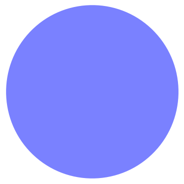
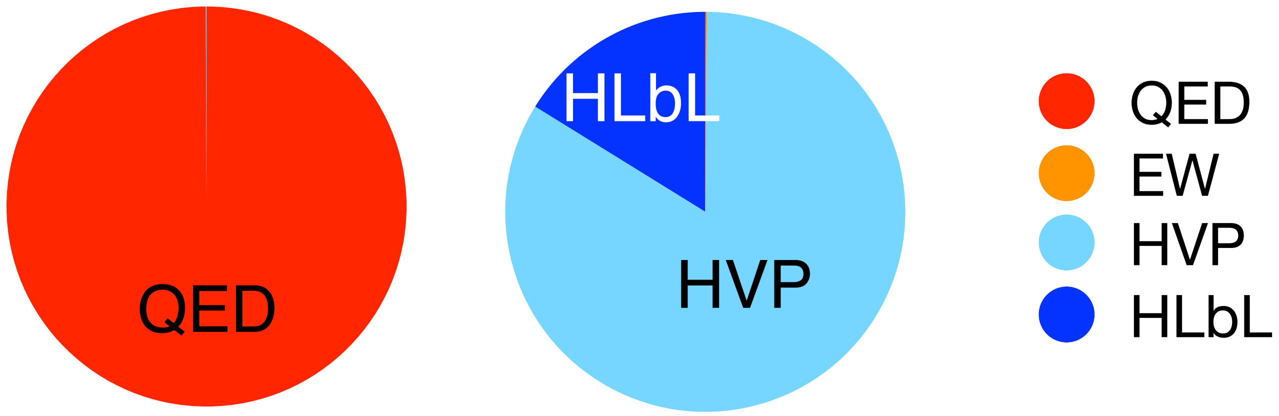


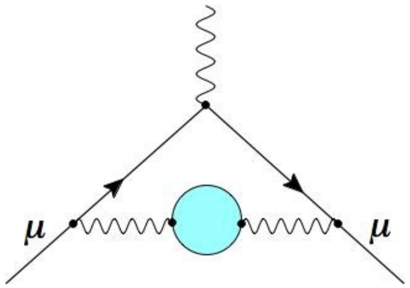
SM theory

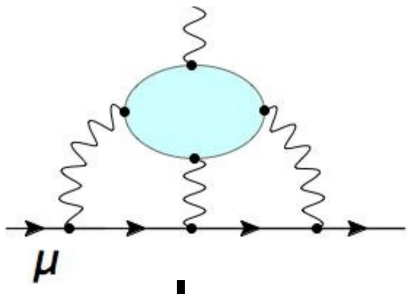


contribution

error^2

coordinated effort:
Muon g-2 Theory Initiative

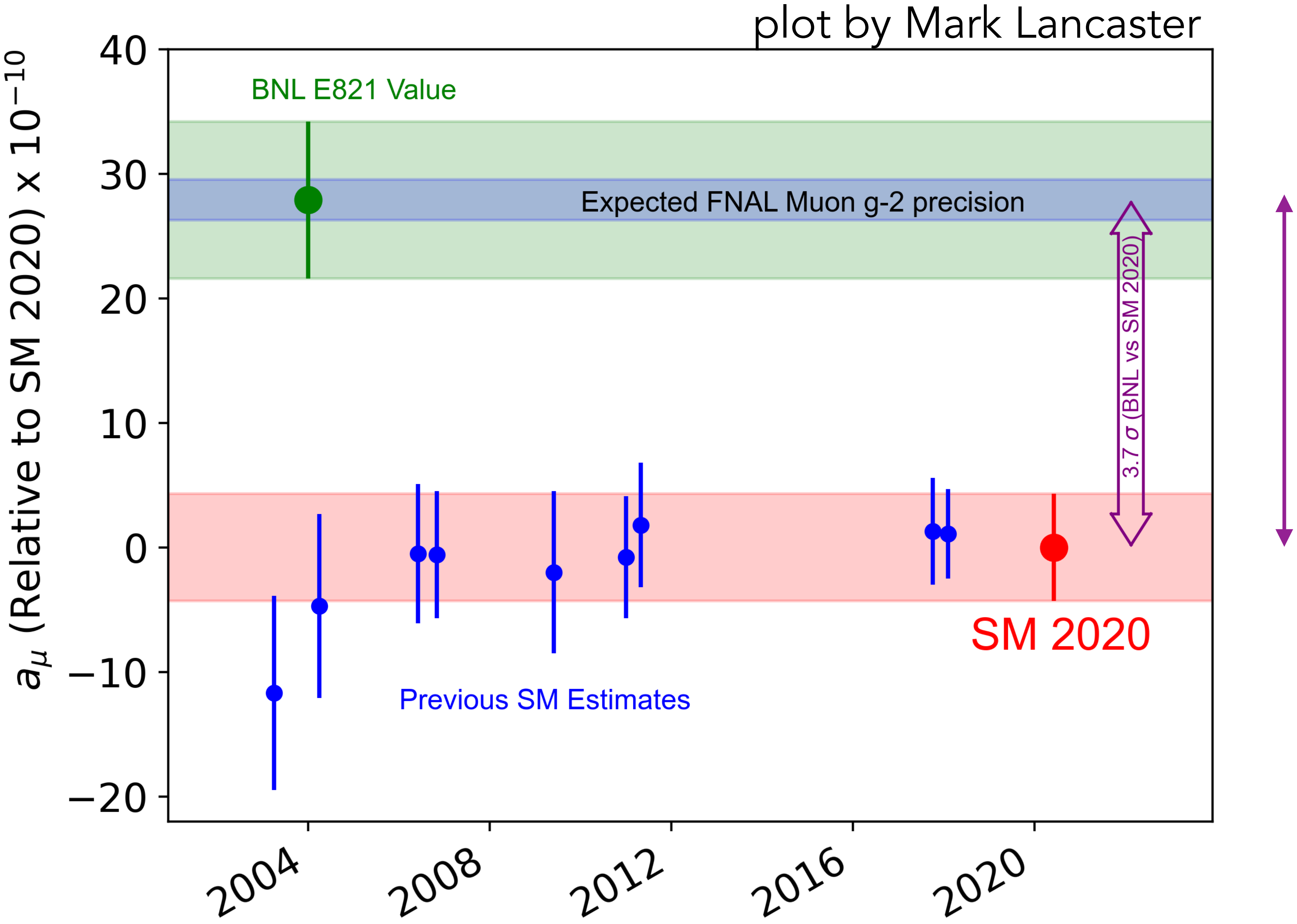




~0.2 %

10 %

Muon g-2: experiment vs theory



Without theory improvements:

$\Delta a_\mu \sim 6\sigma$

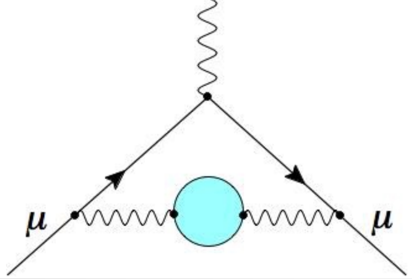
With theory improvements:

$\Delta a_\mu > 10\sigma$

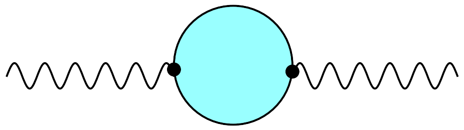
Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Fourth plenary workshop @ KEK: (1-5 June 2020) **postponed to 2021**

- White Paper posted 10 June 2020: [T. Aoyama et al, arXiv:2006.04822]
  132 authors, 82 institutions, 21 countries



Hadronic vacuum polarization

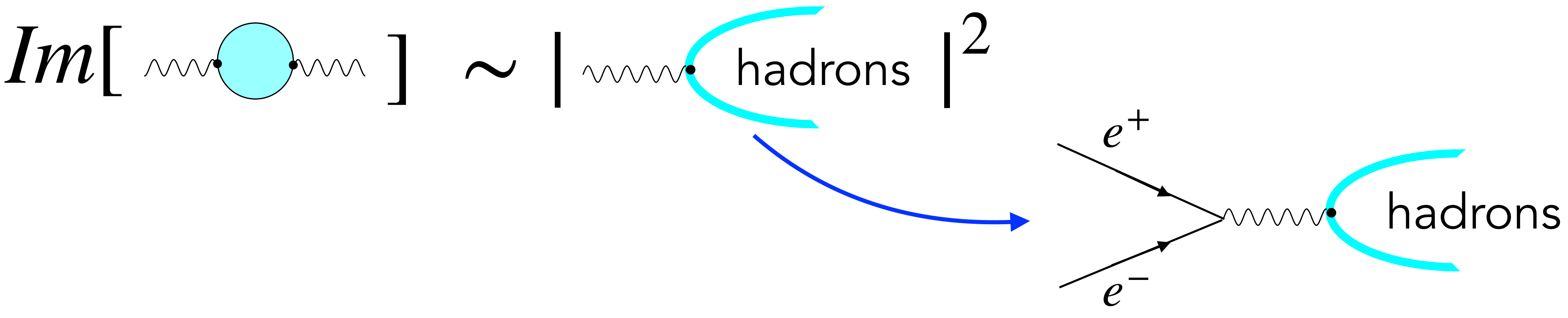


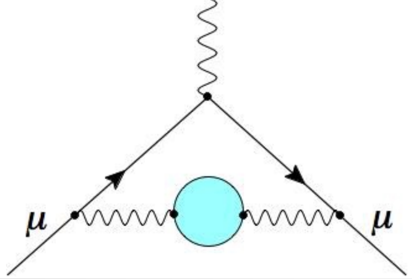
$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

Leading order HVP correction: $\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)}$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:





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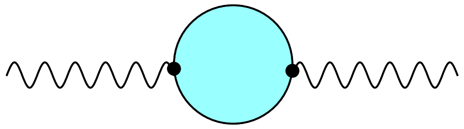

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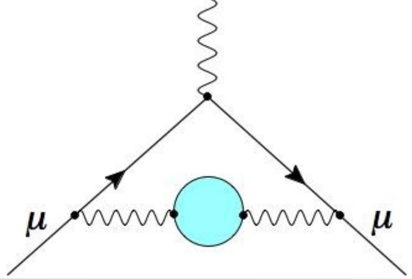
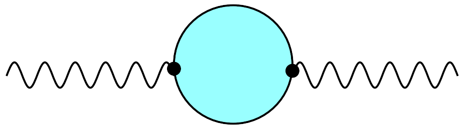
$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\mathrm{exp}}(s)}$$



Hadronic vacuum polarization



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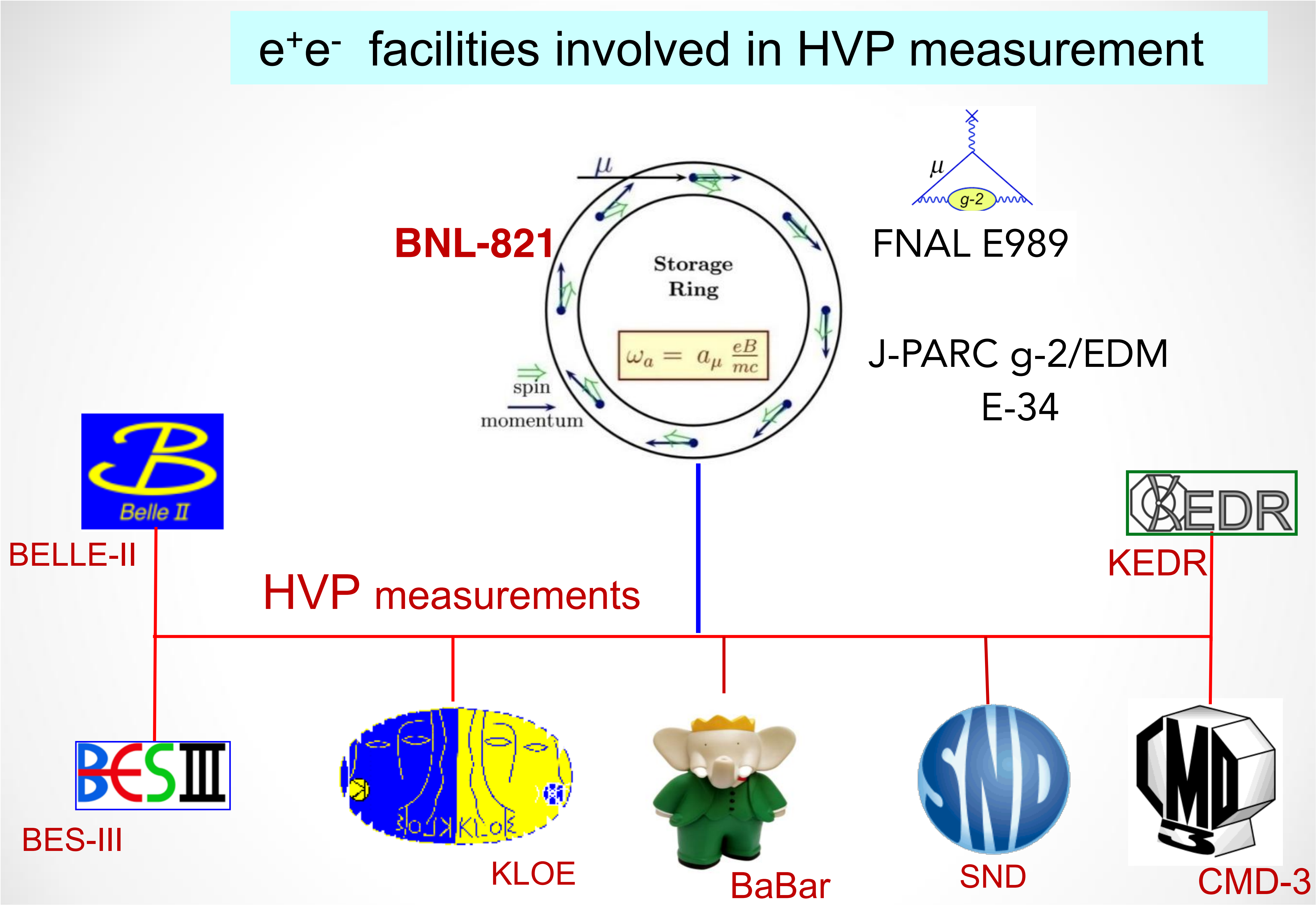
Dominant contributions from low energies

$\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{HVP,LO}}$

Experimental Inputs to HVP

S. Serednyakov (for SND) @ HVP KEK workshop



e^+e^- facilities involved in HVP measurement

BNL-821

FNAL E989

J-PARC g-2/EDM
E-34

HVP measurements

BELLE-II

KEDR

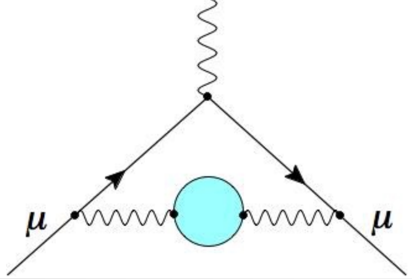
BES-III

KLOE

BaBar

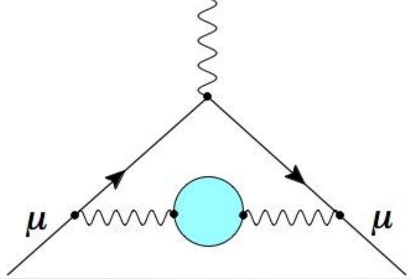
SND

CMD-3



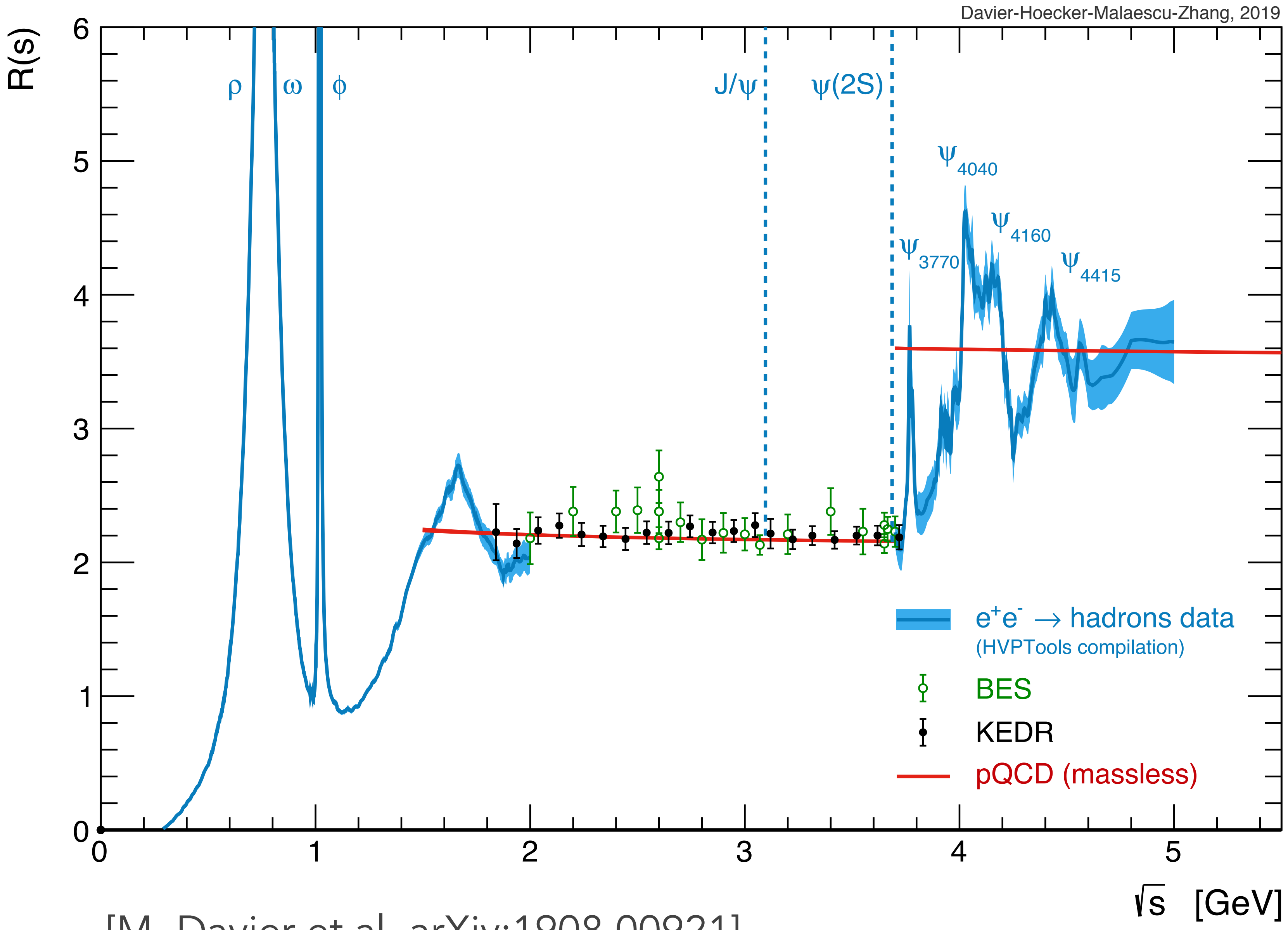
Hadronic vacuum polarization

- ✦ Target: ~0.2% total error
- ✦ Dispersion relation + experimental data for $e^+e^- \to \mathrm{hadrons}$ (and τ data)
  - current uncertainty ~0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,….
- ✦ Challenges:
  - below ~2 GeV: sum > 30 exclusive channels: 2π, 3π, 4π, 5π, 6π, $2K$, $2K\pi$, $2K2\pi$, $\eta\pi$,…. (use isospin relations for missing channels)
  - above ~1.8 GeV:
    inclusive, pQCD (away from flavor thresholds)
    + narrow resonances (J/ψ, Υ,..)
  - Combine data from different experiments/measurements:
    understanding correlations, sources of sys. error, tensions…
  - include FS radiative corrections

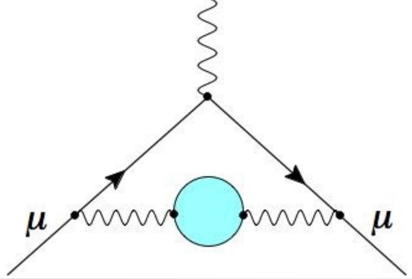


Hadronic vacuum polarization

Z. Zhang for DHMZ @ INT g-2 workshop:



[M. Davier et al, arXiv:1908.00921]



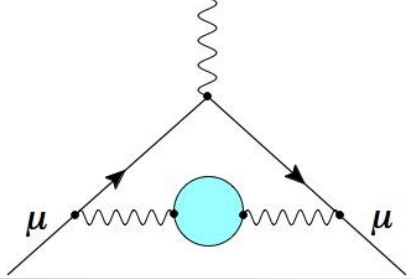
Hadronic vacuum polarization

Z. Zhang for DHMZ @ INT g-2 workshop: [M. Davier et al, arXiv:1908.00921]

	Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$	$\Delta\alpha(m_Z^2)[10^{-4}]$
→	$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
	$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
→	$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
	$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
	$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
	$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
	$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
→	$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
	$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
	$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
→	$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
→	$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
	$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
→	$\eta\pi^+\pi^-\pi^0$(non-ω, ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
	$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
	$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
	$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
	$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
→	ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
→	K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
	$K_S K_L$	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
	ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
→	$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
	$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
	$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.00 \pm 0.00$
→	$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
	$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
	$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
	$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
→	7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
	J/ψ (BW integral)	6.28 ± 0.07	7.09 ± 0.08
	$\psi(2S)$ (BW integral)	1.57 ± 0.03	2.50 ± 0.04
	R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
	R_{QCD} $[1.8 - 3.7\ \text{GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$	$24.27 \pm 0.18 \pm 0.28_{\text{dual}}$
	R_{QCD} $[5.0 - 9.3\ \text{GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.17
	R_{QCD} $[9.3 - 12.0\ \text{GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
	R_{QCD} $[12.0 - 40.0\ \text{GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.12
	R_{QCD} $[> 40.0\ \text{GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
	R_{QCD} $[> 40.0\ \text{GeV}]_{t}$	0.00 ± 0.00	-0.72 ± 0.01
	Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$	$275.42 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_\psi \pm 0.55_{\text{QCD}}$

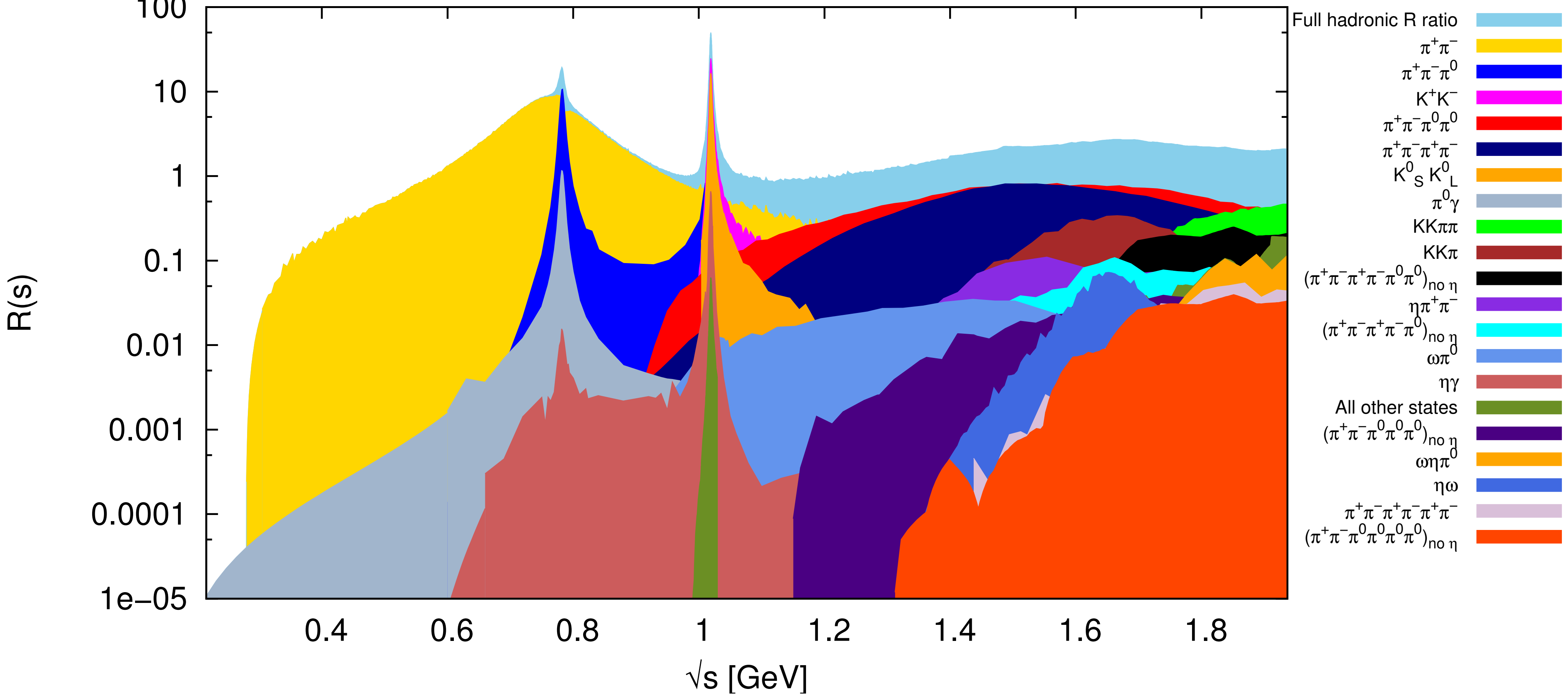
Essentially all exclusive channels (>30) below 1.8 GeV are included thanks mainly to measurements in many modes from BABAR (including the recent $\pi^+\pi^-3\pi^0$)

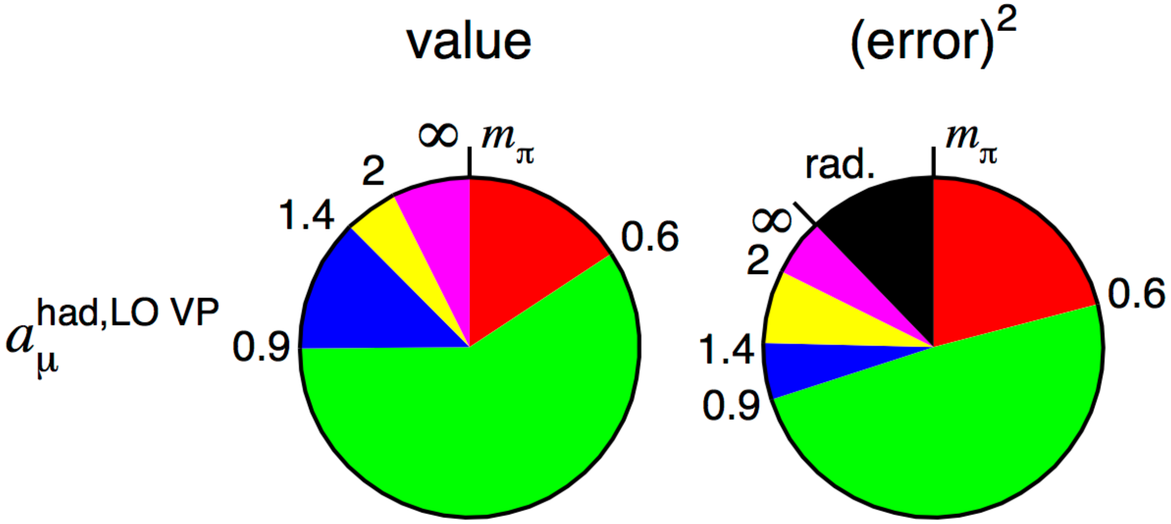
Estimation for missing modes based on isospin constraints becomes negligible (0.016%)

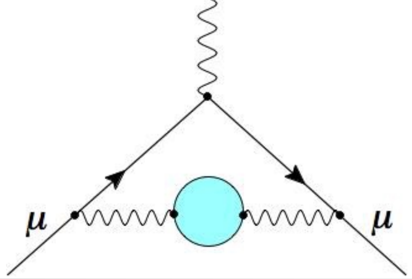


Hadronic vacuum polarization

[A. Keshavarzi et al, arXiv:1802.02995]



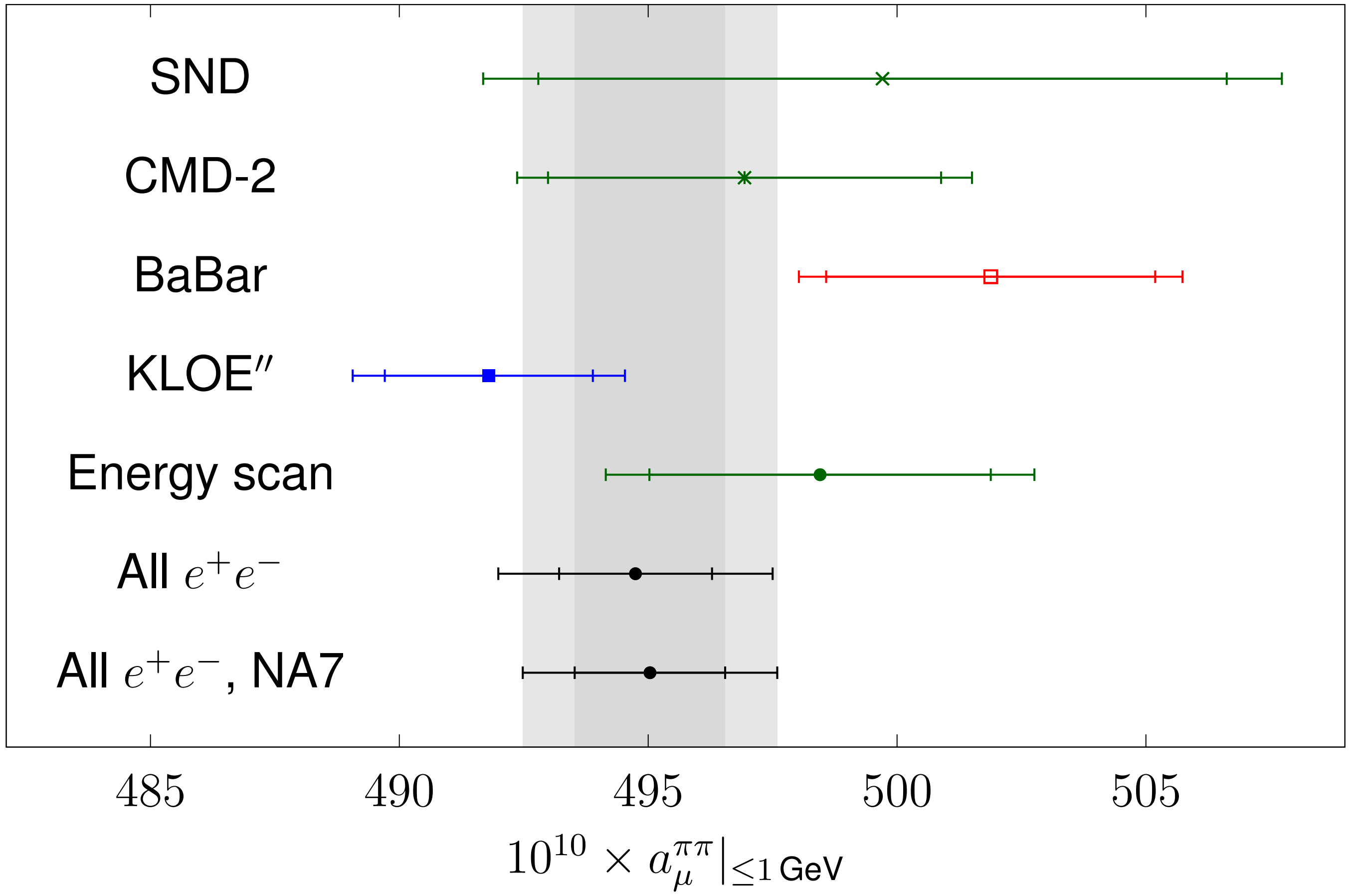




Hadronic vacuum polarization

P. Stoffer @ INT g-2 workshop:

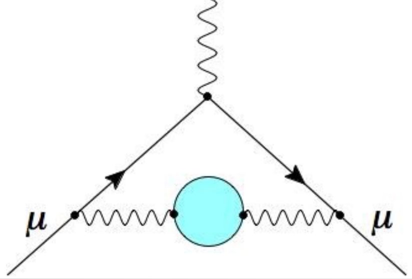
[CHS, G. Colangelo et al, arXiv:1810.00007]



- unitarity and analyticity:
  ➠ relation between pion form factor and $\pi\pi$ scattering
- global fit function
- test of direct integration methods
- also yields better determinations of P-wave phase shift and pion charge radius

Similar analysis also for $\pi\pi\pi$ channel

[HHKS, Hoferichter et al, arXiv:1907.01556]



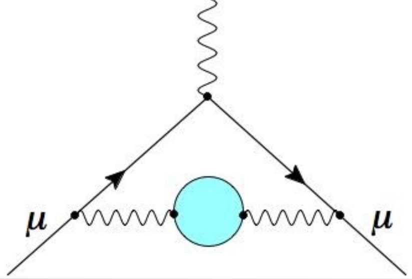
Hadronic vacuum polarization

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, HHKS 2019]



Hadronic vacuum polarization

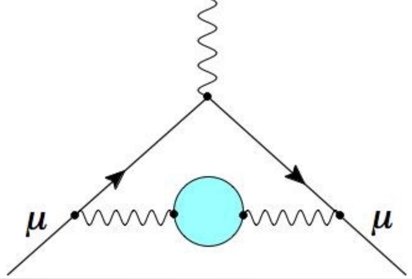
Conservative merging procedure

[B. Malaescu @ INT g-2 workshop]

to obtain a realistic assessment of the underlying uncertainties:

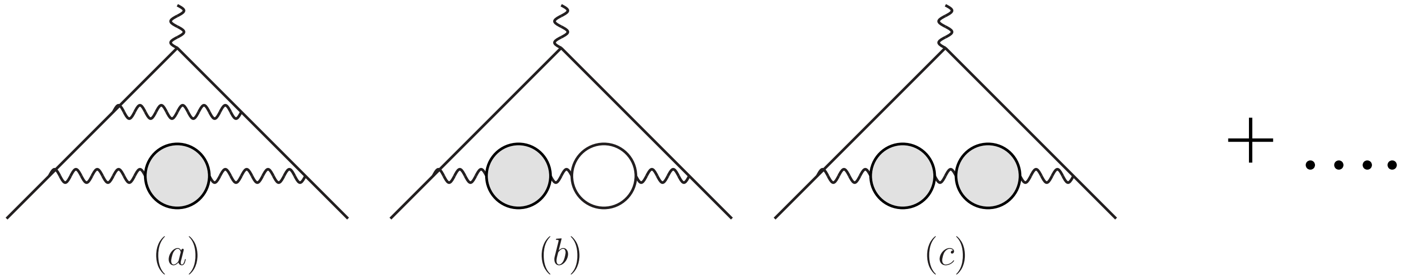
- account for differences in results from the same experimental inputs
- include correlations between systematic errors

➠ $a_\mu^{\mathrm{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$



Hadronic vacuum polarization

NLO and N²LO HVP contributions

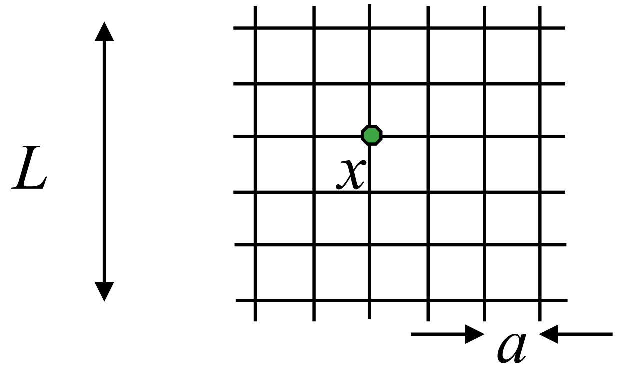


➠ $a_\mu^{\mathrm{HVP,NLO}} = -\,9.83(7) \times 10^{-10}$ [based on KNT 2019]

➠ $a_\mu^{\mathrm{HVP,NNLO}} = 1.24(1) \times 10^{-10}$ [Kurz et al, arXiv:1403.6400, PLB 2014]

Lattice QCD Introduction

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\text{tr}F_{\mu\nu}F^{\mu\nu}$$



- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

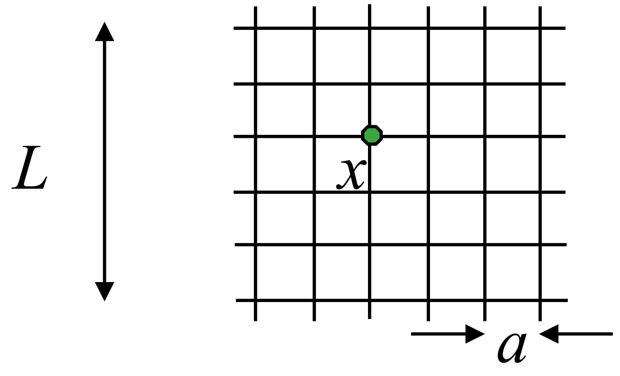
adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\text{lat}} = M_{H,\text{exp}}$
  tune using hadron masses $m_f \to m_{f,\text{phys}}$
  extrapolations/interpolations

m_{ud} m_s m_c m_b

Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$

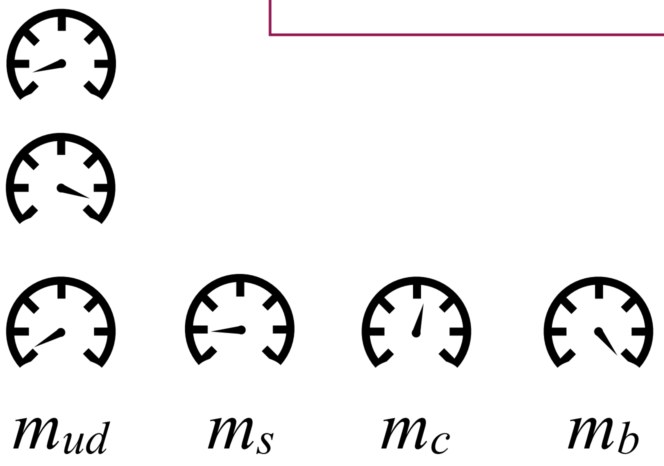


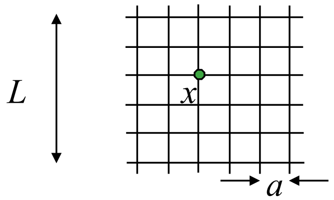
- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty, T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses $m_f \to m_{f,\rm phys}$
  extrapolations/interpolations





Lattice QCD Introduction

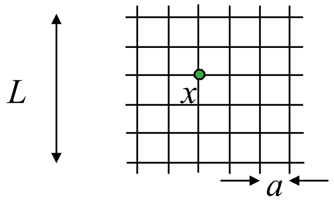
The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) , in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

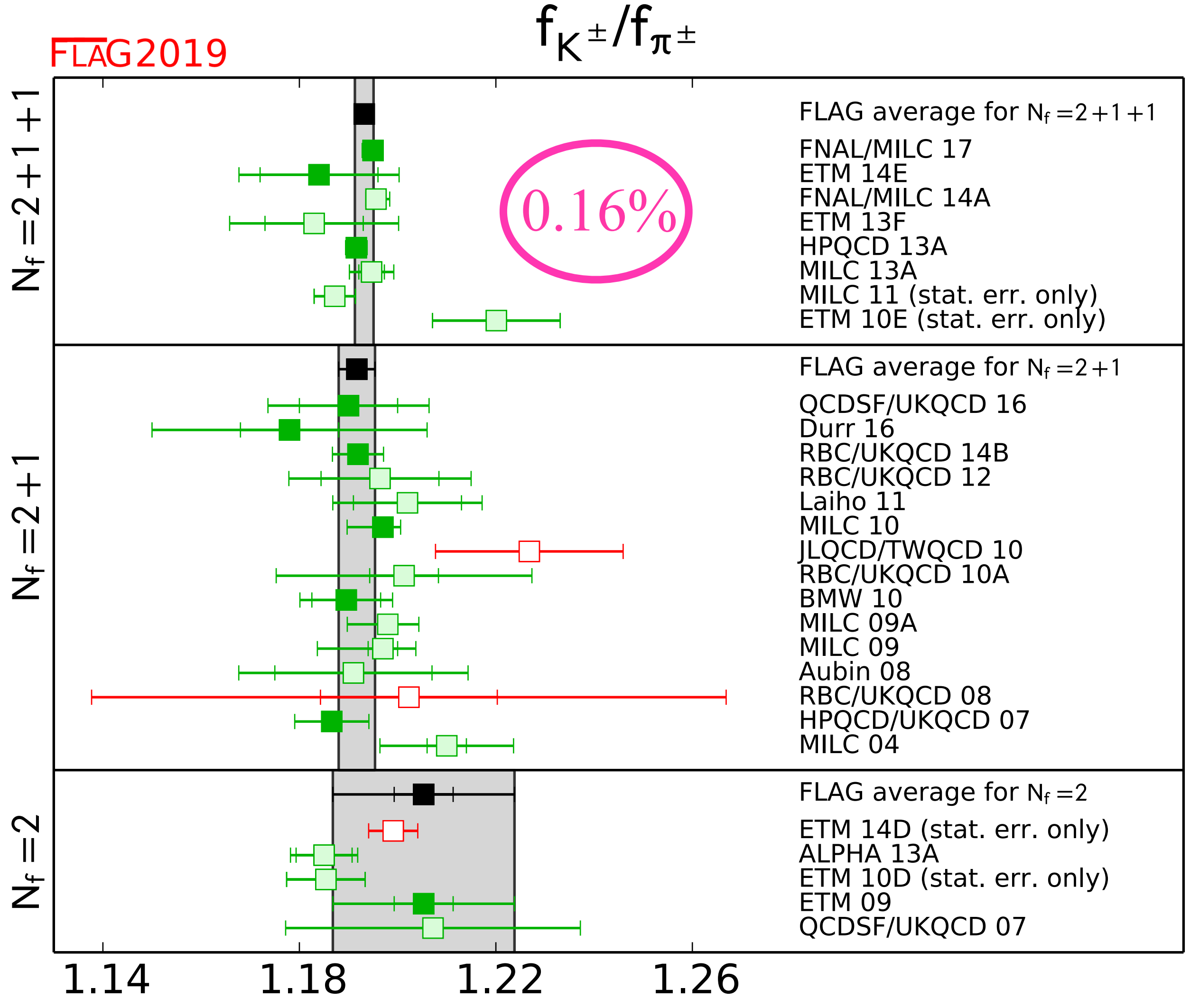
- nucleons and other baryons
- nonleptonic decays ($K \to \pi\pi$, …)
- resonances, scattering, long-distance effects, …
- QED effects
- radiative decay rates …

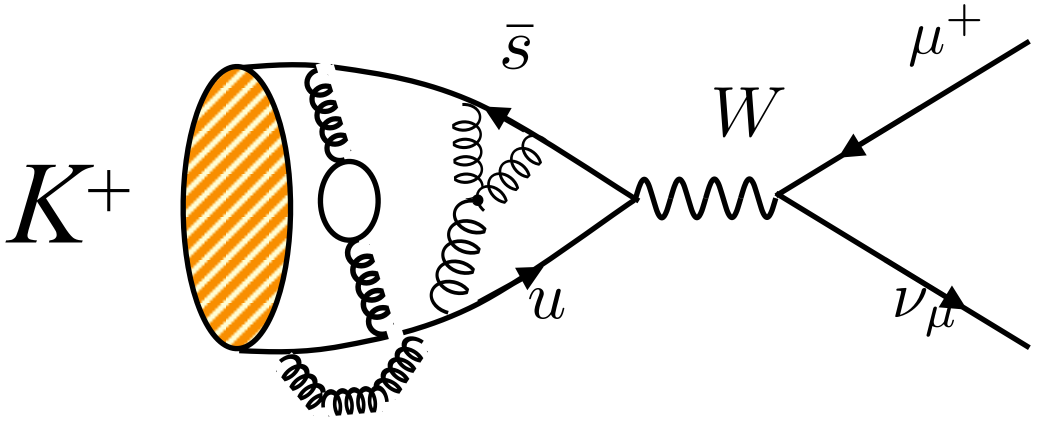


Lattice QCD Introduction

The State of the Art

f_{K^+}/f_{π^+}

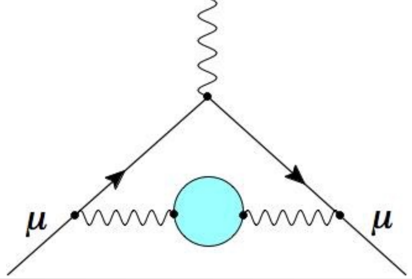




small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ NPR or no renormalization

[S. Aoki et al, FLAG-4 review, arXiv:1902.08191]



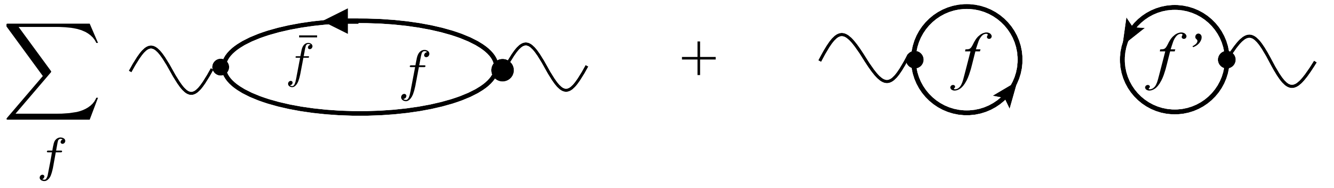
Lattice HVP: Introduction

Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\mathrm{HLO}} \equiv a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

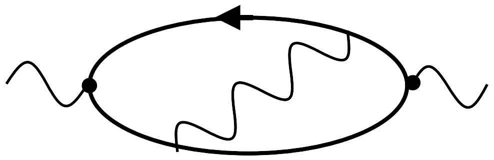
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$



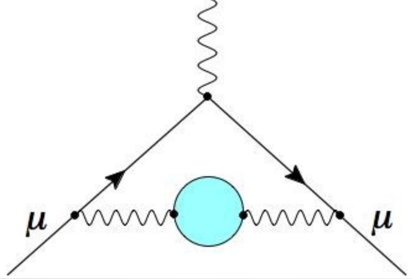
f= ud, s, c, b

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



$+ \ldots$

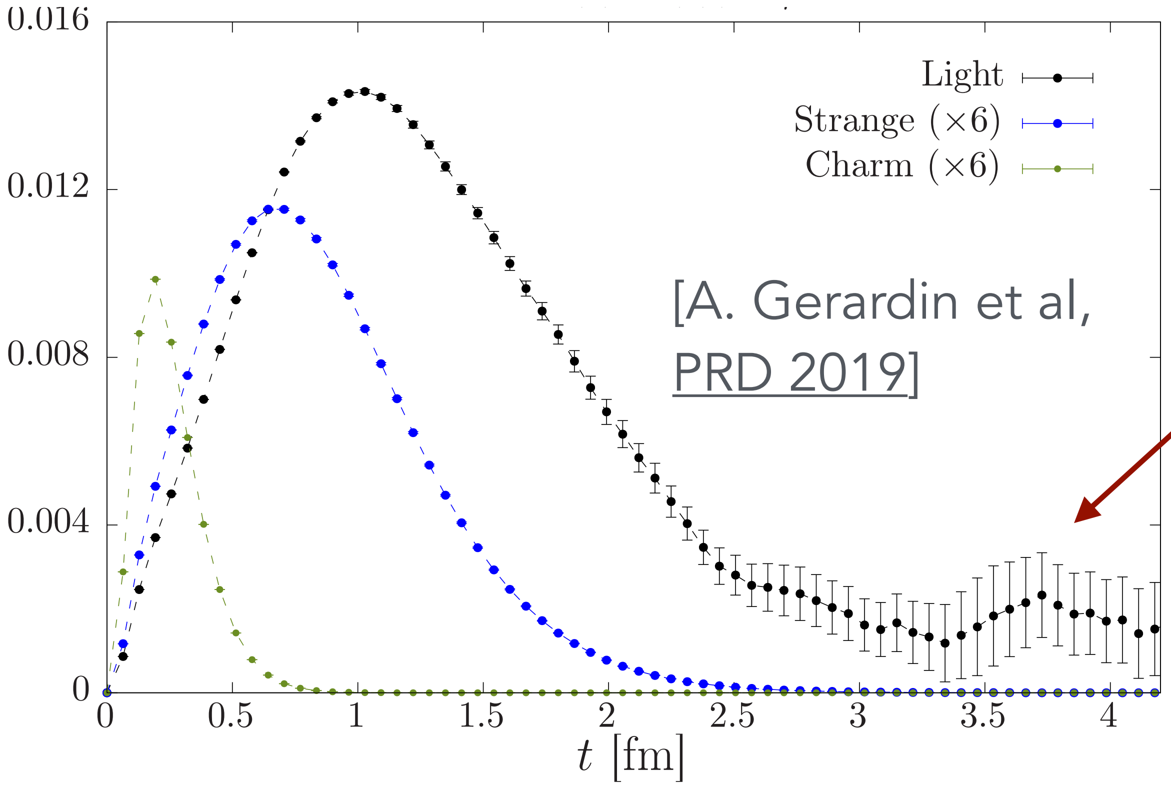
- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$



Lattice HVP: Introduction

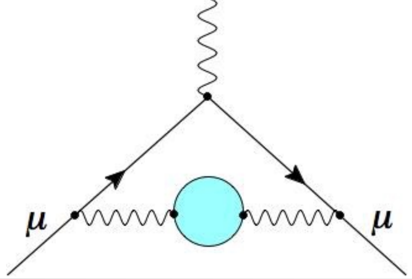
Leading order HVP correction: $$\boxed{a_\mu^{\rm HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2\, \omega(q^2)\, \hat{\Pi}(q^2)}$$

- Calculate $a_\mu^{\rm HLO}$ in Lattice QCD:
  - ✦ Time-momentum representation: [Bernecker & Meyer, EPJ 12]
    reorder the integrations with $G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$
    $$a_\mu^{\rm HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dt\, \tilde{\omega}(t)\, G(t)$$



[A. Gerardin et al, PRD 2019]

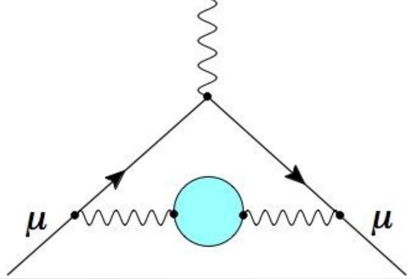
- noise reduction methods to control growth of statistical errors at large t needed for light-quark contribution



Lattice HVP: Introduction

- Target: < 0.5% total error

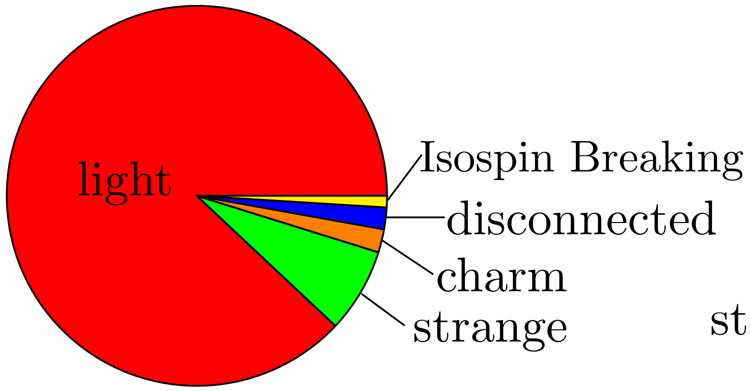
- Challenges:
  ✓needs ensembles with (light sea) quark masses at their physical values
  ✓ finite volume corrections, continuum extrapolation:
  guided by EFT
  - include QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - growth of statistical errors at large Euclidean times
    ➠ statistical noise reduction methods
    include guidance from EFT
    ➠ include two-pion channels into analysis



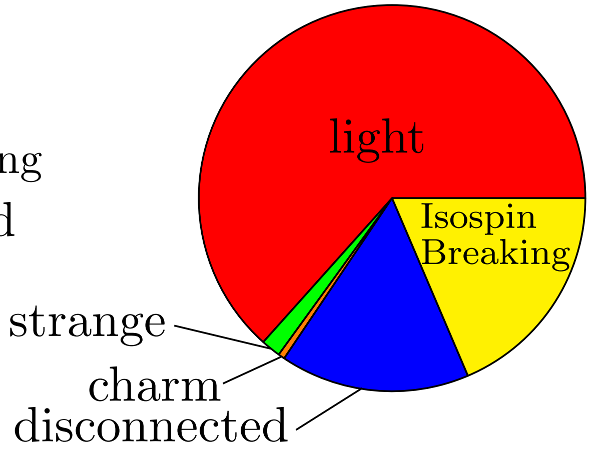
Lattice HVP: Introduction

- Target: < 0.5% total error
- light-quark connected contribution, $a_{\mu,ud}^{\rm HLO}$:
  ~90% of total, with 1-3% error
- ``heavy" flavor contributions, $a_{\mu,s}^{\rm HLO}$, $a_{\mu,c}^{\rm HLO}$, $a_{\mu,b}^{\rm HLO}$:
  ~8%, 2%, 0.05% of total $a_{\mu}^{\rm HLO}$, can be calculated with sufficient precision
- disc. contribution:
  ~2% of total $a_{\mu}^{\rm HLO}$, contributes ~0.3-1% error to $a_{\mu}^{\rm HLO}$
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  ~1% of total $a_{\mu}^{\rm HLO}$, contribute ~0.3-1% error

contribution



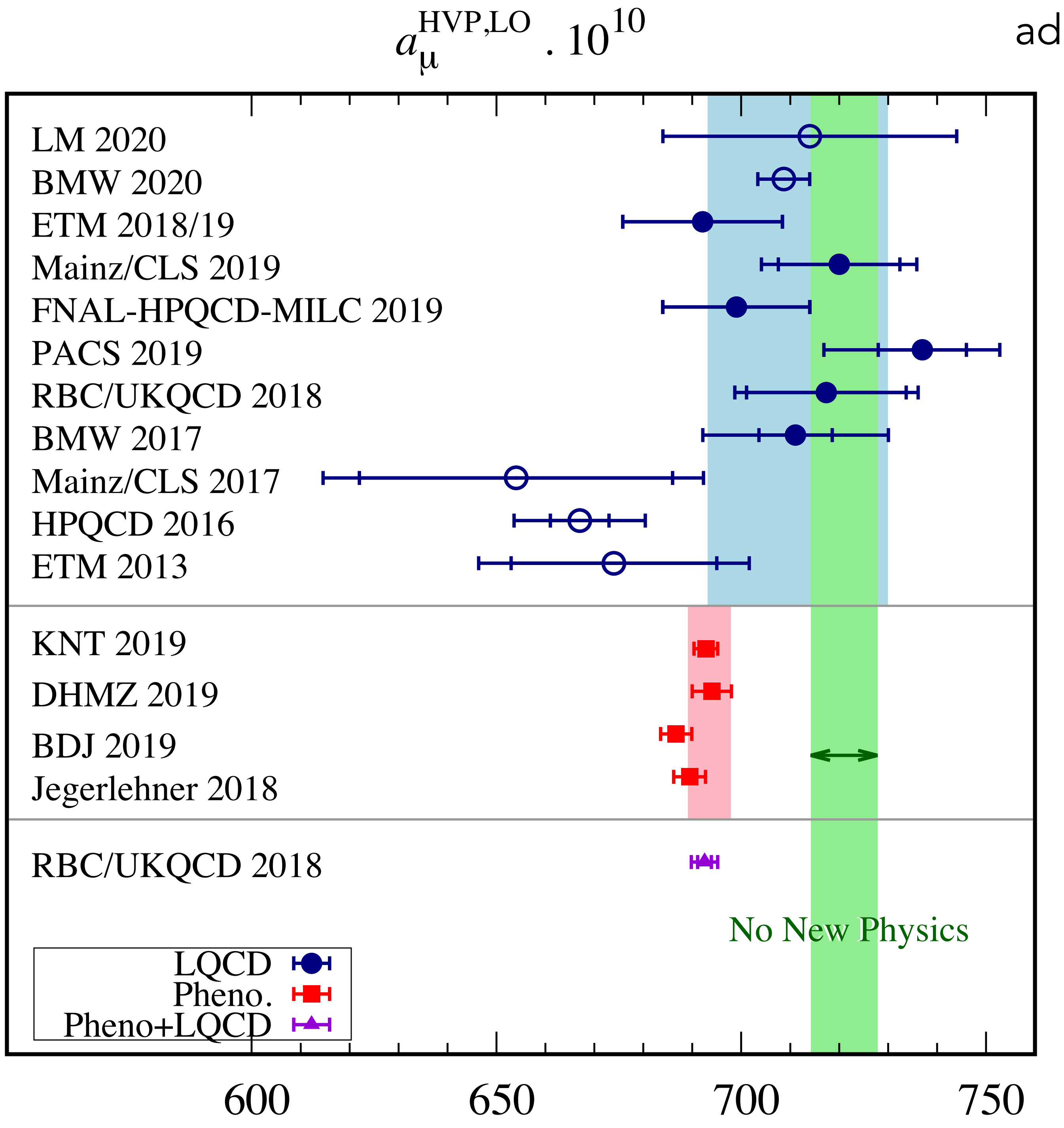
error



[V. Gülpers, adapted for WP from talk @ Lattice 2019, arXiv:2001.11898]

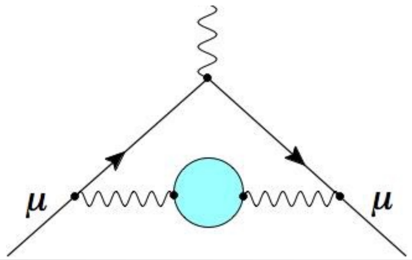
Complete $a_\mu^{\mathrm{HVP,LO}}$: Comparison

adapted from [T. Aoyama et al, arXiv:2006.04822]



- The errors in (all but one of the) lattice QCD results are still large
- All results include contributions from connected ud, s, c, b + disconnected, QED + strong isospin breaking, and finite volume corrections.
- Lattice combination: included results shown with filled circles

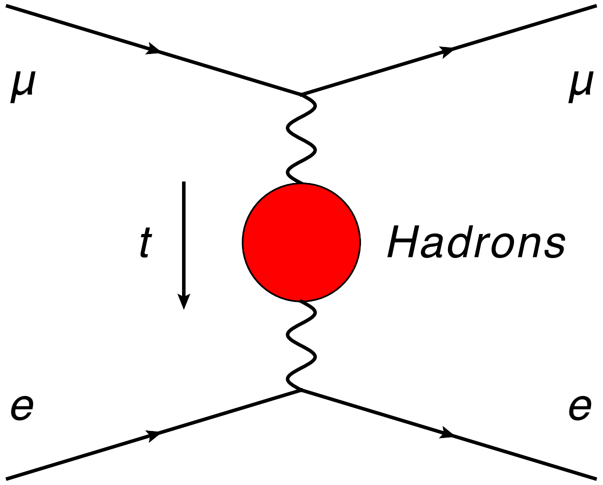
$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{-10}$$



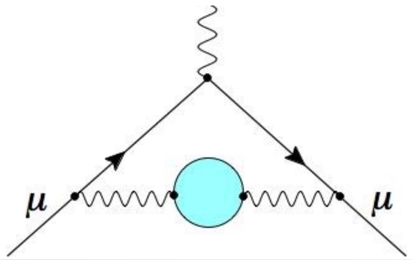
Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

M. Passera @ HVP KEK 2018 [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]



- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- Jan 2020: SPSC recommends pilot run in 2021
- goal: run with full apparatus in 2023-2024



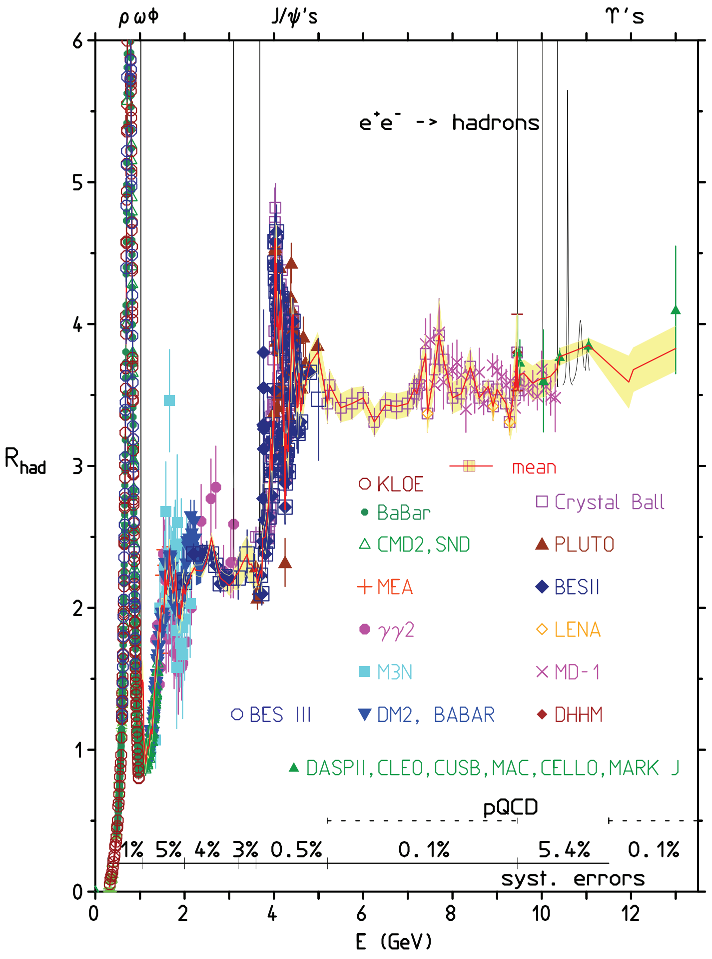
Hadronic vacuum polarization

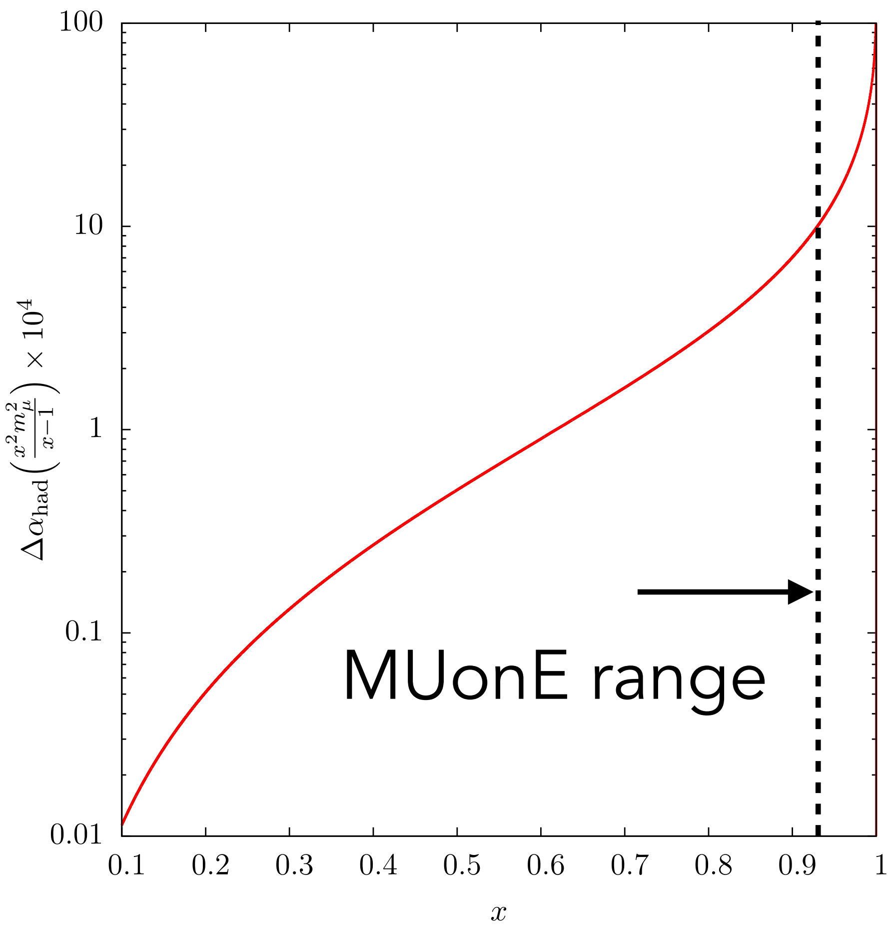
μ-e elastic scattering to measure a_μ^{HVP}

C. Carloni @ g-2 INT workshop [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]

Time-like ↦ **Space-like**

F. Jegerlehner, EPJ Web Conf. 118

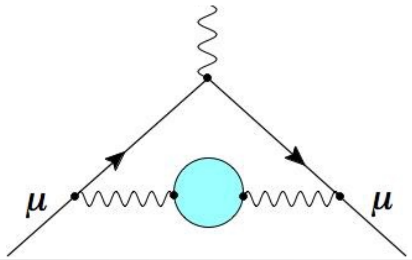




Smooth function

CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- requires calculations of radiative corrections [M. Fael @ g-2 INT workshop]
- complement region not accessible to experiment with LQCD calculation [M. Marinkovic @ g-2 INT workshop]



Hadronic vacuum polarization

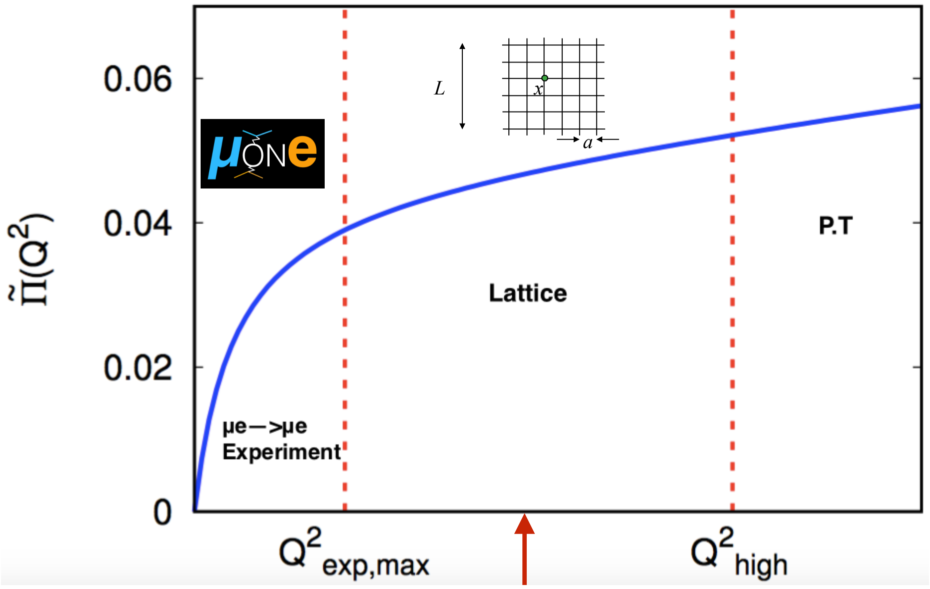
$\mu - e$ elastic scattering to measure a_μ^{HVP}

M. Marinkovic @ HVP KEK 2018:

- complement region not accessible to experiment with LQCD calculation

Hybrid method

Phys. Rev. D 90, 074508 (2014), [Golterman,Maltman,Peris]



$$a_\mu^{had,LO} = \underbrace{\frac{\alpha}{\pi}\int_0^{0.93\ldots} dx(1-x)\Delta\alpha_{had}[Q^2(x)]}_{I_0} + \boxed{\underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q^2_{max}} dQ^2 f(Q^2)\times\hat{\Pi}(Q^2)}_{I_1}} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{max}}^{\infty} dQ^2 f(Q^2)\times\hat{\Pi}_{pert.}(Q^2)}_{I_2}$$

First lattice results for I_1 [Marinkovic & Cardosso, arXiv:1910.06467]

and $I_1 + I_2$ [D. Giusti @ Lattice 2019, arXiv:1910.03874]

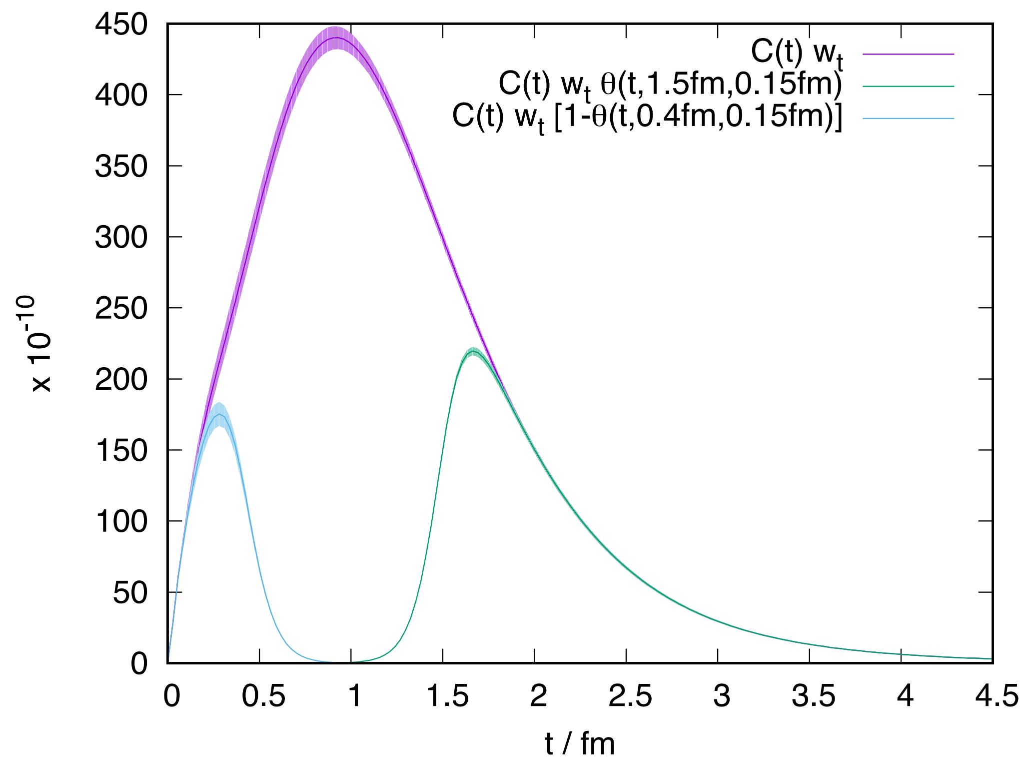
A hybrid method: Euclidean windows

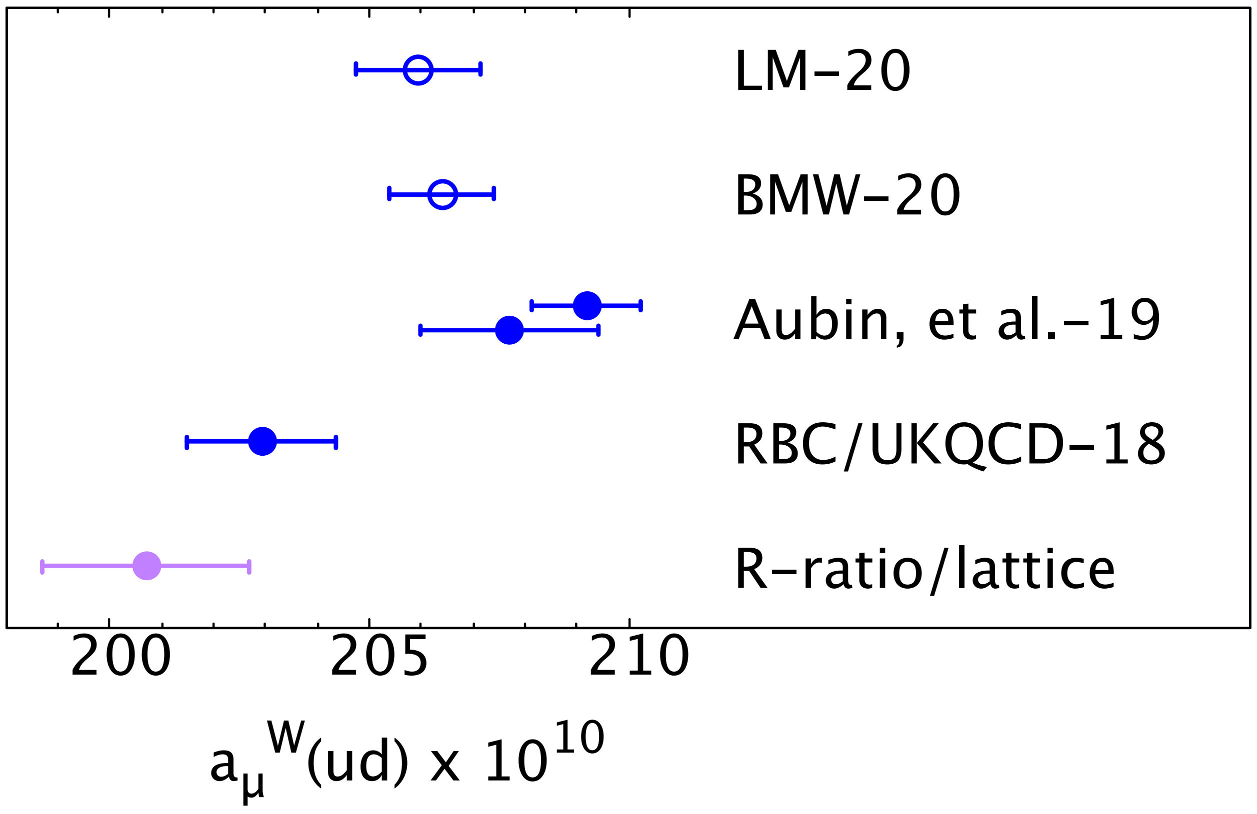
Hybrid method: combine LQCD with R-ratio data

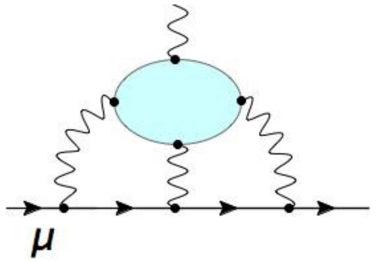
[T. Blum et al, arXiv:1801.07224, 2018 PRL]

Direct LQCD calculations of HVP are still less precise than dispersive methods.

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral) and compare with lattice results for windows in Euclidean time
- intermediate window:
  expect reduced FV effects and discretization errors



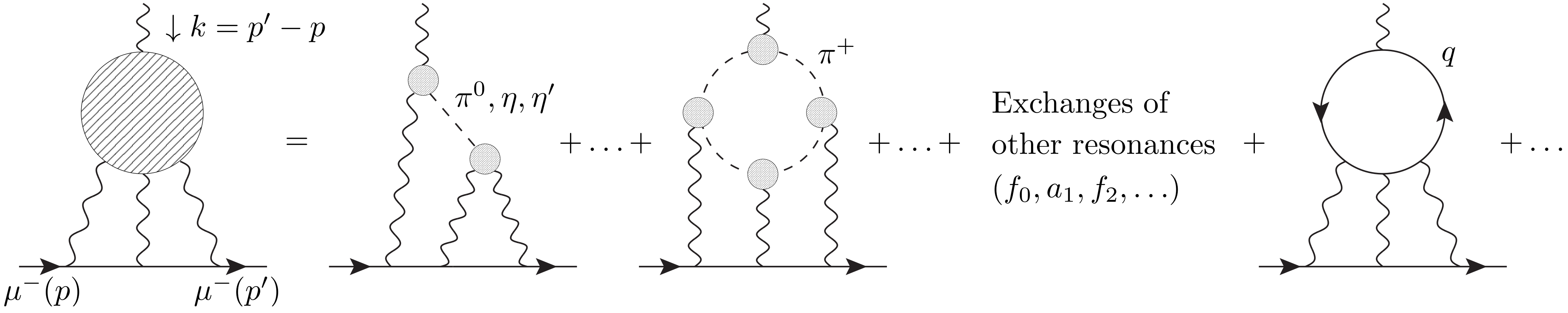




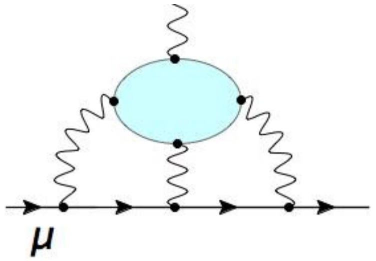
Hadronic Light-by-light

Hadronic light-by-light:

- ✦ Target: ≲ 10% total error



- ✦ previous estimate "Glasgow consensus" based on models of QCD
- ✦ used to evaluate individual contributions to HLbL scattering tensor
- ✦ theory error not well determined and not improvable



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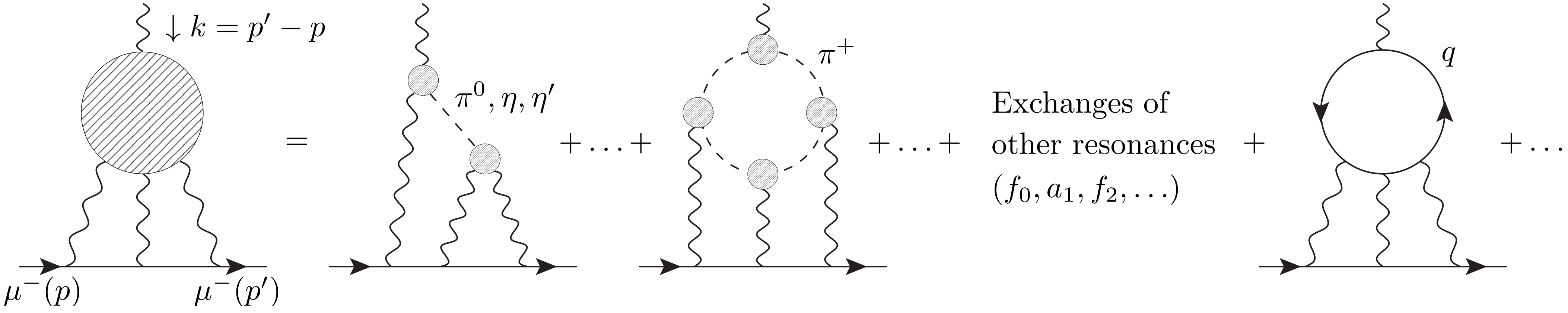
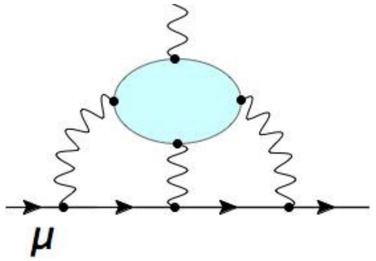


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs



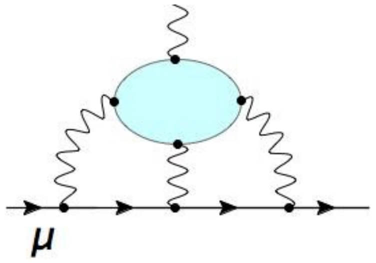
Hadronic Light-by-light: dispersive

Experimental input wish list:

issue	experimental input [I] or cross-checks [C]
axials, tensors, higher pseudoscalars	$\gamma^{(*)}\gamma^* \to 3\pi,\ 4\pi,\ K\bar{K}\pi,\ \eta\pi\pi,\ \eta'\pi\pi$ [I]
missing states	inclusive $\gamma^{(*)}\gamma^* \to$ hadrons at 1–3 GeV [I]
dispersive analysis of $\eta^{(\prime)}$ TFFs	$e^+e^- \to \eta\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-\pi^+\pi^-$ [I]
	$\eta' \to \pi^+\pi^-e^+e^-$ [I]
	$\gamma\pi^- \to \pi^-\eta$ [C]
dispersive analysis of π^0 TFF	$\gamma\pi \to \pi\pi$ [I]
	high accuracy Dalitz plot $\omega \to \pi^+\pi^-\pi^0$ [C]
	$e^+e^- \to \pi^+\pi^-\pi^0$ [C]
	$\omega, \phi \to \pi^0 l^+ l^-$ [C]
pseudoscalar TFF	$\gamma^{(*)}\gamma^* \to \pi^0, \eta, \eta'$ at arbitrary virtualities [I,C]
pion, kaon, $\pi\eta$ loops (including scalars and tensors)	$\gamma^{(*)}\gamma^* \to \pi\pi,\ K\bar{K},\ \pi\eta$ at arbitrary virtualities, partial waves [I,C]

[talk by A. Kupsc @ INT g-2 workshop]

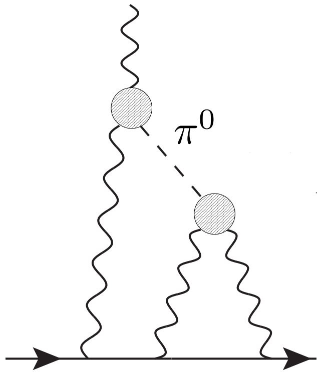
Active experimental programs by BESIII, Belle, BaBar, KLOE-2, PrimEx, JLAB, CMD, SND, …



Hadronic Light-by-light: dispersive

Three independent results for the pion pole contribution:

[G. Colangelo @ INT g-2 workshop]



- Dispersive calculation of the pion TFF — Hoferichter et al. (18)

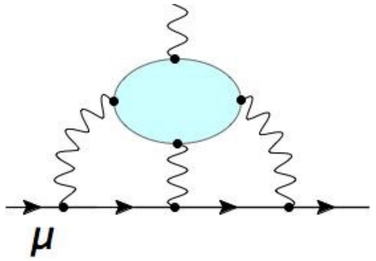
$$a_\mu^{\pi^0} = 63.0^{+2.7}_{-2.1} \times 10^{-11}$$

- Padé-Canterbury approximants — Masjuan & Sanchez-Puertas (17)

$$a_\mu^{\pi^0} = 63.6(2.7) \times 10^{-11}$$

- Lattice — Gérardin, Meyer, Nyffeler (19)

$$a_\mu^{\pi^0} = 62.3(2.3) \times 10^{-11}$$



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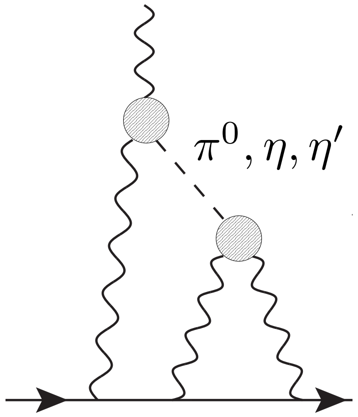


PS-poles: conclusion [G. Colangelo @ INT g-2 workshop]

Dispersive (π^0) + Canterbury (η, η'):

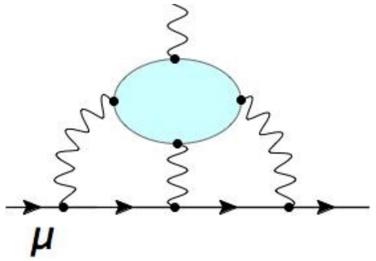
$$a_\mu^{\pi^0+\eta+\eta'} = 93.8^{+4.0}_{-3.6} \times 10^{-11}$$

Canterbury:

$$a_\mu^{\pi^0+\eta+\eta'} = 94.3(5.3) \times 10^{-11}$$

Outlook:

Dispersive evaluation of the η, η' contributions will give two fully independent evaluations ⇒ better control over systematics



Hadronic Light-by-light: dispersive

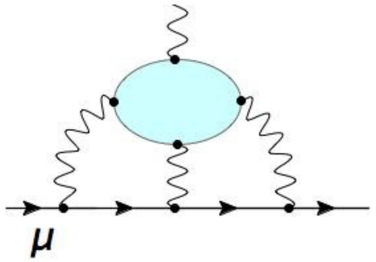
G. Colangelo @ INT g-2 workshop

Contributions to $10^{11} \cdot a_\mu^{\mathrm{HLbL}}$

▶ Pseudoscalar poles	$= 93.8^{+4.0}_{-3.6}$
▶ pion box (kaon box ~ -0.5)	$= -15.9(2)$
▶ S-wave $\pi\pi$ rescattering	$= -8(1)$
▶ scalars and tensors with $M_R > 1$ GeV	= -1(3)
▶ axial vectors	= 6(6)
▶ short-distance contribution	= 15(10)
▶ charm, etc…	~ 3(1)

- more work is needed for higher scalars, tensors and axial vectors as well as for the SDC

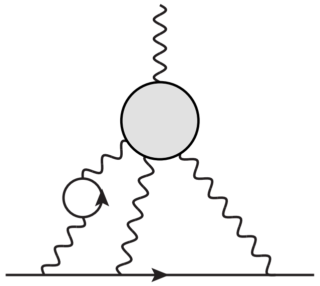
$$\boxed{\text{Total: } 10^{11} \cdot a_\mu^{\mathrm{HLbL}} = 92\,(19)}$$



Hadronic Light-by-light: dispersive

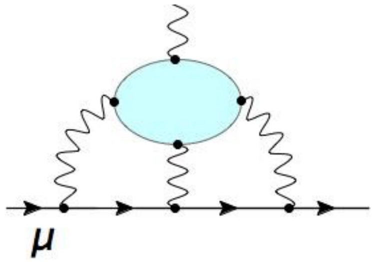
Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} – 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



NLO HLbL contribution:

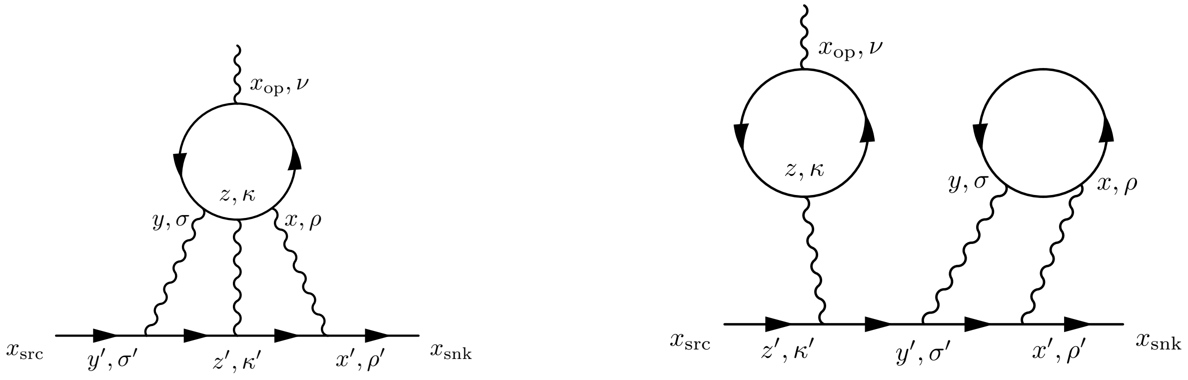
$$a_\mu^{\mathrm{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$



Hadronic Light-by-light

Hadronic light-by-light:

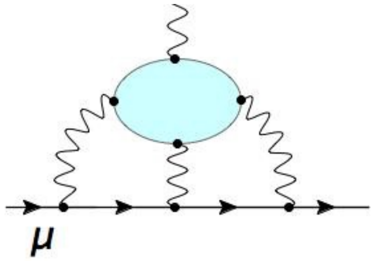
- ✦ Target: ≲ 10% total error



+ SU(3) suppressed

Direct lattice QCD calculations:

- ✦ QCD + QED$_L$ (finite volume)
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123)]
- ✦ QCD + QED (infinite volume & continuum)
  [E. Chao et al, arXiv:2006.16224; Asmussen @ Lattice 2017; Asmussen et al, arXiv:1911.05573, Green et al, arXiv:PRL 2015; T. Blum et al, arXiv:1705.01067, 2017 PRD]
- ✦ dominant contribution from pion pole (transition form factors)
  [Gerardin et al, arXiv:1607.08174, 2016 PRD; Lattice 2017]



Hadronic Light-by-light: lattice

RBC/UKQCD [T. Blum et al, arXiv:1911.08123, PRL 2020]:
First complete LQCD calculation of connected and leading disconnected contribution with continuum and finite volume extrapolation

$$\boxed{a_\mu^{\mathrm{HLbL}} = 7.87\,(3.06)\,(1.77) \times 10^{-10}}$$

✦ uses QCD + $\mathrm{QED_L}$ (finite volume)

✦ C. Lehner @ INT g-2 workshop:
Cross checks between RBC/UKQCD and Mainz at unphysical pion mass

QCD + QED (infinite volume):

✦ RBC/UKQCD:
calculation in progress (can reuse QCD part from QCD+$\mathrm{QED_L}$ calculation)

✦ Mainz group:
first complete calculation at SU(3) symmetric point (E. Chao at al, arXiv:2006.16224), work in progress (Asmussen at al, arXiv:1911.05573, Asmussen @ Lattice 2017,…)

Summary Table

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	–98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)	

Citation References

website: https://muon-gm2-theory.illinois.edu

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HLbL NLO (phenomenology)	bib, cite
HLbL (lattice, uds)	bib, cite
HLbL (phenomenology + lattice	bib, cite
QED	bib, cite
Electroweak	bib, cite
HVP $(e^+e^-$, LO + NLO + NNL	bib, cite
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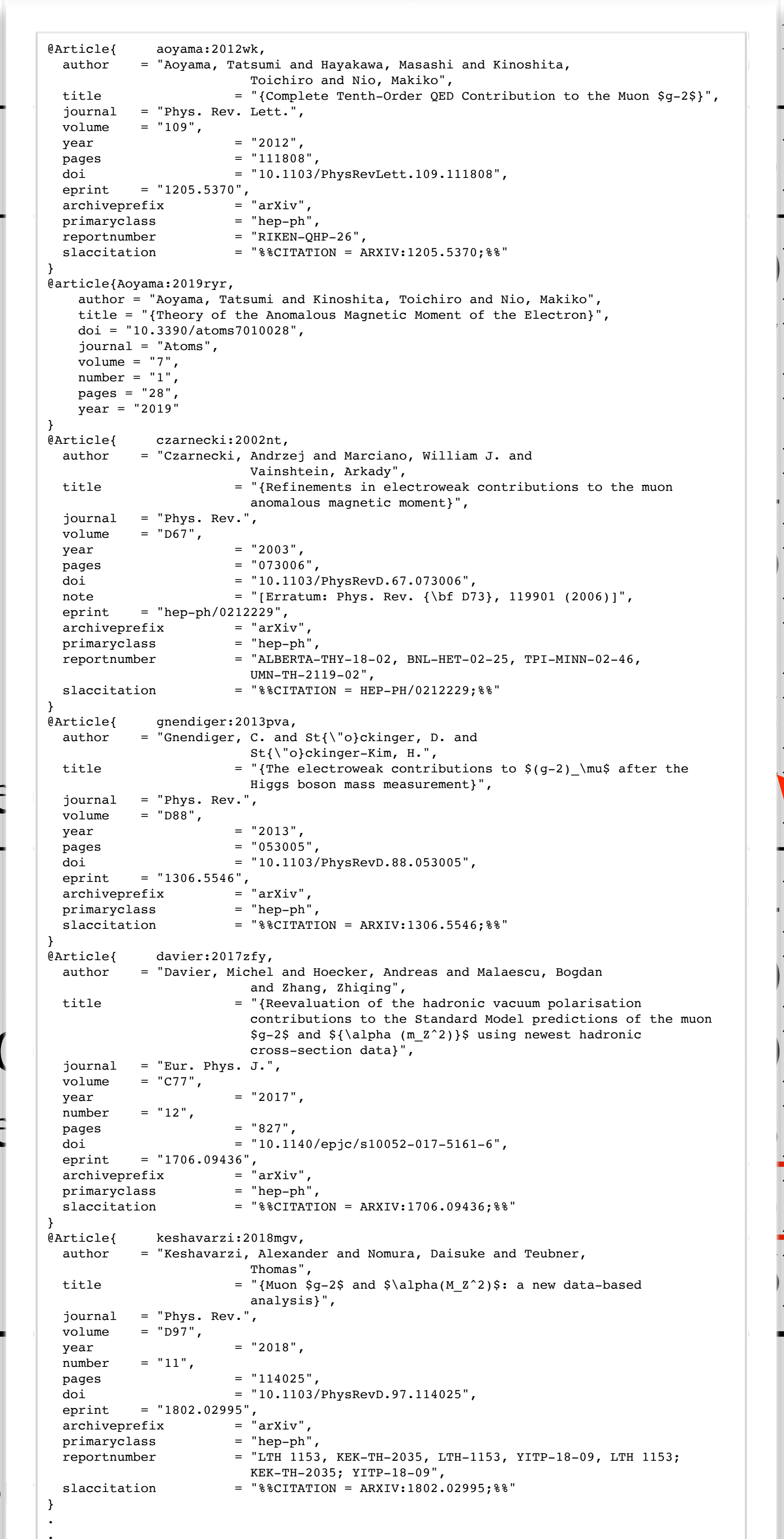

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Electroweak	bib, cite
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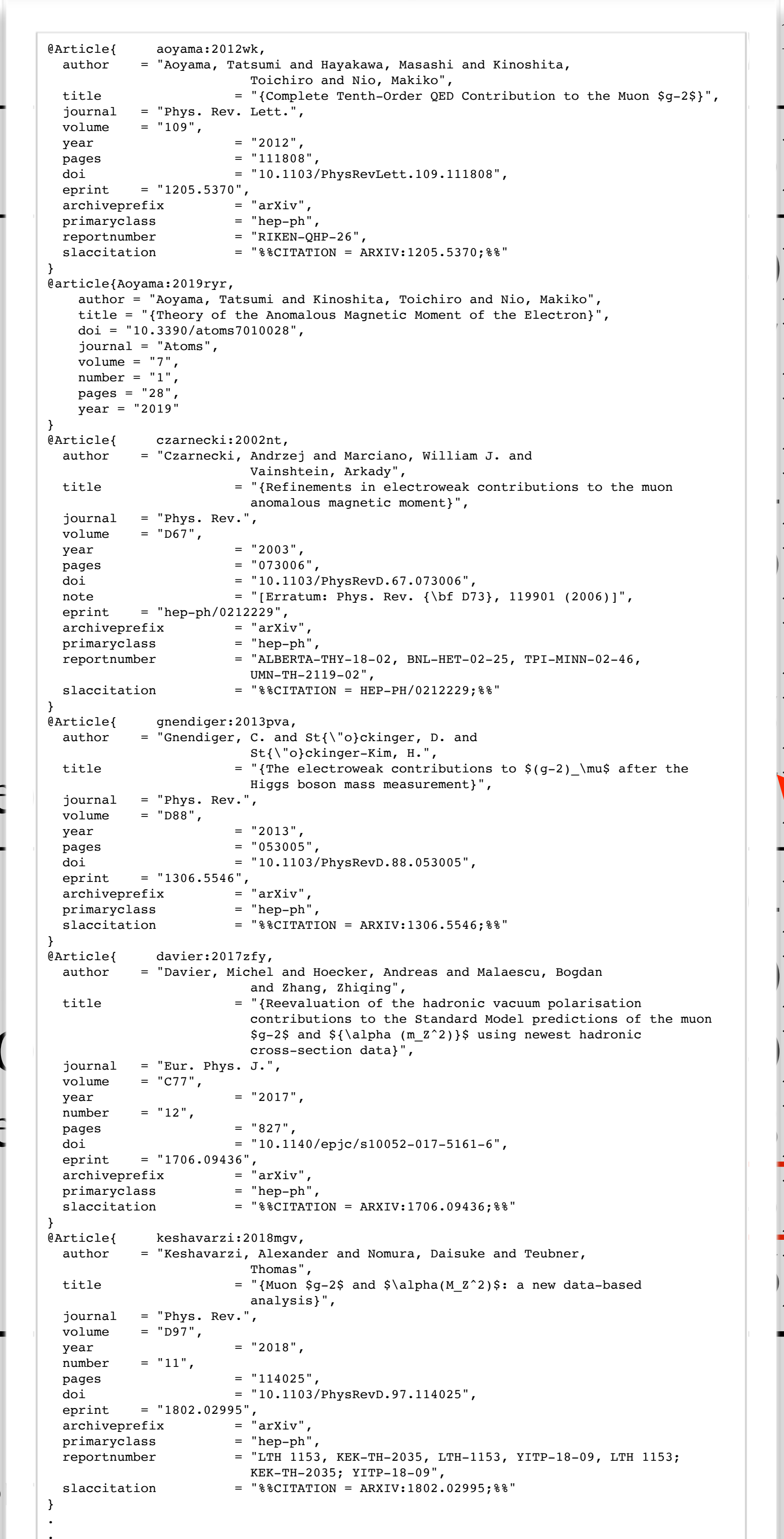

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    author = "Aoyama, Tatsumi and Kinoshita, Toichiro and Nio, Makiko",
    title = "{Theory of the Anomalous Magnetic Moment of the Electron}",
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  author    = "Czarnecki, Andrzej and Marciano, William J. and
                          Vainshtein, Arkady",
  title                   = "{Refinements in electroweak contributions to the muon
                          anomalous magnetic moment}",
  journal   = "Phys. Rev.",
  volume    = "D67",
  year                    = "2003",
  pages                   = "073006",
  doi                     = "10.1103/PhysRevD.67.073006",
  note                    = "[Erratum: Phys. Rev. {\bf D73}, 119901 (2006)]",
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  primaryclass            = "hep-ph",
  reportnumber            = "ALBERTA-THY-18-02, BNL-HET-02-25, TPI-MINN-02-46,
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@Article{     gnendiger:2013pva,
  author    = "Gnendiger, C. and St{\"o}ckinger, D. and
                          St{\"o}ckinger-Kim, H.",
  title                   = "{The electroweak contributions to $(g-2)_\mu$ after the
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  year                    = "2013",
  pages                   = "053005",
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                          contributions to the Standard Model predictions of the muon
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  doi                     = "10.1140/epjc/s10052-017-5161-6",
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  title                   = "{Muon $g-2$ and $\alpha(M_Z^2)$: a new data-based
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  journal   = "Phys. Rev.",
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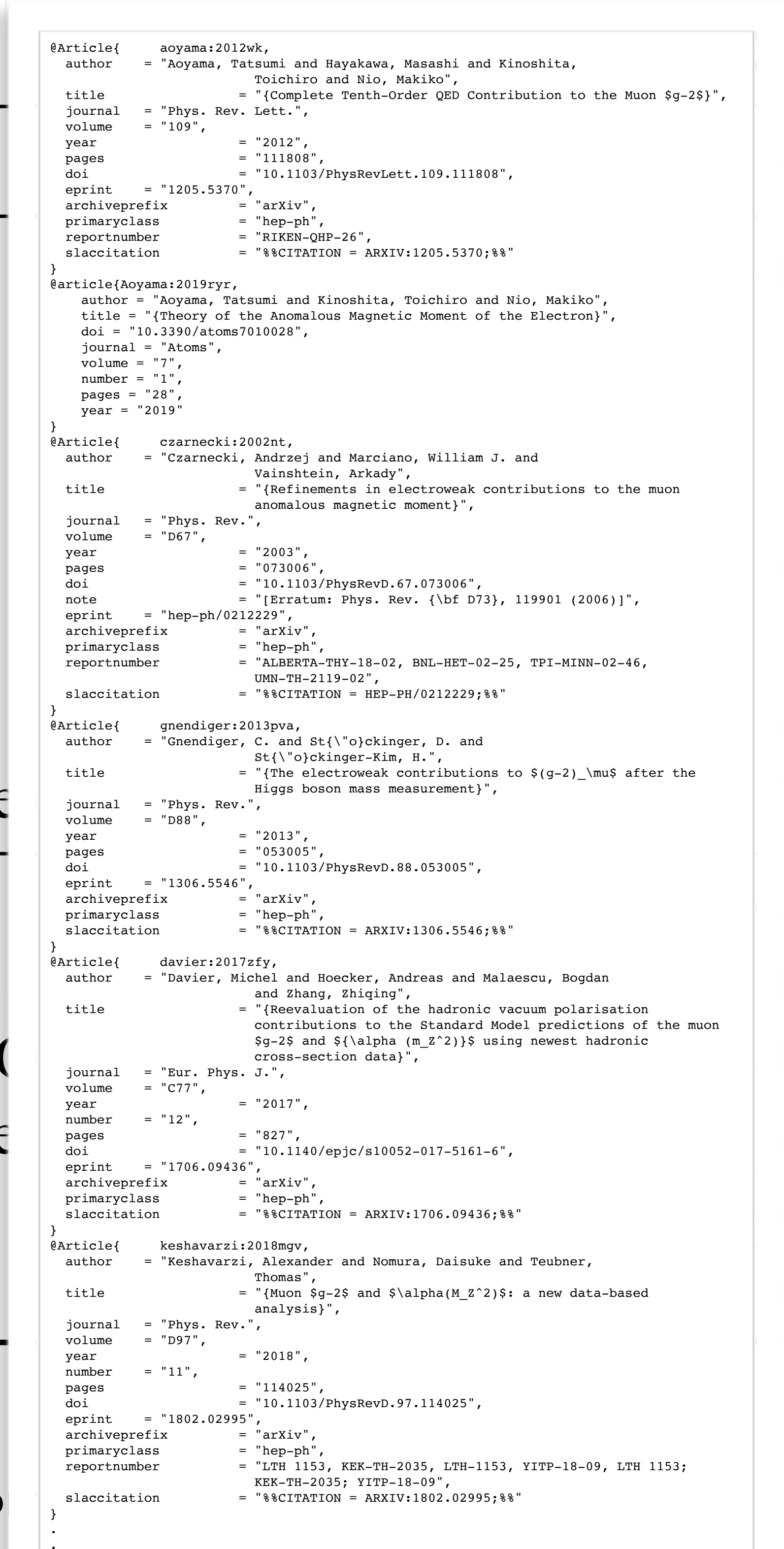
}
.
.
```

Citation References

website: https://muon-gm2-theory.illinois.edu

Contribution	Citation References
Experiment (E821)	bib, cite
HVP LO (e^+e^-)	bib, cite
HVP NLO (e^+e^-)	bib, cite
HVP NNLO (e^+e^-)	b
HVP LO (lattice, $udsc$)	b
HLbL (phenomenology)	b
HLbL NLO (phenomenology)	b
HLbL (lattice, uds)	b
HLbL (phenomenology + lattice	b
QED	bib, cite
Electroweak	bib, cite
HVP (e^+e^-, LO + NLO + NNL	bib, cite
HLbL (phenomenology + lattice	bib, cite
Total SM Value	bib, cite
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	



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  author    = "Aoyama, Tatsumi and Hayakawa, Masashi and Kinoshita,
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  title                  = "{Complete Tenth-Order QED Contribution to the Muon $g-2$}",
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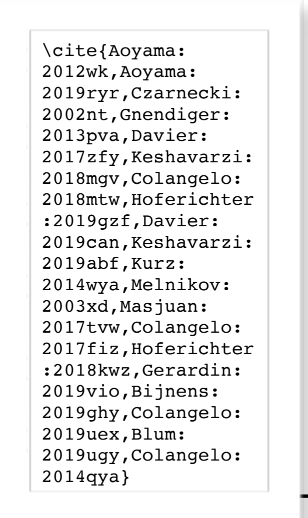

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\cite{Aoyama:
2012wk,Aoyama:
2019ryr,Czarnecki:
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2014qya}
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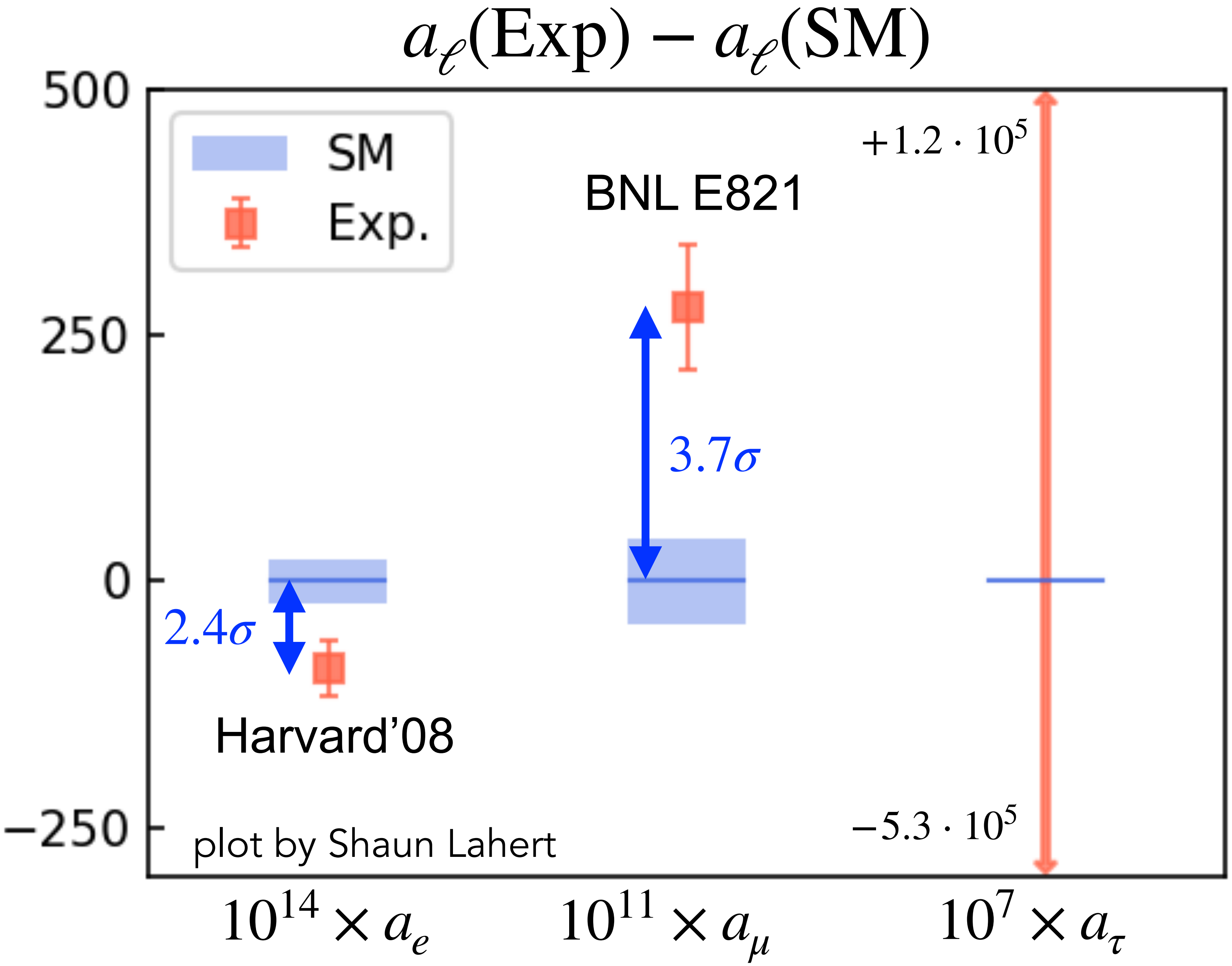
Lepton moments summary



Sensitivity to heavy new physics:

$$a_\ell^{\text{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

Ongoing experimental programs for improved measurements of α

[S. Guellati-Khelifa (Paris), Z. Pagel (Berkeley) @ INT workshop]

Summary: HVP

data-driven methods

- almost all channels are now measured
  - ➠ remaining channels (estimated using isospin) contribute ~0.02%
- Current tensions in experimental inputs for the $\pi\pi$ channel limit precision on a_μ
- New measurements with higher precision are expected from BaBar, Novosibirsk, Belle II,… experiments

Lattice QCD+QED

- Methods have been developed for complete calculations, including sub-leading contributions, but current uncertainties are large (in all but one case)
- no roadblocks towards reducing errors to < 1%, thanks to ensembles with light sea quarks at their **physical masses**
  - ➠ expect meaningful tests of data driven methods HVP

Summary: HVP

Connections

- lattice QCD+QED calculation of IB contributions to τ decays

  [M. Bruno @ INT g-2 workshop, M. Bruno et al arXiv:1811.00508]
- MUonE: experimental measurement in space-like region
  - plans to start running in 2023
  - lattice inputs at intermediate Q^2
- Lattice QCD+QED calculations of related quantities, $\Delta_{\text{had}}\alpha(Q^2)$, etc..

  [M. Cè @INT g-2 workshop, M. Cè et al, arXiv:1910.09525]

Summary: HLbL

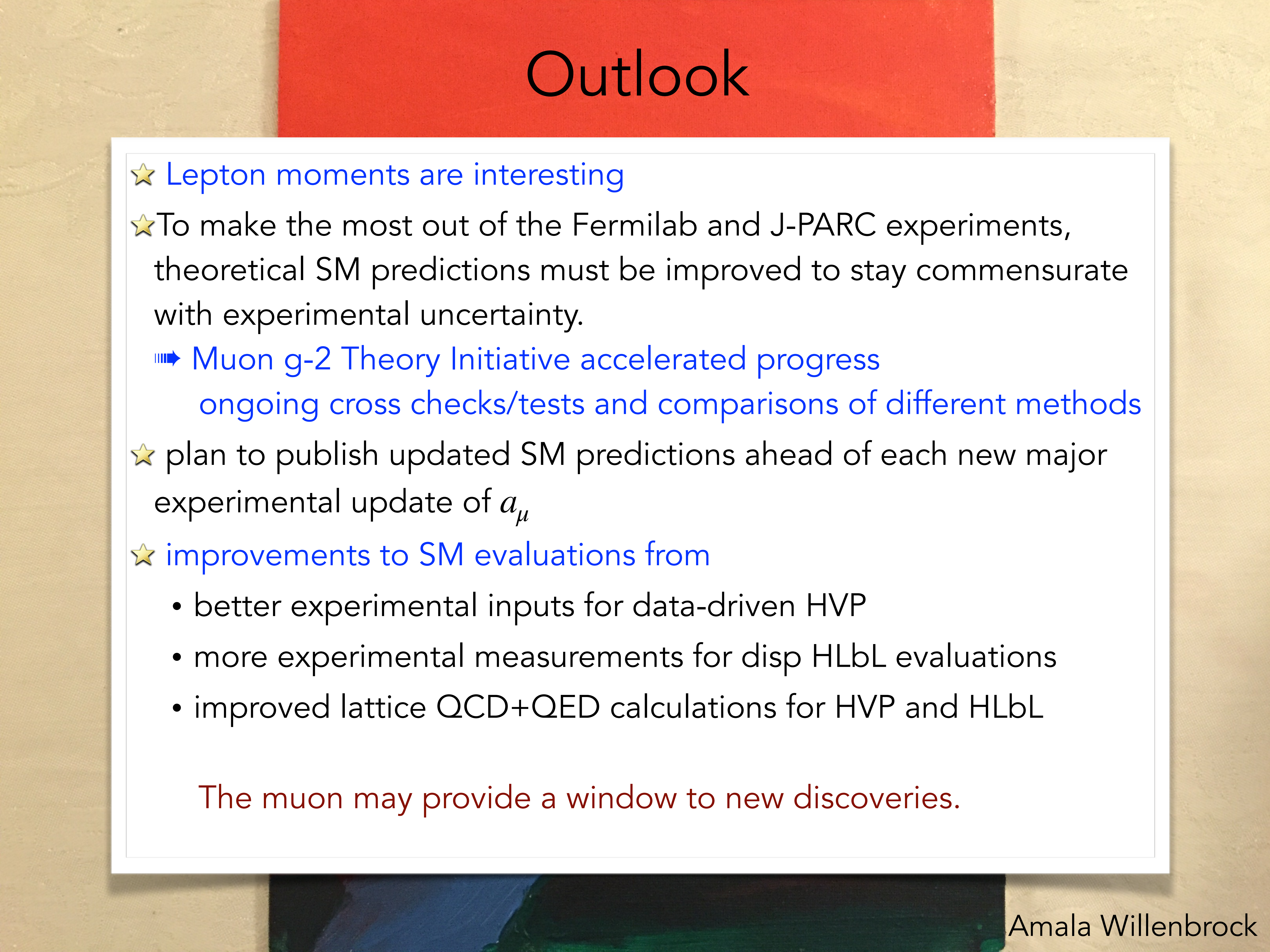
Dispersive methods

- ☆ ~80% of contributions determined reliably (pion pole three ways)
- ☆ more work needed for remaining 20%
- ☆ dispersive HLbL results have similar values compared to old "Glasgow" consensus ➠ a_μ^{HLbL} cannot "rescue" the SM

Lattice QCD+QED

- ☆ first complete QCD+QED calculation, errors are large, but improvable
- ☆ cross checks between RBC/UKQCD and Mainz groups
- ☆ lattice QCD calculation of pion TFF, forward scattering amplitude by Mainz group
- ☆ lattice HLbL result is consistent with new dispersive HLbL results
  - ➠ a_μ^{HLbL} cannot "rescue" the SM
  - ➠ combine disp and lattice HLbL for SM prediction

Outlook

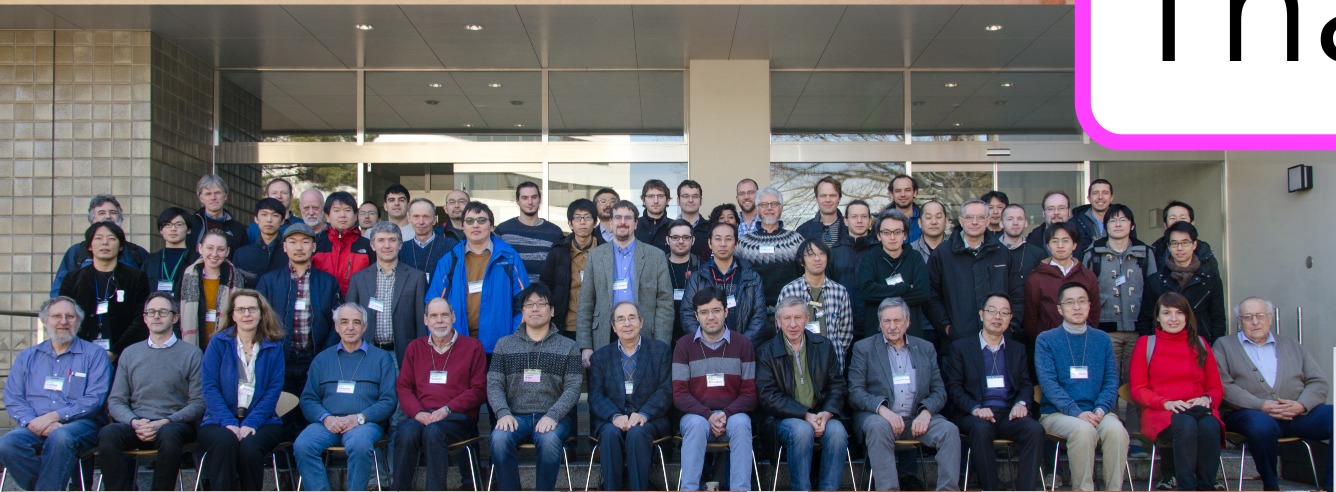


- Lepton moments are interesting
- To make the most out of the Fermilab and J-PARC experiments, theoretical SM predictions must be improved to stay commensurate with experimental uncertainty.
  - ➠ Muon g-2 Theory Initiative accelerated progress ongoing cross checks/tests and comparisons of different methods
- plan to publish updated SM predictions ahead of each new major experimental update of a_μ
- improvements to SM evaluations from
  - better experimental inputs for data-driven HVP
  - more experimental measurements for disp HLbL evaluations
  - improved lattice QCD+QED calculations for HVP and HLbL

The muon may provide a window to new discoveries.

Amala Willenbrock

Thank you!

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URA UNIVERSITIES RESEARCH ASSOCIATION

Thank you!

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

KEK

NEC

HIM HELMHOLTZ Helmholtz-Institut Mainz

PRISMA

UCONN DEPARTMENT OF PHYSICS

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Appendix

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- Michel Davier (Orsay)
- Simon Eidelman (Novosibirsk)
- Aida El-Khadra (UIUC & Fermilab)
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL)
- Tsutomu Mibe (KEK) J-PARC E34 experiment
- Andreas Nyffeler (Mainz)
- Lee Roberts (Boston) Fermilab E989 experiment
- Thomas Teubner (Liverpool)

Muon g-2 Theory Initiative

WP section authors:

- Section 2: Data-driven evaluations of HVP
  M. Benayoun, C. M. Carloni Calame, H. Czyz, M. Davier, S. I. Eidelman, M. Hoferichter, F. Jegerlehner, A. Keshavarzi, B. Malaescu, D. Nomura, M. Passera, T. Teubner, G. Venanzoni, Z. Zhang
- Section 3: Lattice QCD calculations of HVP
  T. Blum, M. Bruno, M. Ce, C. T. H. Davies, M. Della Morte, A. X. El-Khadra, D. Giusti, Steven Gottlieb, V. Guelpers, G. Herdoiza, T. Izubuchi, C. Lehner, L. Lellouch, M. K. Marinkovic, A. S. Meyer, K. Miura, A. Portelli, S. Simula, R. Van de Water, G. von Hippel, H. Wittig
- Section 4: Data-driven and dispersive approach to HLbL
  J. Bijnens, G. Colangelo, F. Curciarello, H. Czyz, I. Danilkin, F. Hagelstein, M. Hoferichter, B. Kubis, A. Kupsc, A. Nyffeler, V. Pascalutsa, E. Perez del Rio, M. Procura, C. F. Redmer, P. Sanchez-Puertas, P. Stoffer, M. Vanderhaeghen
- Section 5: Lattice approaches to HLbL
  N. Asmussen, T. Blum, A. Gerardin, M. Hayakawa, R. J. Hudspith, T. Izubuchi, L. Jin, C. Lehner, H. B. Meyer, A. Nyffeler
- Section 6: The QED contributions to a_μ
  T. Aoyama, T. Kinoshita, M. Nio
- Section 7: The electroweak contributions to a_μ
  D. Stoeckinger, H. Stoeckinger-Kim

First Workshop of the Muon g-2 Theory Initiative

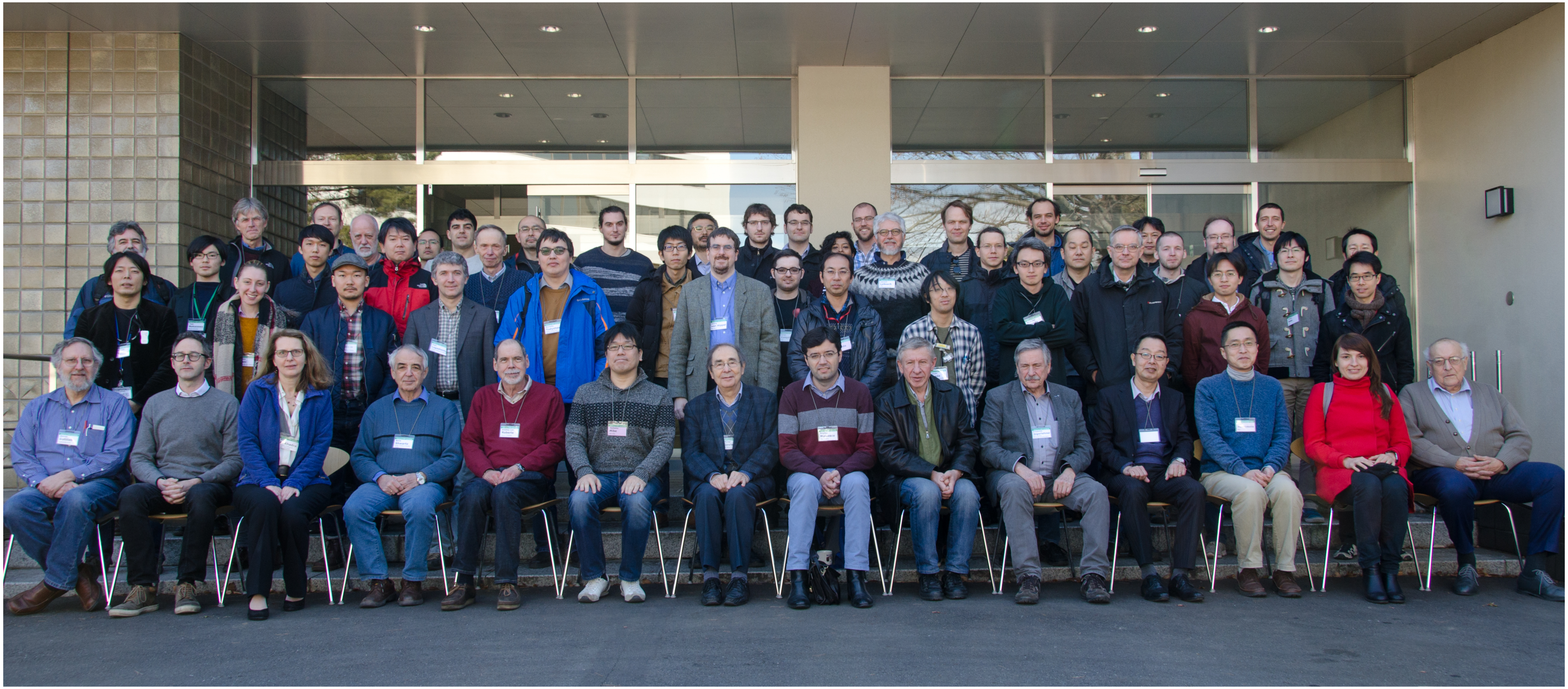
took place near Fermilab, 3-6 June 2017:

66 registered participants, 40 talks, 15 discussion sessions (525 minutes)

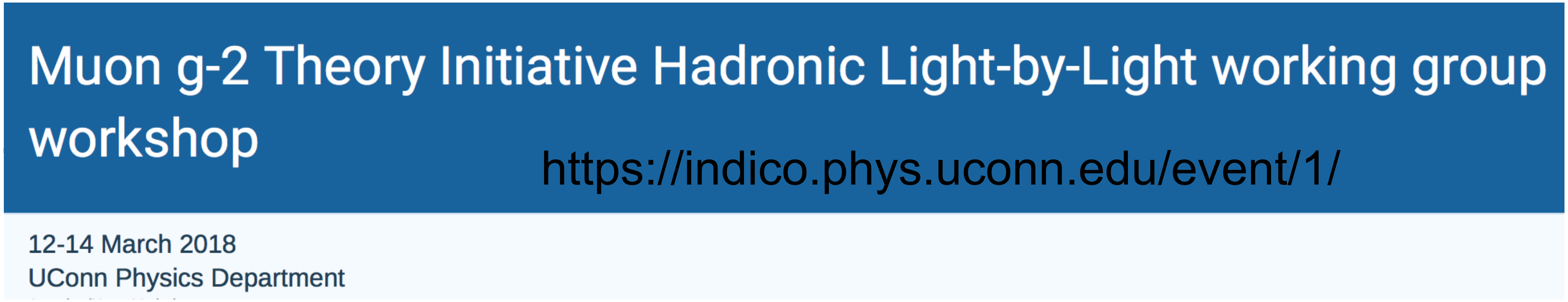
Workshop on hadronic vacuum polarization contributions to muon g-2

February 12-14, 2018
KEK, Tsukuba, Japan

http://www-conf.kek.jp/muonHVPws/index.html



70 registered participants, 28 talks, 6 discussion sessions (330 minutes)



Muon g-2 Theory Initiative Hadronic Light-by-Light working group workshop

https://indico.phys.uconn.edu/event/1/

12-14 March 2018
UConn Physics Department

21 registered participants, 22 talks, 4 discussion sessions (160 minutes)

Second Workshop of the Muon g - 2 Theory Initiative

18-22 June 2018

https://indico.him.uni-mainz.de/event/11/

71 registered participants, 4 days of talks and discussion sessions, and 1/2 day of white paper planning sessions

INT workshop: Hadronic contributions to Third Plenary Meeting of the Muon g-2 Theory Initiative

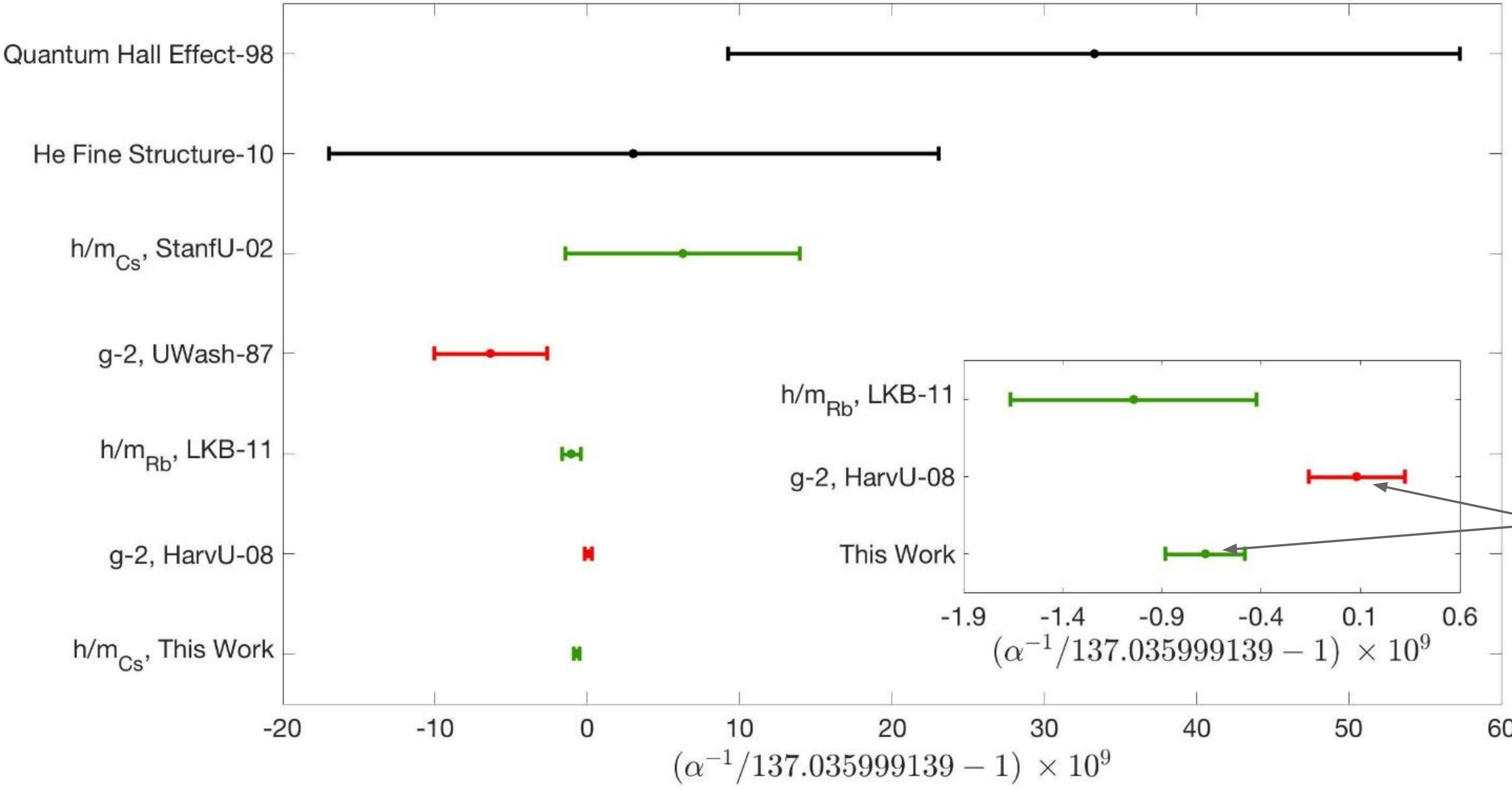
9-13 September 2019

https://indico.fnal.gov/event/21626/

73 participants, 5 days of talks and discussion sessions

Electron g-2: experiment vs theory

[Zachary Pagel (UC Berkeley) @ INT workshop



2.5σ tension

Electron g-2: experiment vs theory

T. Aoyama @ INT g-2 workshop:

- We obtain the theoretical prediction of a_e as

$$a_e(\text{theory: } \alpha(\text{Rb})) = 1\ 159\ 652\ 182.037\ (720)(11)(12) \times 10^{-12}$$
$$a_e(\text{theory: } \alpha(\text{Cs})) = 1\ 159\ 652\ 181.606\ (229)(11)(12) \times 10^{-12}$$

where uncertainties are due to fine-structure constant α, QED 10th order, and hadronic contribution.

- The measurement of a_e is

$$a_e(\text{expt.}) = 1\ 159\ 652\ 180.73\ (28) \times 10^{-12}$$

- The differences between theory and measurement are

$$a_e(\text{theory: } \alpha(\text{Rb})) - a_e(\text{expt.}) = 1.31\ (77) \times 10^{-12}\ [1.7\sigma]$$
$$a_e(\text{theory: } \alpha(\text{Cs})) - a_e(\text{expt.}) = 0.88\ (36) \times 10^{-12}\ [2.4\sigma]$$