On gauge-dependence of gravitational waves from 1st-order phase transition

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Outline

- Motivation

  - Gauge-dependence (ξ) of the effective potential

- Impact of ξ on 1st-order phase transition in classical scale-inv. U(1) models: T_C, T_N, GW
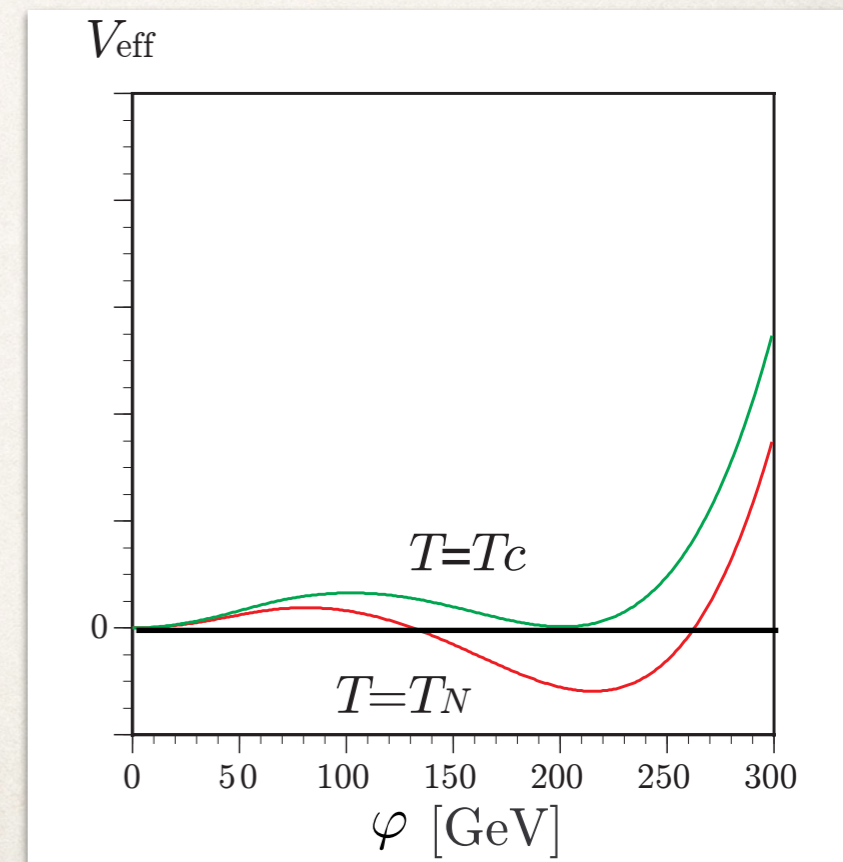
- Summary

Introduction

- 1ˢᵗ-order phase transition (PT) has interesting physical implications:
  Electroweak Baryogenesis, Gravitational Waves (GW), etc.

- Mostly, effective potential is used for such calculations.

problem

- Effective potential inherently depends on gauge-fixing parameter (ξ).

- Nucleation temperature (T_N), GW can be ξ dependent.
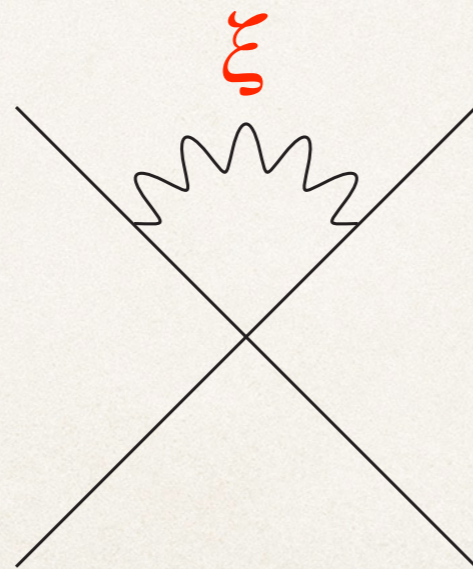
  Q. How (numerically) serious?

1ˢᵗ-order PT

Thorny problem

Effective potential is gauge dependent!!

Jackiw, PRD9,1686 (1974)

Because

$$V_{eff} \ni \qquad \xi \qquad \neq \qquad \xi$$

1PI diagrams only

Leg corrections are needed to remove the ξ dependence.
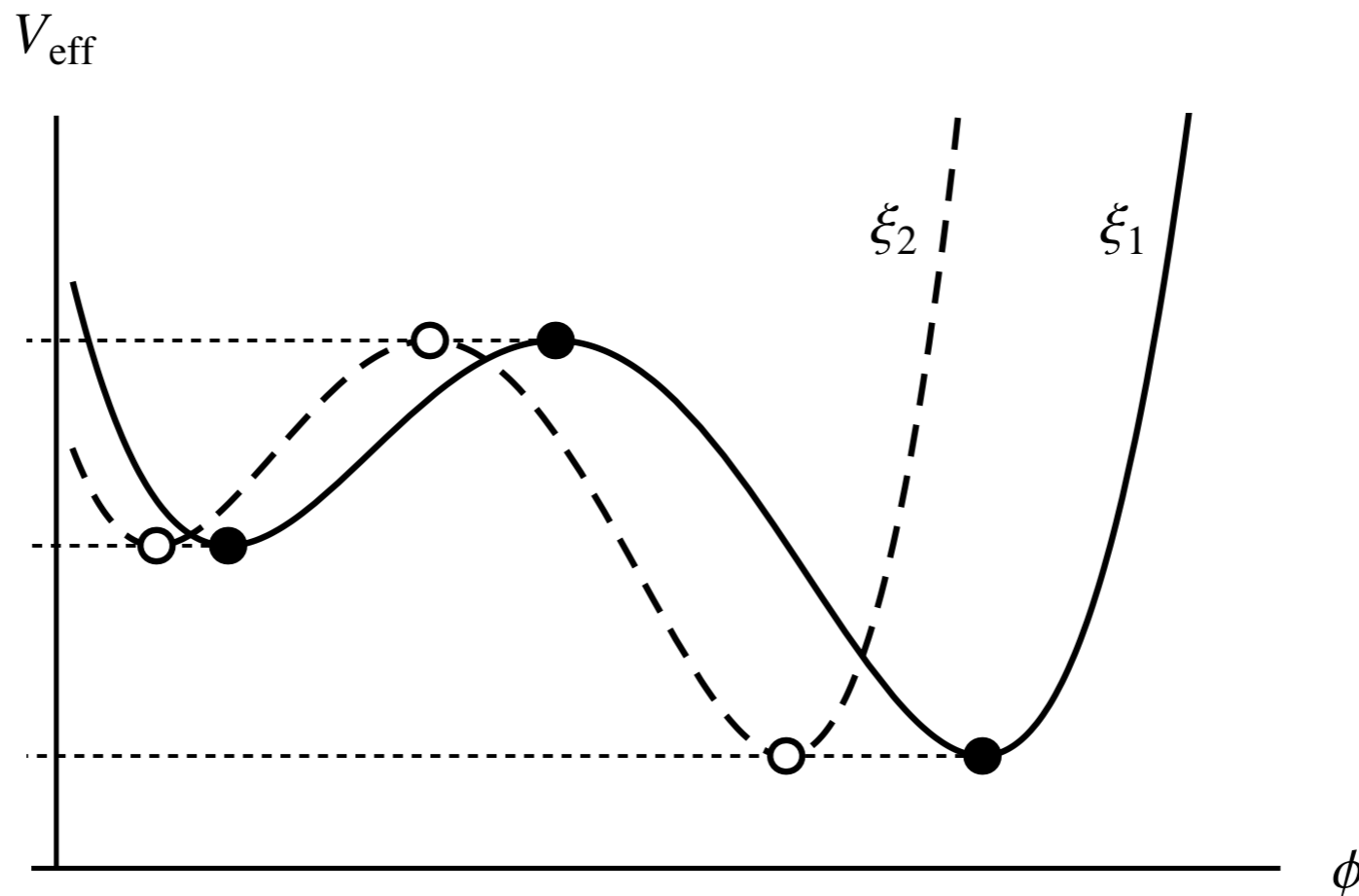
Gauge dependence of V_{eff}

Fig. taken from H. Patel and M. Ramsey-Musolf, JHEP,07(2011)029

- Generally, VEV depends on a gauge parameter ξ

- Energies at stationary points do not depend on ξ

$$\frac{\partial V_{\text{eff}}}{\partial \xi} = C(\varphi, \xi)\frac{\partial V_{\text{eff}}}{\partial \varphi}$$ (Nielsen-Fukuda-Kugo (NFK) identity)

Abelian–Higgs model

~ an example ~

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\Phi|^2 - V(|\Phi|^2),$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu\Phi = (\partial_\mu - ieA_\mu)\Phi,$$

$$V(|\Phi|^2) = -\nu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4, \quad \Phi(x) = \frac{1}{\sqrt{2}}\big(v + h(x) + iG(x)\big).$$

gauge boson: $D_{\mu\nu}^{-1}(k) = (-k^2 + \bar{m}_A^2)\Pi_{\mu\nu}^T(k) + \frac{1}{\xi}(-k^2 + \xi\bar{m}_A^2)\Pi_{\mu\nu}^L(k),$

$$\Pi_{\mu\nu}^T(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \quad \Pi_{\mu\nu}^L(k) = \frac{k_\mu k_\nu}{k^2},$$

NG boson: $\Delta_G^{-1}(k) = k^2 - \bar{m}_G^2 - \xi\bar{m}_A^2$

ghost: $\Delta_c^{-1}(k) = i(k^2 - \xi\bar{m}_A^2)$

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1-loop effective potential

e.g., Abelian-Higgs model

$$
\begin{aligned}
\mu^\epsilon V_1^{A+G+c}(\varphi, \xi) &= -\frac{i}{2}\mu^\epsilon \int \frac{d^D k}{(2\pi)^D}\Bigg[(D-1)\ln(-k^2 + \bar{m}_A^2) + \ln(-k^2 + \xi\bar{m}_A^2) \\
&\quad + \ln(-k^2 + \xi\bar{m}_A^2) + \ln\left(1 + \frac{\bar{m}_G^2}{-k^2 + \xi\bar{m}_A^2}\right) \\
&\quad - 2\ln(-k^2 + \xi\bar{m}_A^2)\Bigg] \\
&= -\frac{i}{2}\mu^\epsilon \int \frac{d^D k}{(2\pi)^D}\left[(D-1)\ln(-k^2 + \bar{m}_A^2) + \ln\left(1 + \frac{\bar{m}_G^2}{-k^2 + \xi\bar{m}_A^2}\right)\right].
\end{aligned}
$$

ξ –dependence disappears at $\bar{m}_G^2(\varphi = v) = 0$, $\quad \left.\dfrac{\partial V_0}{\partial \varphi}\right|_{\varphi = v} = 0$

NFK identity at 1-loop level:

$$
\frac{\partial V_1(\varphi, \xi)}{\partial \xi} = C(\varphi, \xi)\frac{\partial V_0(\varphi)}{\partial \varphi}
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1-loop

$$\frac{\partial V_1(\varphi, \xi)}{\partial \xi} = C(\varphi, \xi)\frac{\partial V_0(\varphi)}{\partial \varphi}$$

1-loop effective potential

e.g., Abelian-Higgs model

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$$\mu^\epsilon V_1^{A+G+c}(\varphi, \xi) = -\frac{i}{2}\mu^\epsilon \int \frac{d^D k}{(2\pi)^D}\left[\boxed{(D-1)\ln(-k^2 + \bar{m}_A^2) + \ln(-k^2 + \xi \bar{m}_A^2)} \right.$$

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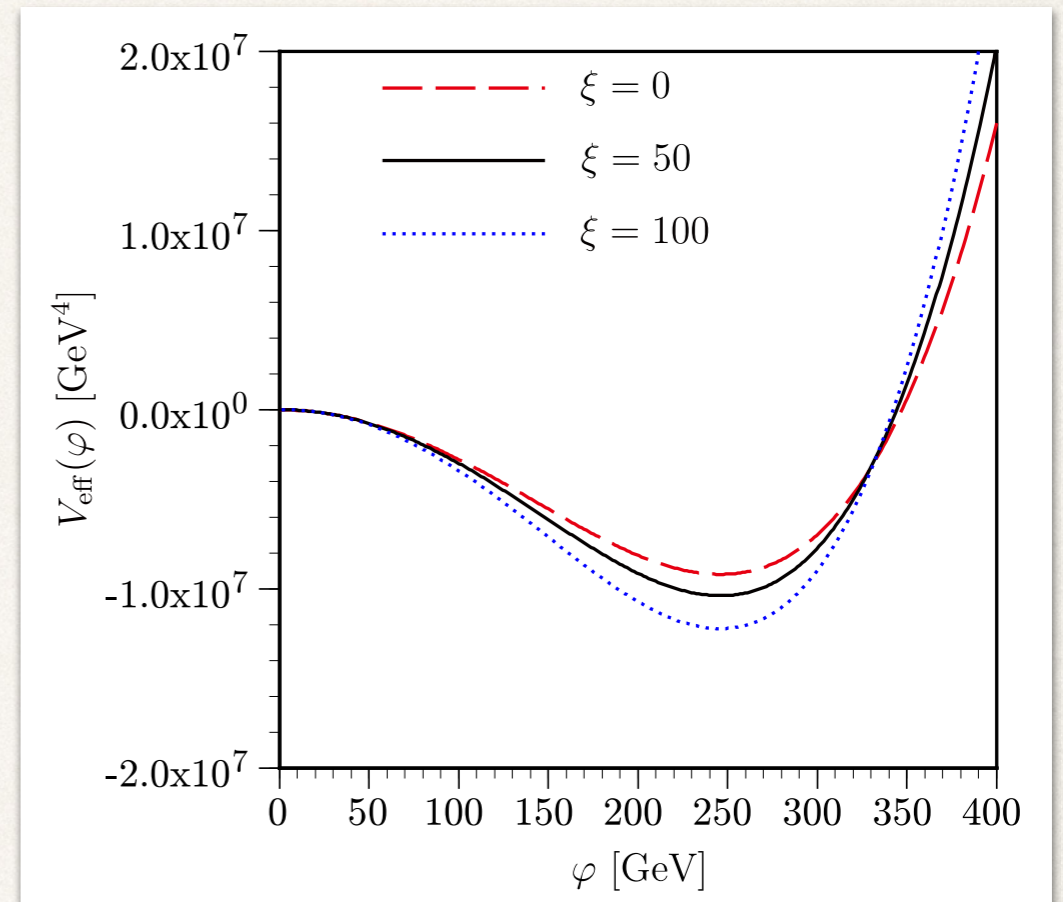
NFK identity at 1-loop level:

$$\frac{\partial \boxed{V_1(\varphi, \xi)}}{\partial \xi} = C(\varphi, \xi)\frac{\partial \boxed{V_0(\varphi)}}{\partial \varphi}$$

1-loop   Tree

Comments

(1) vacuum condition at 1 loop:

$$\frac{\partial(V_0 + V_1)}{\partial\varphi} = 0$$

-> vacuum E also depends on ξ.

(2) prescriptions for obtaining physical variables:

$$v(\xi) \quad \longleftrightarrow \quad G_F \quad \text{(fermi constant)}$$

$$m_h^2(\xi) = \left.\frac{\partial^2 V_\text{eff}}{\partial\varphi^2}\right|_{\varphi=v} \quad \longleftrightarrow \quad M_h^2 \quad \text{(on-shell Higgs mass)}$$

Input parameters must be ξ independent.

1-loop effective potential at T≠0

$$V_1(\varphi, \xi; T) = \sum_i \frac{T^4}{2\pi^2} I_B(a_i^2),$$

where

$$I_B(a^2) = \int_0^\infty dx \; x^2 \ln\left[1 - e^{-\sqrt{x^2+a^2}}\right].$$

Using a high-T expansion of $I_B(a^2)$, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

$$= \frac{T^2}{24}(\bar{m}_h^2 + \bar{m}_G^2 + 3\bar{m}_A^2) - \frac{T}{12\pi}\left[(\bar{m}_h^2)^{3/2} + (\bar{m}_G^2 + \xi\bar{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\bar{m}_A^2)^{3/2}\right]$$

$$+ \frac{1}{64\pi^2}\left[\bar{m}_h^4 \ln\frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}_G^2 + \xi\bar{m}_A^2)^2 \ln\frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}_A^4\left(\ln\frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3}\right) - (\xi\bar{m}_A^2)^2 \ln\frac{\alpha_B T^2}{\bar{\mu}^2}\right].$$

1-loop effective potential at $T \neq 0$

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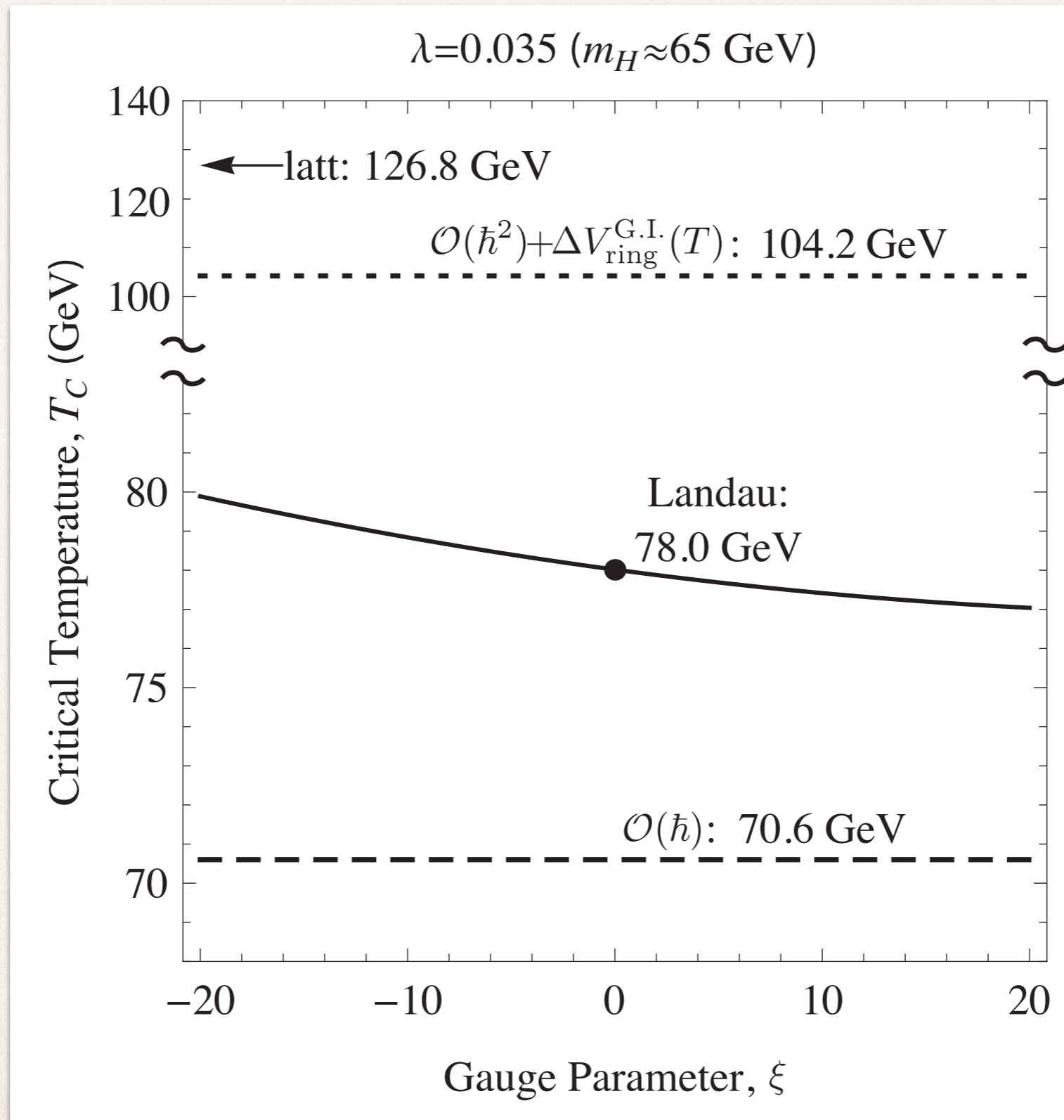$$I_B(a^2) = \int_0^\infty dx \; x^2 \ln\left[1 - e^{-\sqrt{x^2+a^2}}\right].$$

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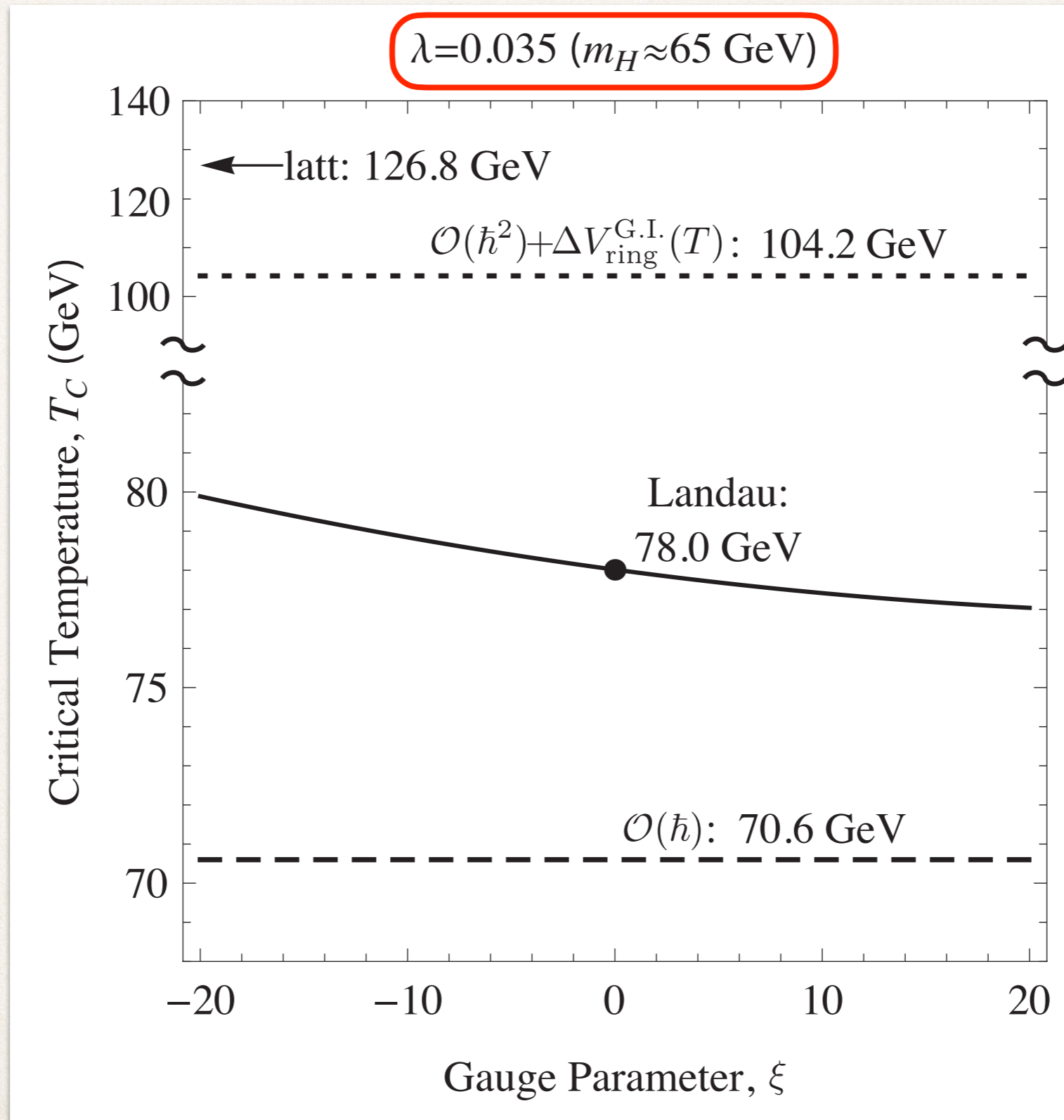$$

V_{eff} at $T \neq 0$ also depends on ξ except "T^2-terms".

SM case

$\lambda=0.035\ (m_H\approx 65\ \mathrm{GeV})$

SM case

[H.Patel, M.Ramsey-Musolf, JHEP,07(2011)029]

$\lambda=0.035$ ($m_H \approx 65$ GeV)

latt: 126.8 GeV

$\mathcal{O}(\hbar^2)+\Delta V_{\mathrm{ring}}^{\mathrm{G.I.}}(T):$ 104.2 GeV

Landau: 78.0 GeV

$\mathcal{O}(\hbar):$ 70.6 GeV

Critical Temperature, T_C (GeV)

140

120

100

80

75

70

−20   −10   0   10   20

Gauge Parameter, ξ

SM case

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Critical Temperature, T_C (GeV)

Gauge Parameter, ξ

Classical scale-inv. U(1) model

SM + U(1)$'$ w/ scale symmetry

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}'} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + |D_\mu S|^2 - V(H,S)$$

$$Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, \; D_\mu S = (\partial_\mu + ig'Q'_S Z'_\mu)S,$$

scalar potential

$$V(H,S) = \lambda_H(H^\dagger H)^2 + \lambda_{HS}H^\dagger H|S|^2 + \lambda_S|S|^4$$

singlet scalar field: $S(x) = \frac{1}{\sqrt{2}}(v_S + h_S(x) + iG(x))$

After U(1) is radiatively broken (<S>≠0), EW symmetry is broken if λ_{HS} < 0. $m_h^2 = -\lambda_{HS}v_S^2 \longrightarrow -\lambda_{HS} = m_h^2/v_S^2 = \mathcal{O}(10^{-3})$

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Classical scale-inv. U(1) model

ξ dependence is different from the massive U(1) model case.

$$V_{\text{eff}}(\varphi_S) = \frac{\lambda_S}{4}\varphi_S^4 + 3\frac{\bar{m}_{Z'}^4}{64\pi^2}\left(\ln\frac{\bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{5}{6}\right)$$
$$+ \frac{\bar{m}_{G,\xi}^4}{64\pi^2}\left(\ln\frac{\bar{m}_{G,\xi}^2}{\bar{\mu}^2} - \frac{3}{2}\right) - \frac{(\xi\bar{m}_{Z'}^2)^2}{64\pi^2}\left(\ln\frac{\xi\bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{3}{2}\right),$$

where $\quad \bar{m}_{Z'}^2 = (g'Q'_S\varphi_S)^2, \quad \bar{m}_{G,\xi}^2 = \lambda_S\varphi_S^2 + \xi\bar{m}_{Z'}^2.$

Minimization condition -> $\lambda_S = O(g'^4/16\pi^2)$

One gets $\quad V_{\text{eff}}(\varphi_S) \simeq \frac{3\bar{m}_{Z'}^4}{64\pi^2}\left(\ln\frac{\varphi_S^2}{v_S^2} - \frac{1}{2}\right), \quad$ ξ independent!!

- Finite-T 1-loop effective potential is also ξ independent.

- ξ dependence will appear from 2-loop order.

Classical scale-inv. U(1) model

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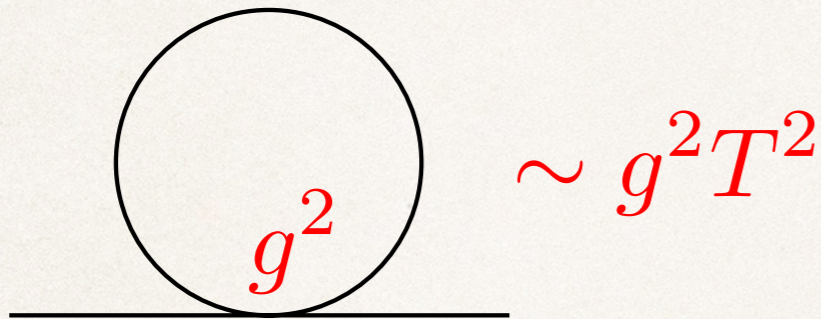
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Thermal resummation

At high T

$$\sim g^2 T^2$$

g^2

Prescription

e.g. NG boson

$$\bar{m}^2_{G,\xi} \to \bar{m}^2_{G,\xi} + \Delta m^2_S$$

to leading order: $\Delta m^2_S = \dfrac{(g' Q'_S)^2}{4} T^2$
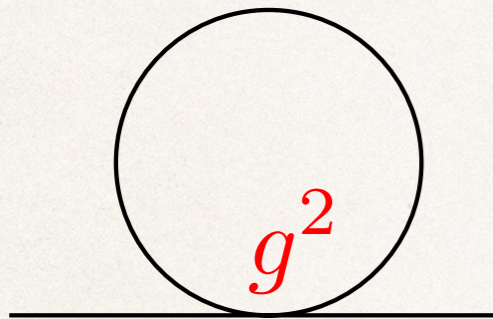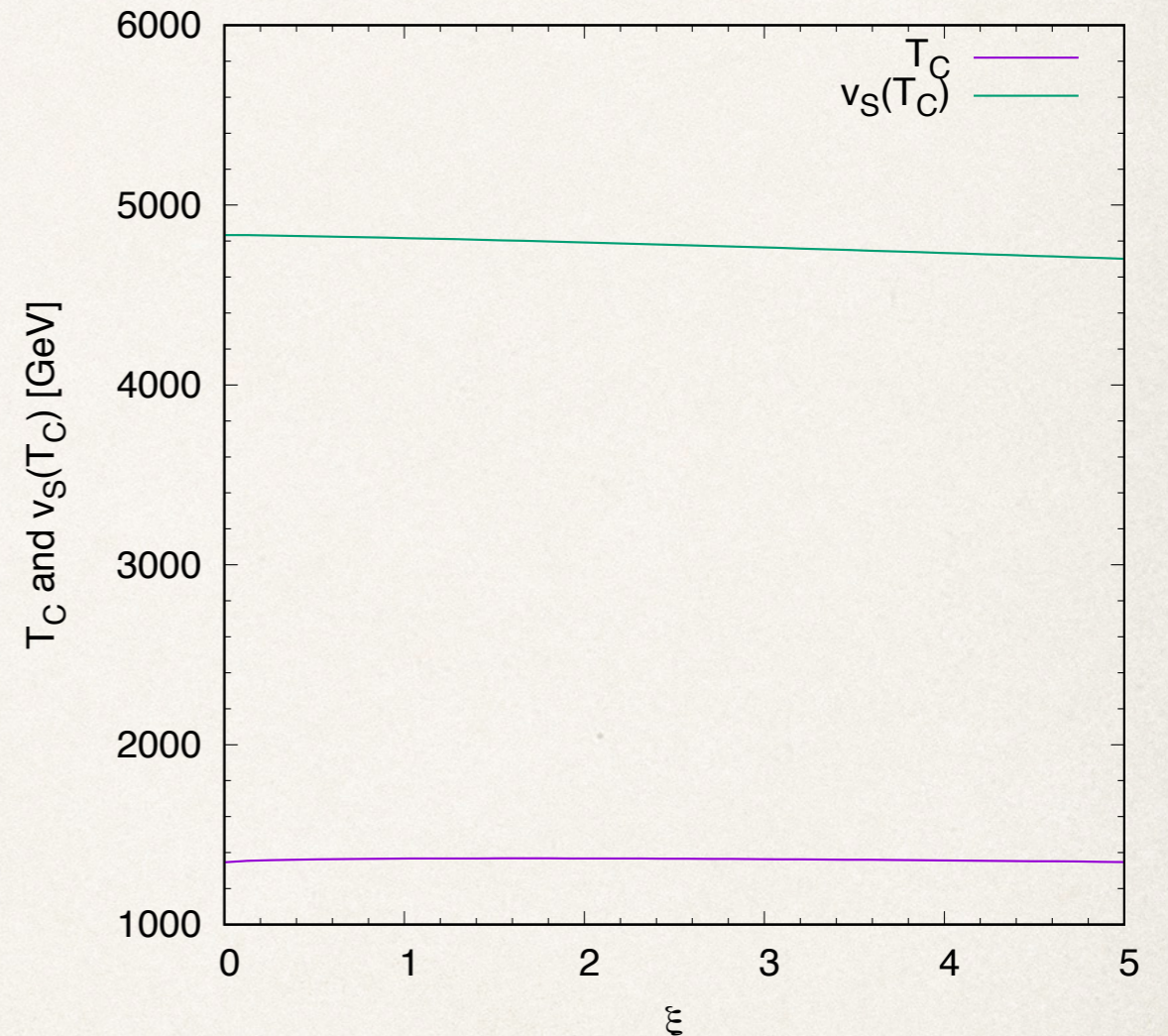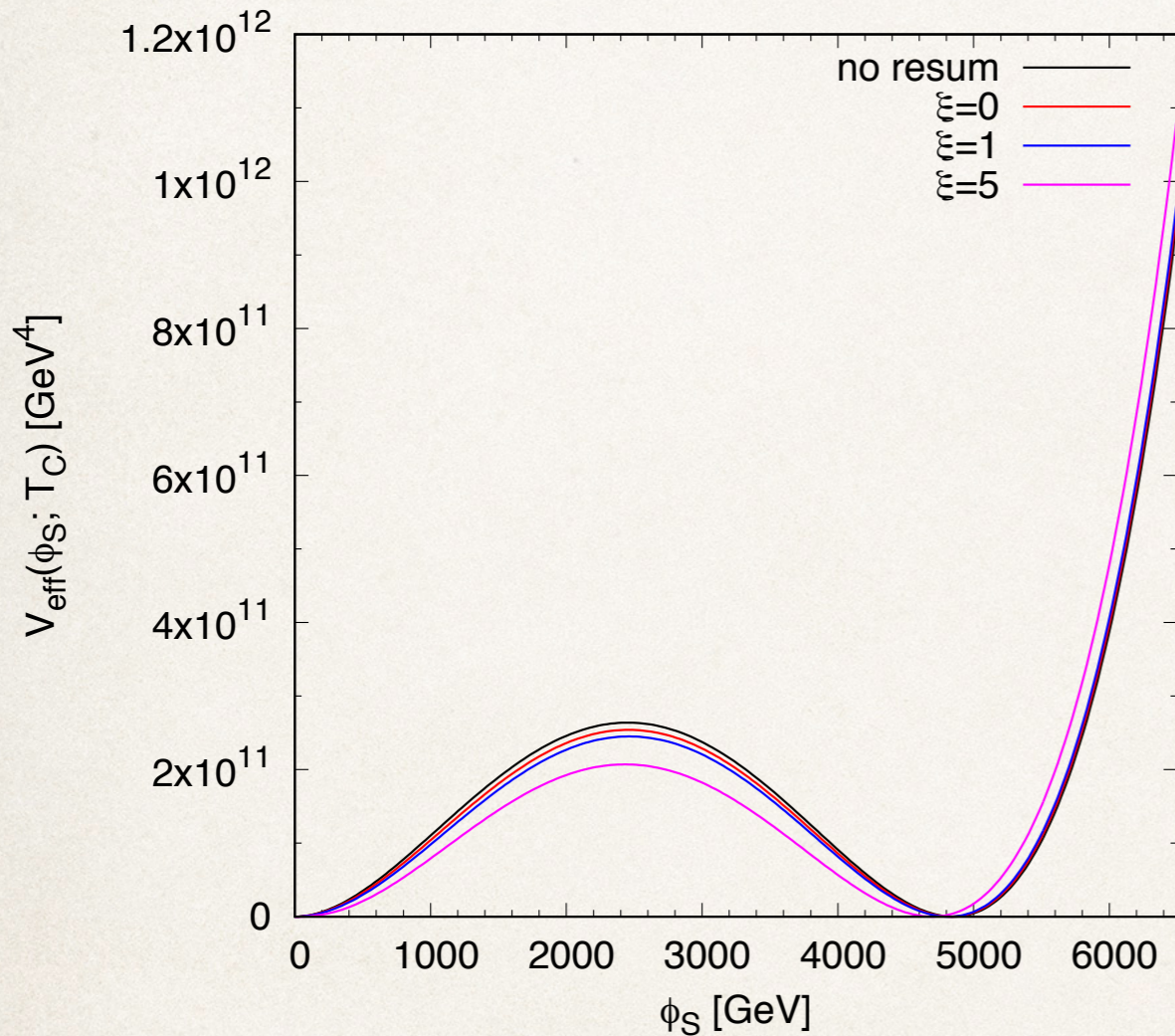
$$V_{\text{eff}}(\varphi_S; T)\Big|_{\xi\text{-part}} = \frac{(\xi \bar{m}^2_{Z'} + \Delta m^2_S)^2}{64\pi^2}\left(\ln \frac{\xi \bar{m}^2_{Z'} + \Delta m^2_S}{\bar{\mu}^2} - \frac{3}{2}\right) - \frac{(\xi \bar{m}^2_{Z'})^2}{64\pi^2}\left(\ln \frac{\xi \bar{m}^2_{Z'}}{\bar{\mu}^2} - \frac{3}{2}\right)$$
$$+ \frac{T^4}{2\pi^2}\left[I_B\left(\frac{\xi \bar{m}^2_{Z'} + \Delta m^2_S}{T^2}\right) - I_B\left(\frac{\xi \bar{m}^2_{Z'}}{T^2}\right)\right].$$

where I$_B$ is a 1-loop thermal function.

V$_{\text{eff}}$ is no longer ξ independent due to $\Delta m^2_S \neq 0$

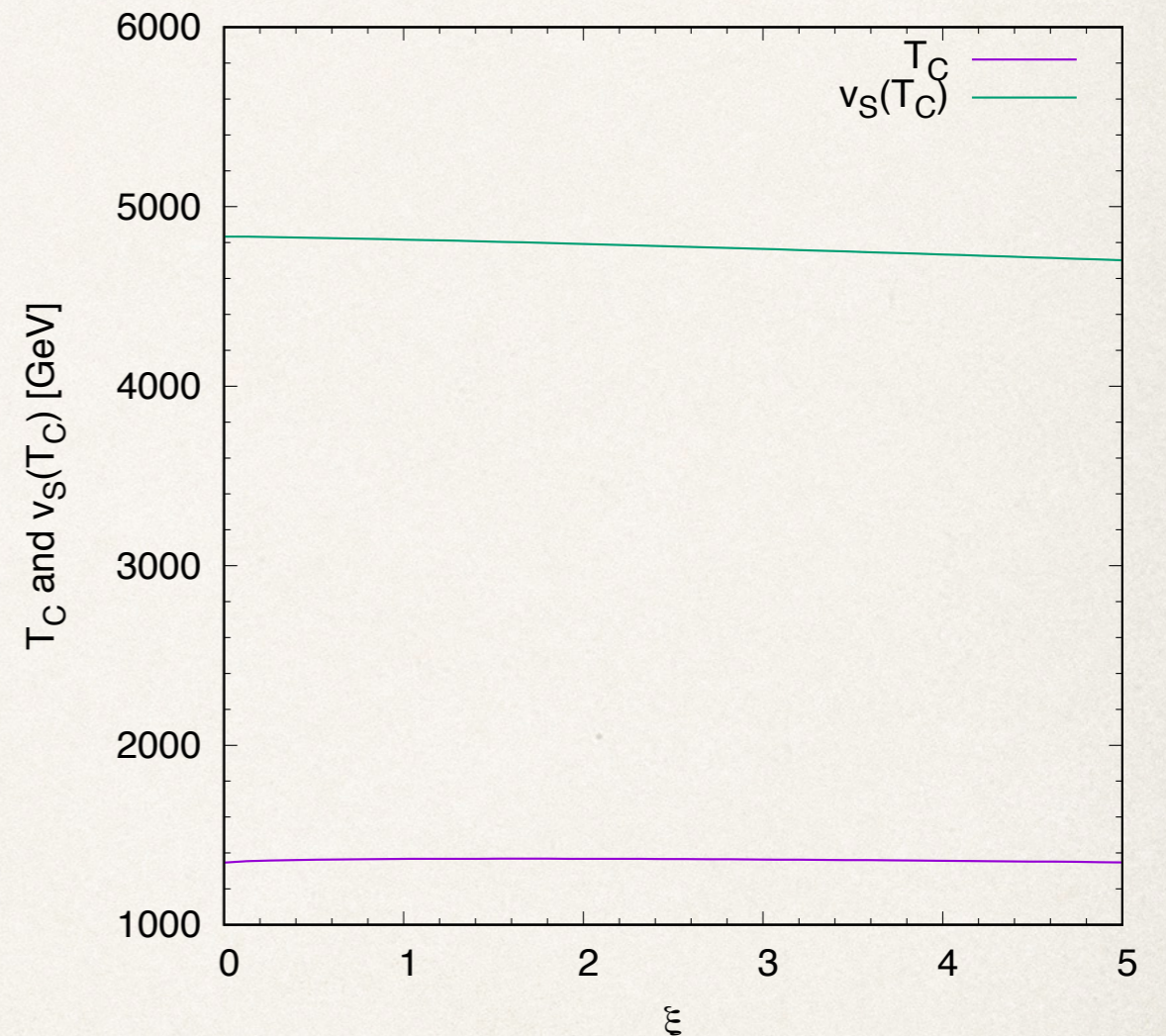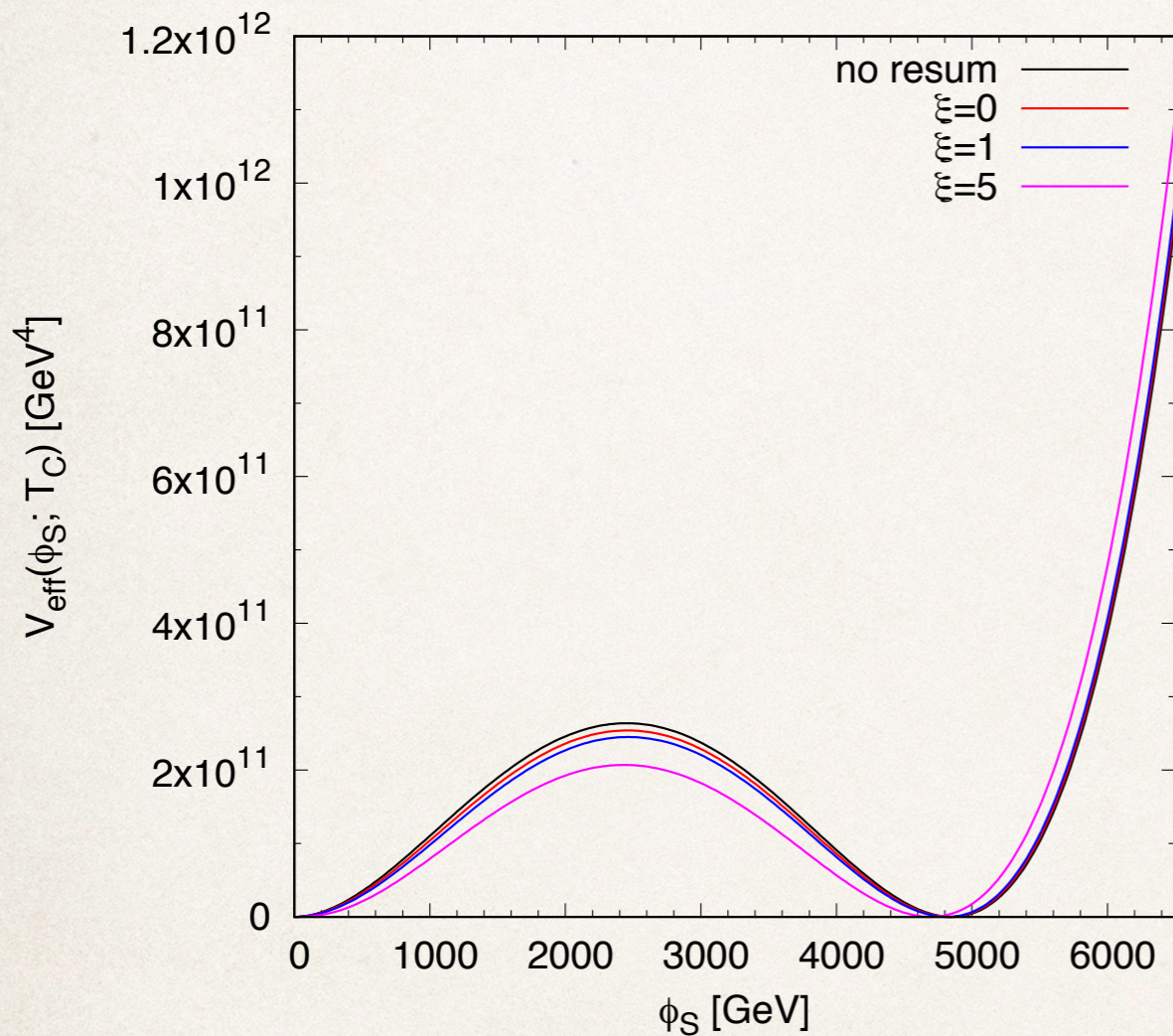# Thermal resummation

At high T

$$\sim g^2 T^2$$

g^2

Prescription

e.g. NG boson

$$\bar{m}^2_{G,\xi} \rightarrow \bar{m}^2_{G,\xi} + \Delta m^2_S$$

to leading order: $\quad \Delta m^2_S = \dfrac{(g' Q'_S)^2}{4} T^2$

$$
V_{\text{eff}}(\varphi_S; T)\Big|_{\xi\text{-part}} = \frac{(\xi \bar{m}^2_{Z'} + \Delta m^2_S)^2}{64\pi^2} \left(\ln \frac{\xi \bar{m}^2_{Z'} + \Delta m^2_S}{\bar{\mu}^2} - \frac{3}{2} \right) - \frac{(\xi \bar{m}^2_{Z'})^2}{64\pi^2} \left(\ln \frac{\xi \bar{m}^2_{Z'}}{\bar{\mu}^2} - \frac{3}{2} \right)
$$
$$
+ \frac{T^4}{2\pi^2} \left[I_B \left(\frac{\xi \bar{m}^2_{Z'} + \Delta m^2_S}{T^2} \right) - I_B \left(\frac{\xi \bar{m}^2_{Z'}}{T^2} \right) \right].
$$

where I$_B$ is a 1-loop thermal function.

V$_{\text{eff}}$ is no longer ξ independent due to $\Delta m^2_S \neq 0$

Impacts of ξ on v/T

$$Q'_S = 2,\ \alpha' = g'^2/4\pi = 0.015,\ m_{Z'} = 4.5 \text{ TeV and } m_{\nu_{R1,2,3}} = 1.0 \text{ TeV}.$$

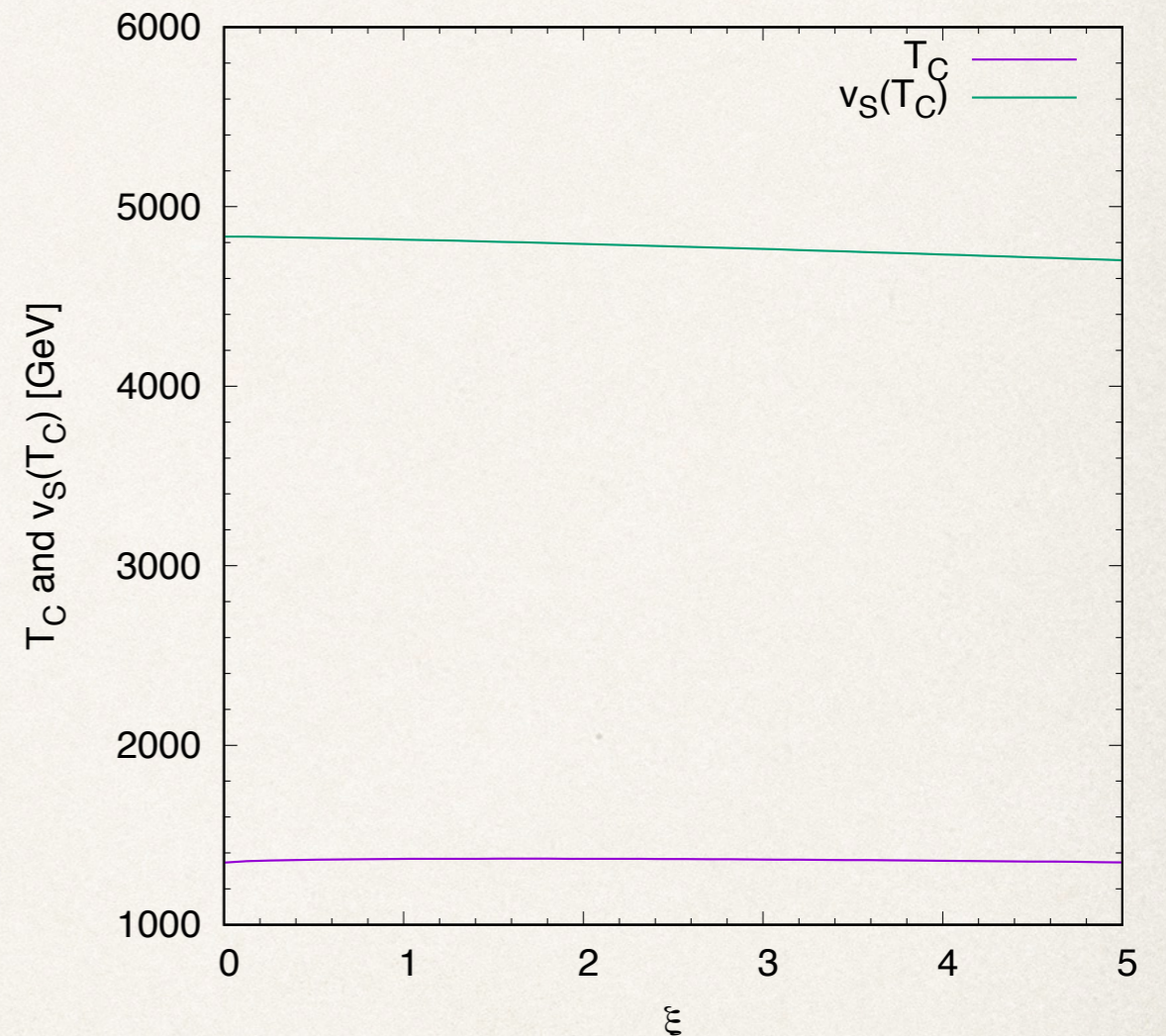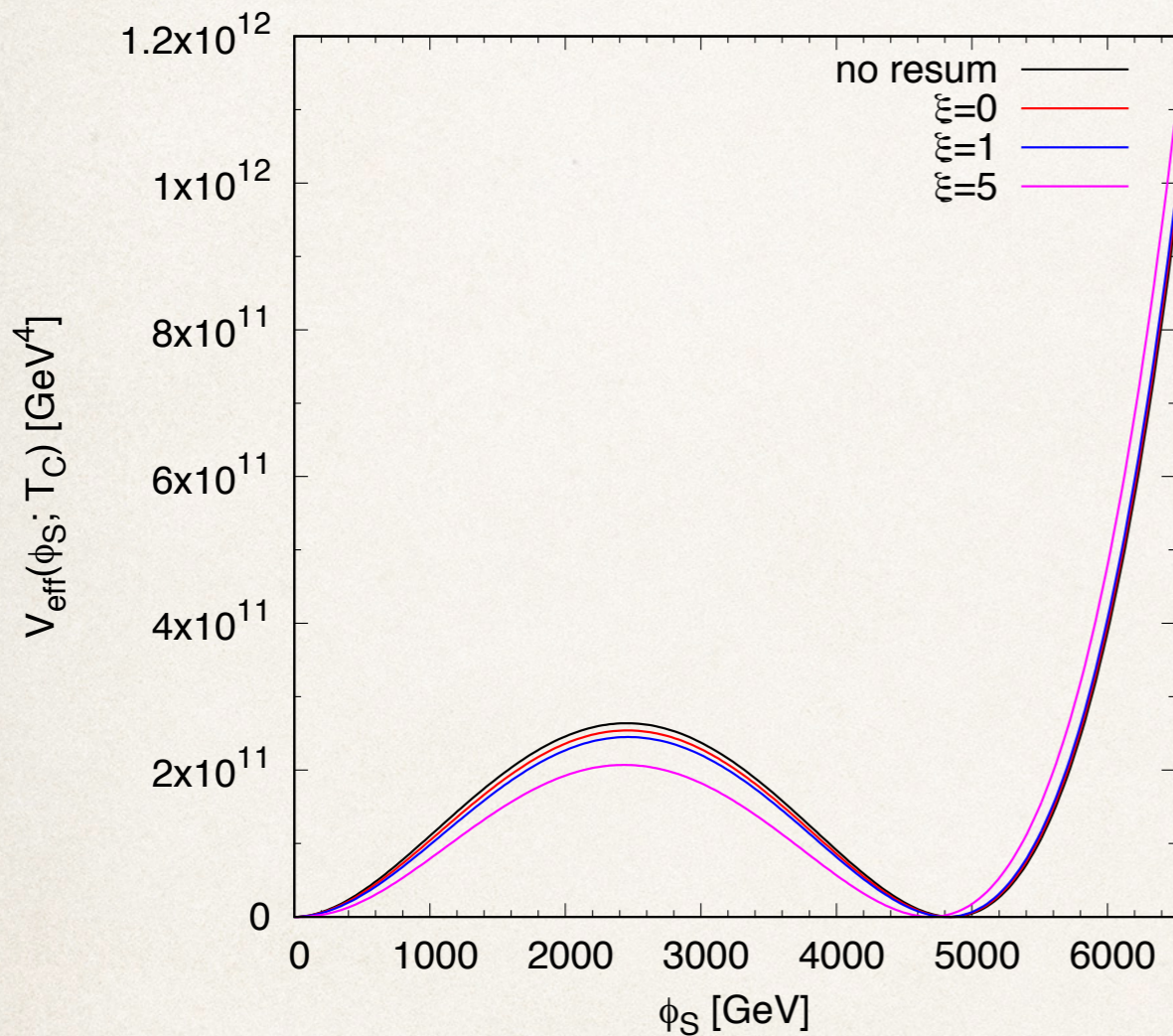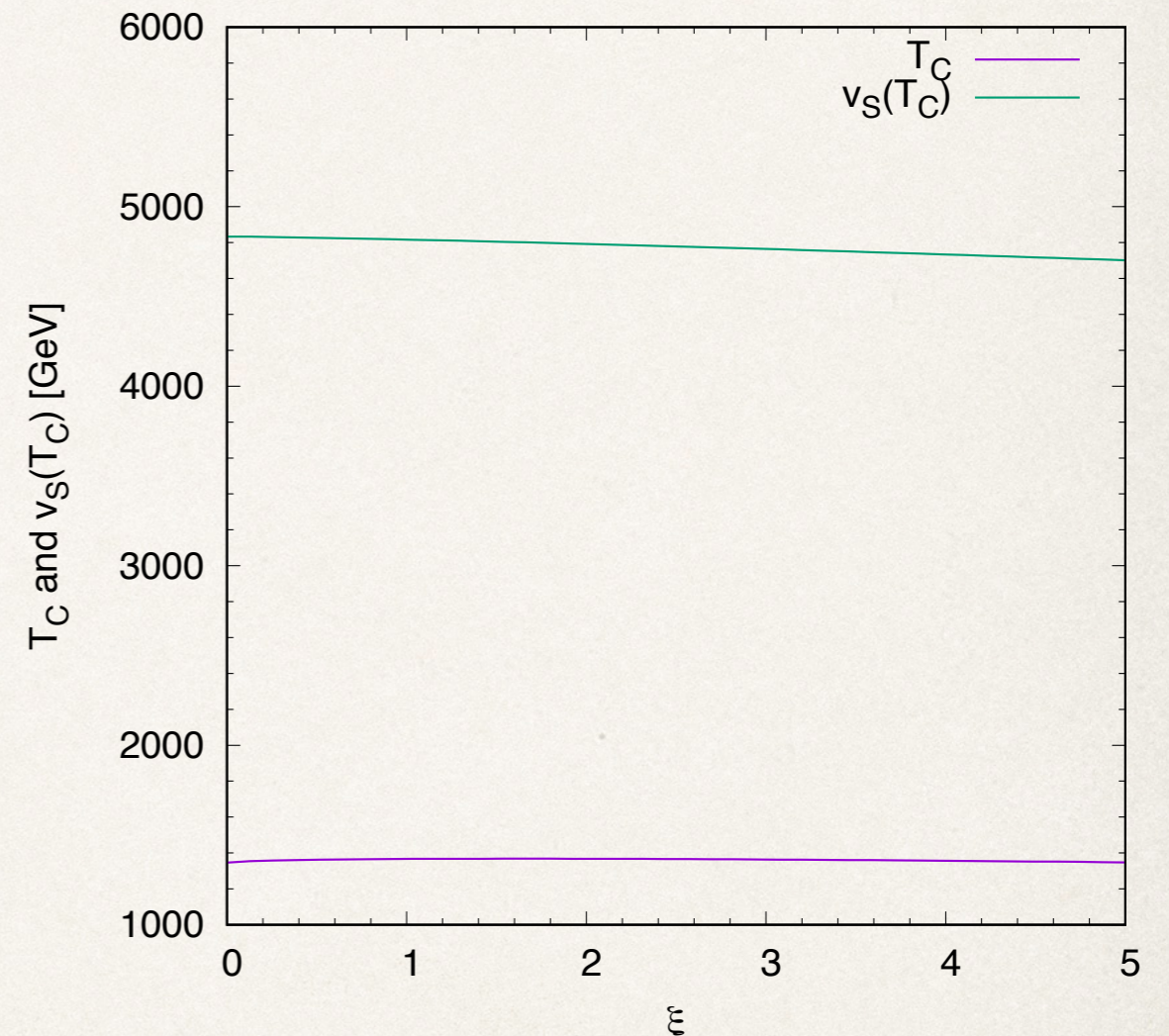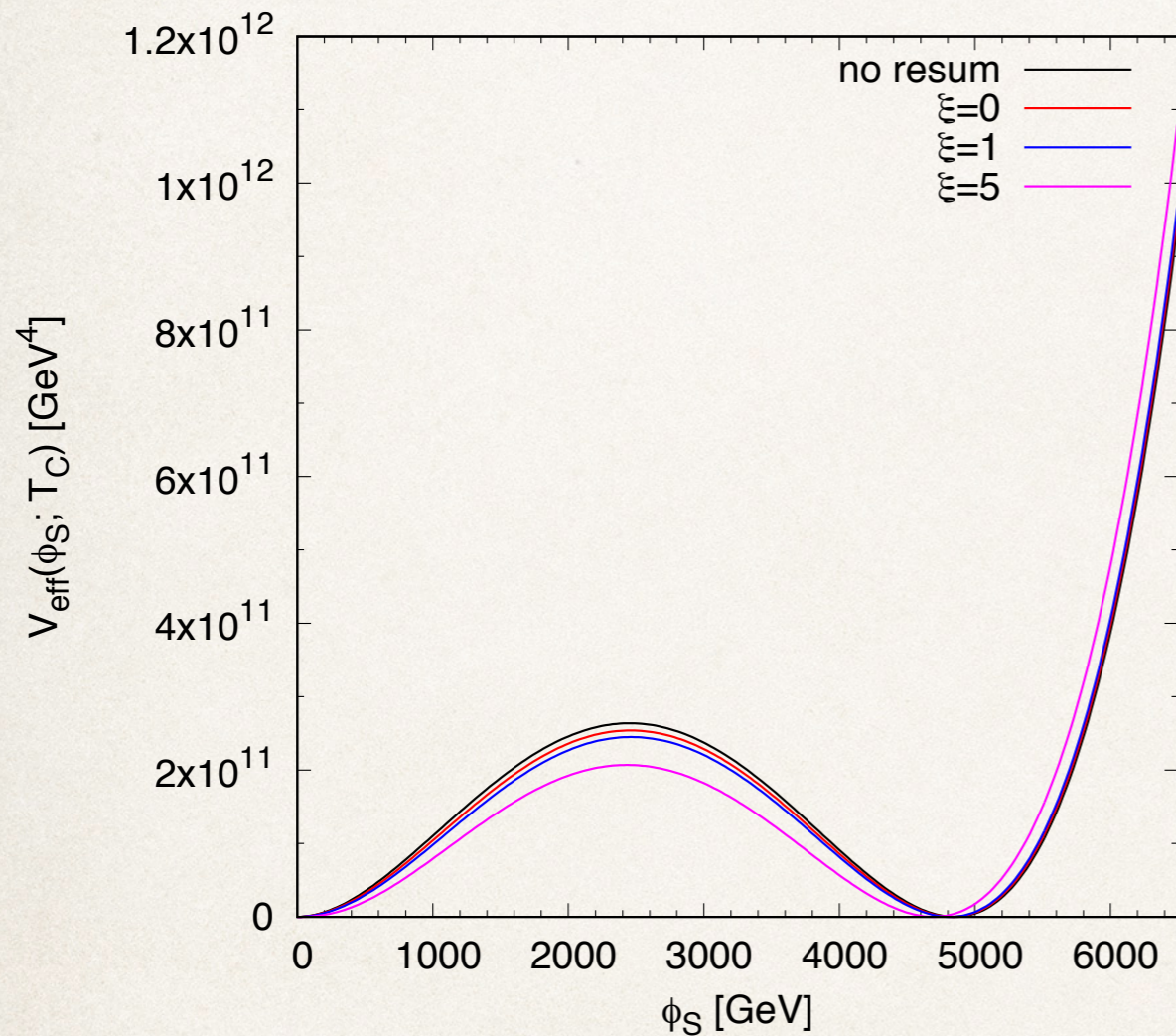|	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
$\dfrac{v_S(T_C)}{T_C}$	$\dfrac{4.851}{1.321} = 3.67$	$\dfrac{4.833}{1.346} = 3.59$	$\dfrac{4.816}{1.368} = 3.52$	$\dfrac{4.695}{1.348} = 3.48$

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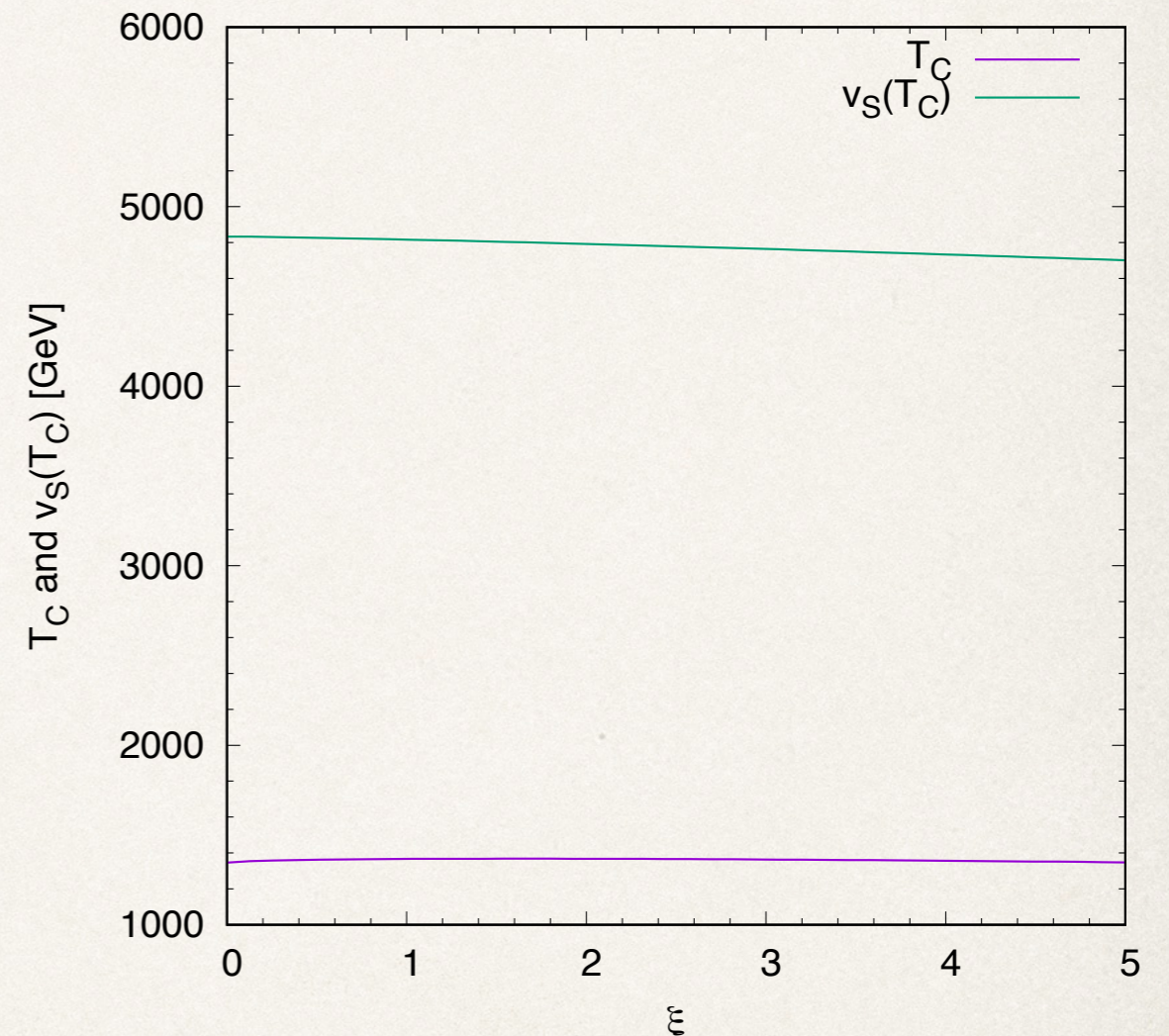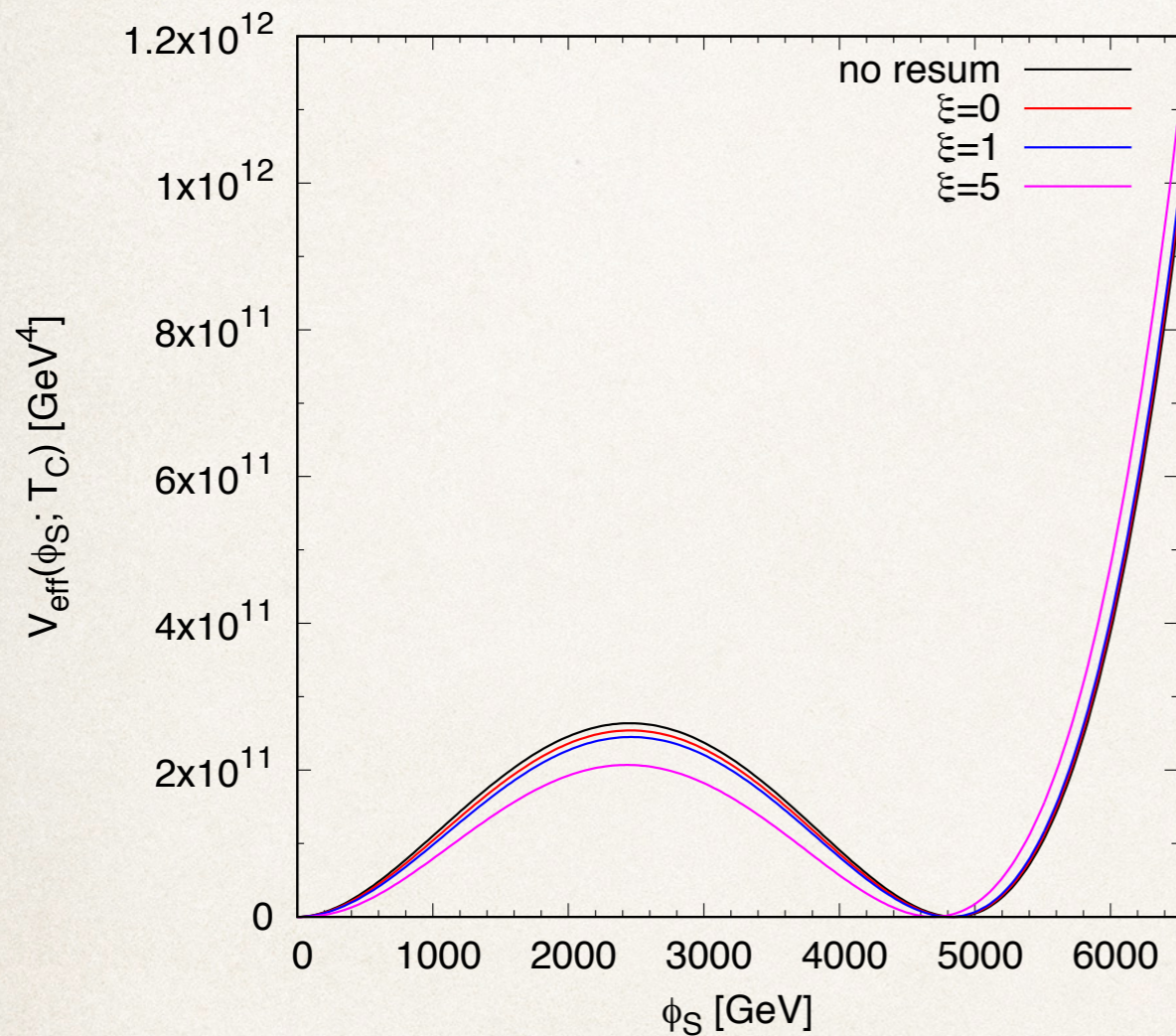|	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
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Impacts of ξ on v/T

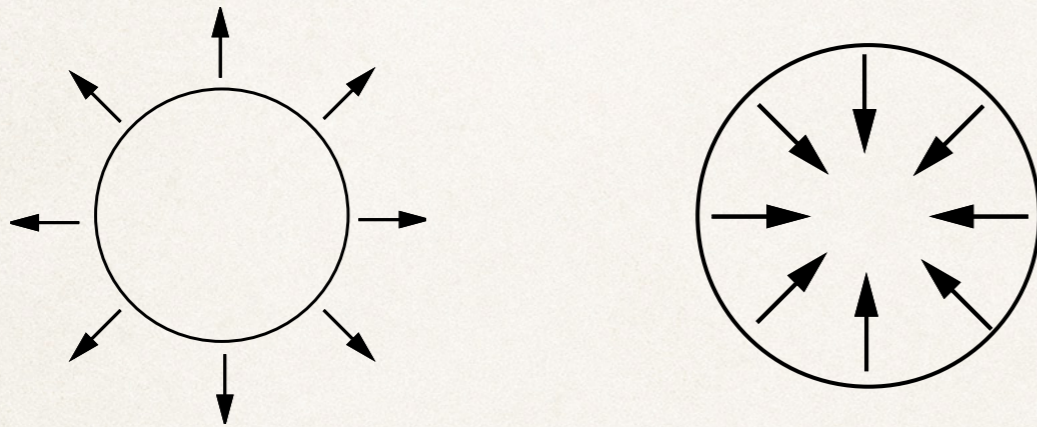$$Q'_S = 2, \; \alpha' = g'^2/4\pi = 0.015, \; m_{Z'} = 4.5 \text{ TeV and } m_{\nu_{R1,2,3}} = 1.0 \text{ TeV.}$$

	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
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Onset of PT

- T_C is not onset of the PT.

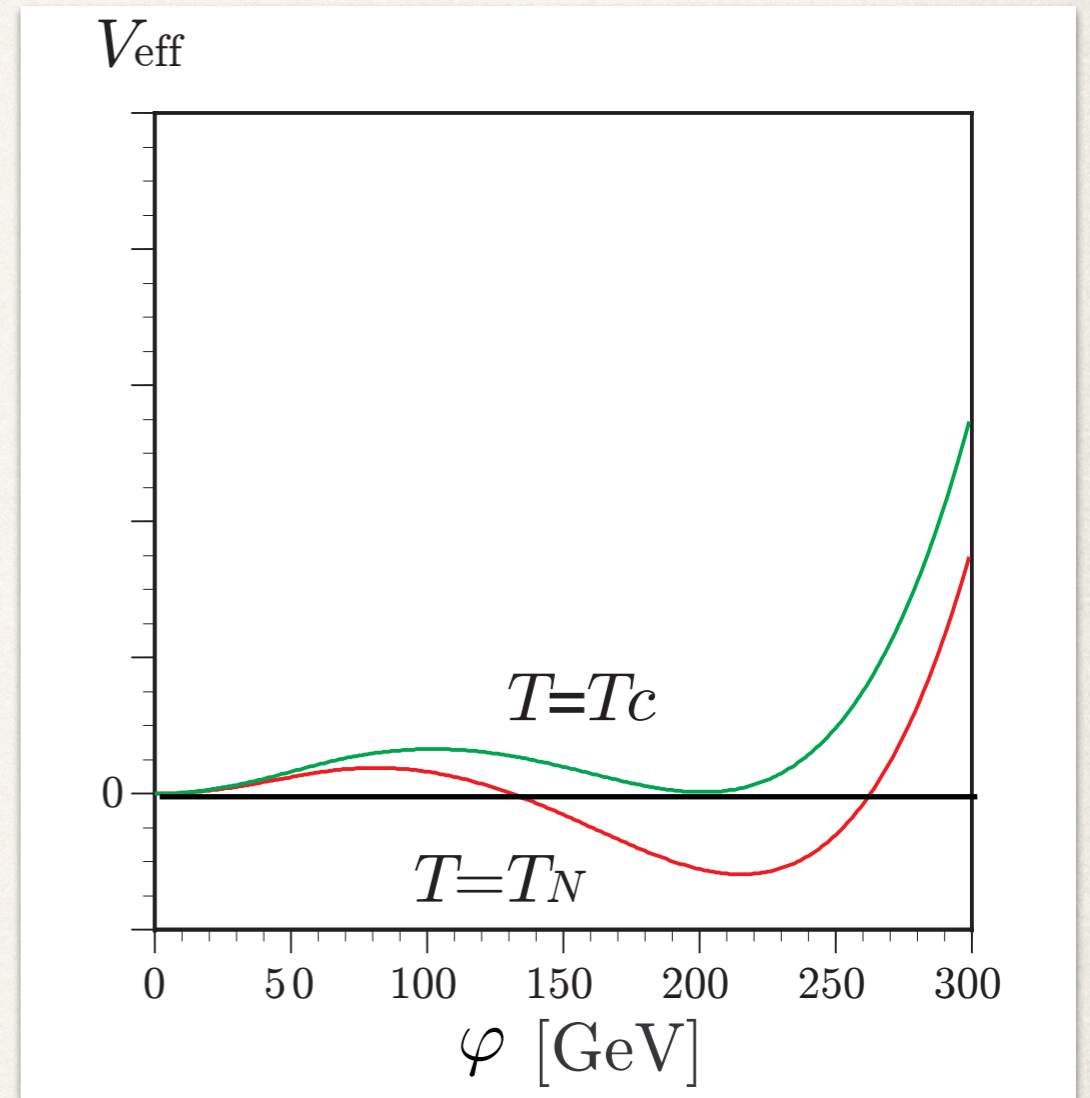- Nucleation starts somewhat below T_C.

"Not all bubbles can grow"

expand?   or   shrink?

volume energy vs. surface energy

$\propto$ (radius)^3 $\propto$ (radius)^2

There is a critical value of radius -> critical bubble

Nucleation temperature

– Nucleation rate per unit time per unit volume

$$\Gamma_N(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

[A.D. Linde, NPB216 ('82) 421]

$S_3(T)$: energy of the critical bubble at T

– Definition of nucleation temperature (T_N)

horizon scale $\simeq H(T)^{-1}$

$$\boxed{\Gamma_N(T_N) H(T_N)^{-3} = H(T_N)}$$

$$\frac{S_3(T_N)}{T_N} - \frac{3}{2} \ln \left(\frac{S_3(T_N)}{T_N} \right) = 152.59 - 2 \ln g_*(T_N) - 4 \ln \left(\frac{T_N}{100 \text{ GeV}} \right)$$
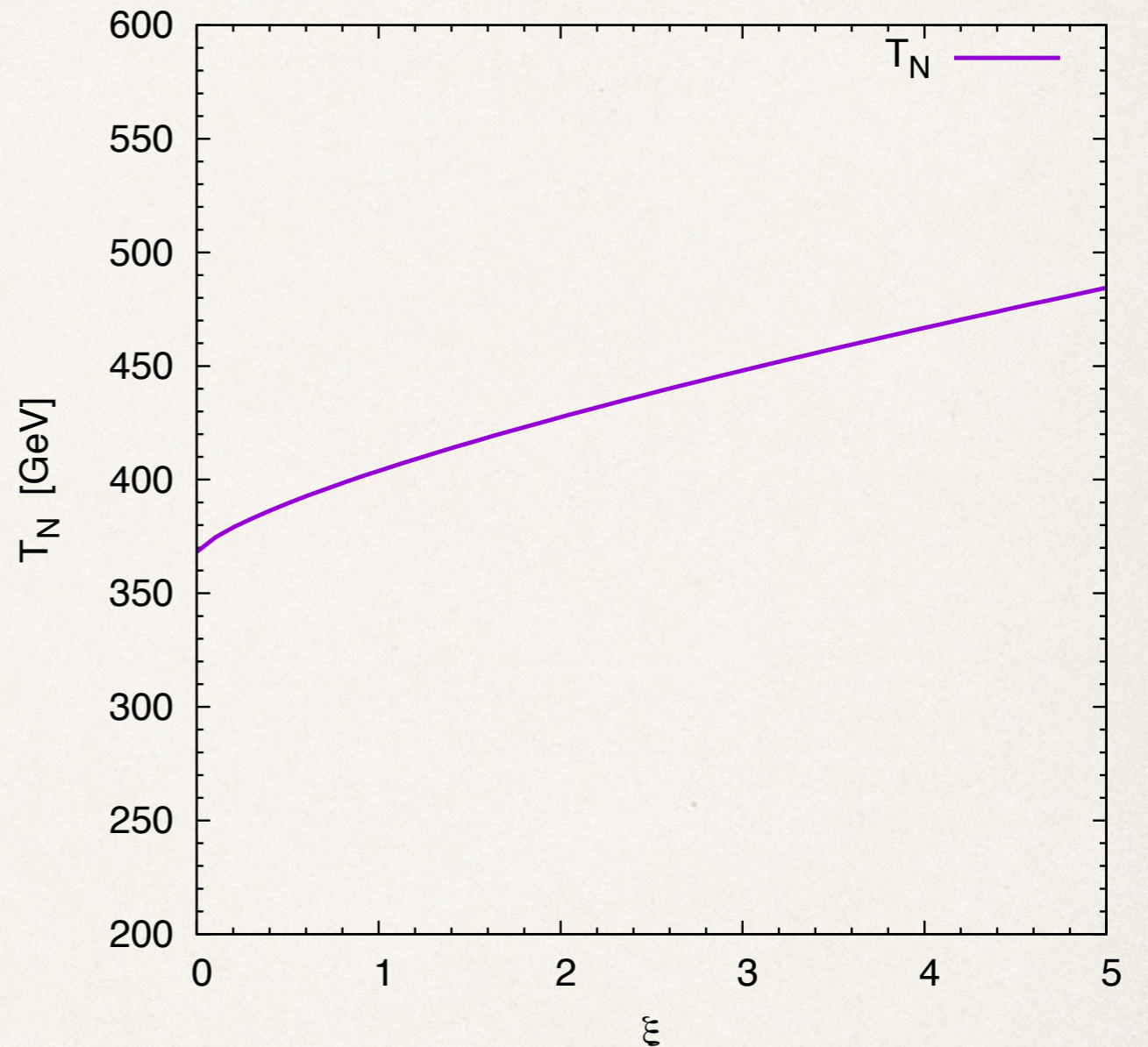
Roughly, $S_3(T)/T \lesssim 150$ is needed for the PT.

Impacts of ξ on T_N

$$S_3 = 4\pi \int_0^\infty dr \; r^2 \left[\frac{1}{2}\left(\frac{d\phi_S}{dr}\right)^2 + V_{\text{eff}}(\phi_S; T) \right]$$

$$\frac{d^2\phi_S}{dr^2} + \frac{2}{r}\frac{d\phi_S}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi_S} = 0$$

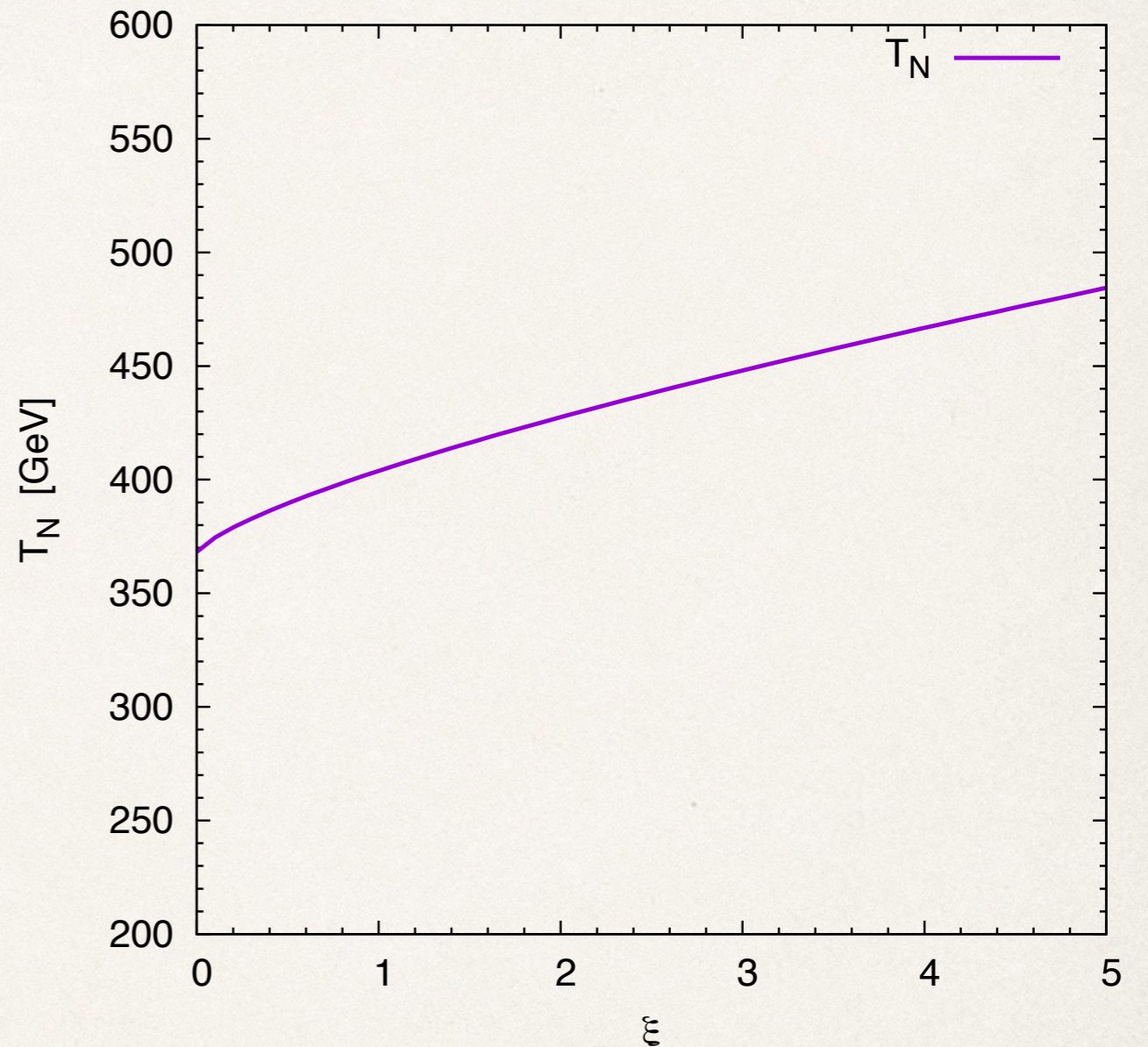$$\lim_{r\to\infty} \phi_S(r) = 0 \text{ and } d\phi_S(r)/dr|_{r=0} = 0.$$

	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
$\dfrac{v_S(T_N)}{T_N}$	$\dfrac{5.181}{0.328} = 15.8$	$\dfrac{5.181}{0.368} = 14.1$	$\dfrac{5.180}{0.405} = 12.8$	$\dfrac{5.163}{0.490} = 10.5$

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ξ dep.

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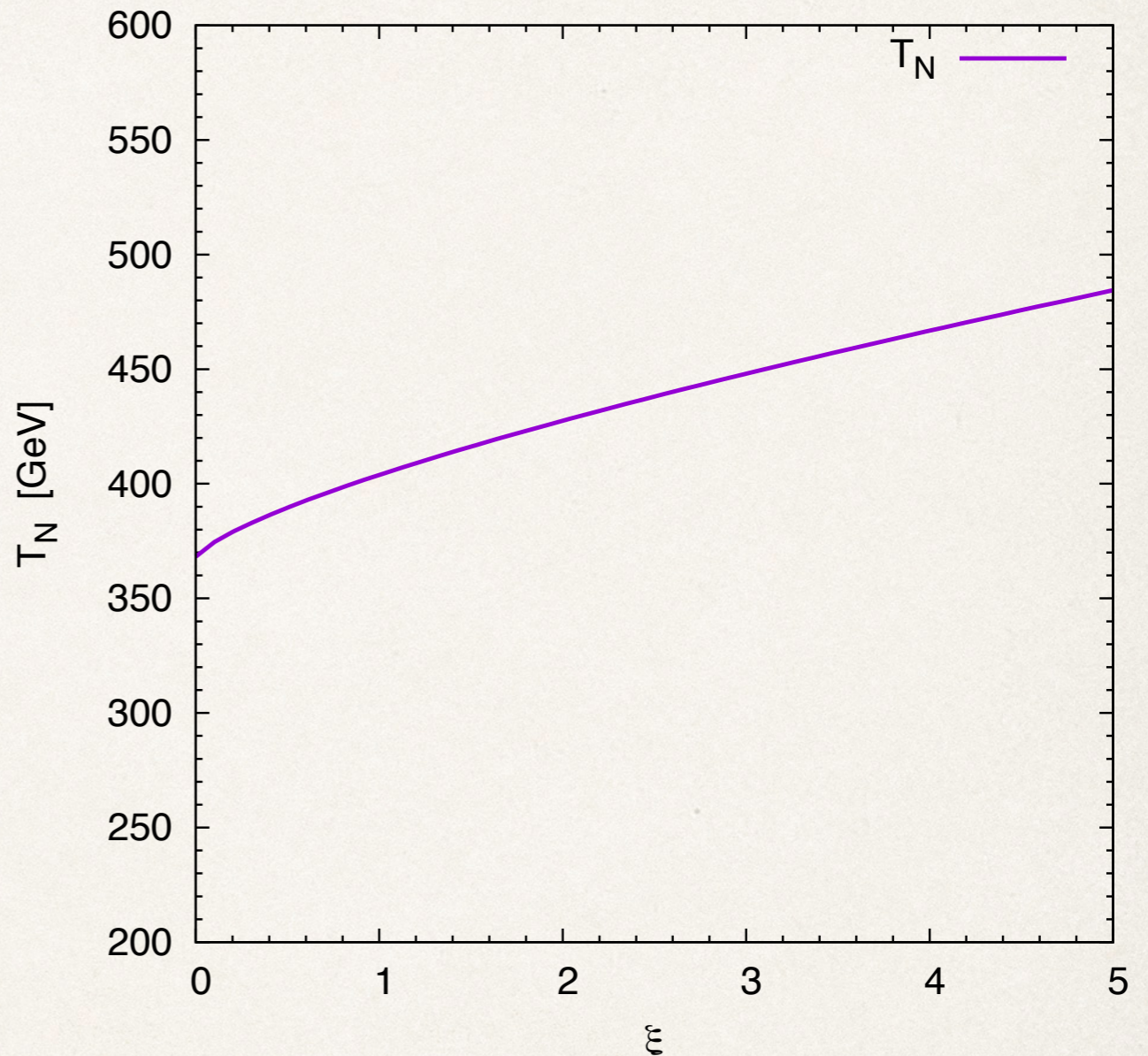$$\lim_{r\to\infty} \phi_S(r) = 0 \text{ and } d\phi_S(r)/dr|_{r=0} = 0.$$

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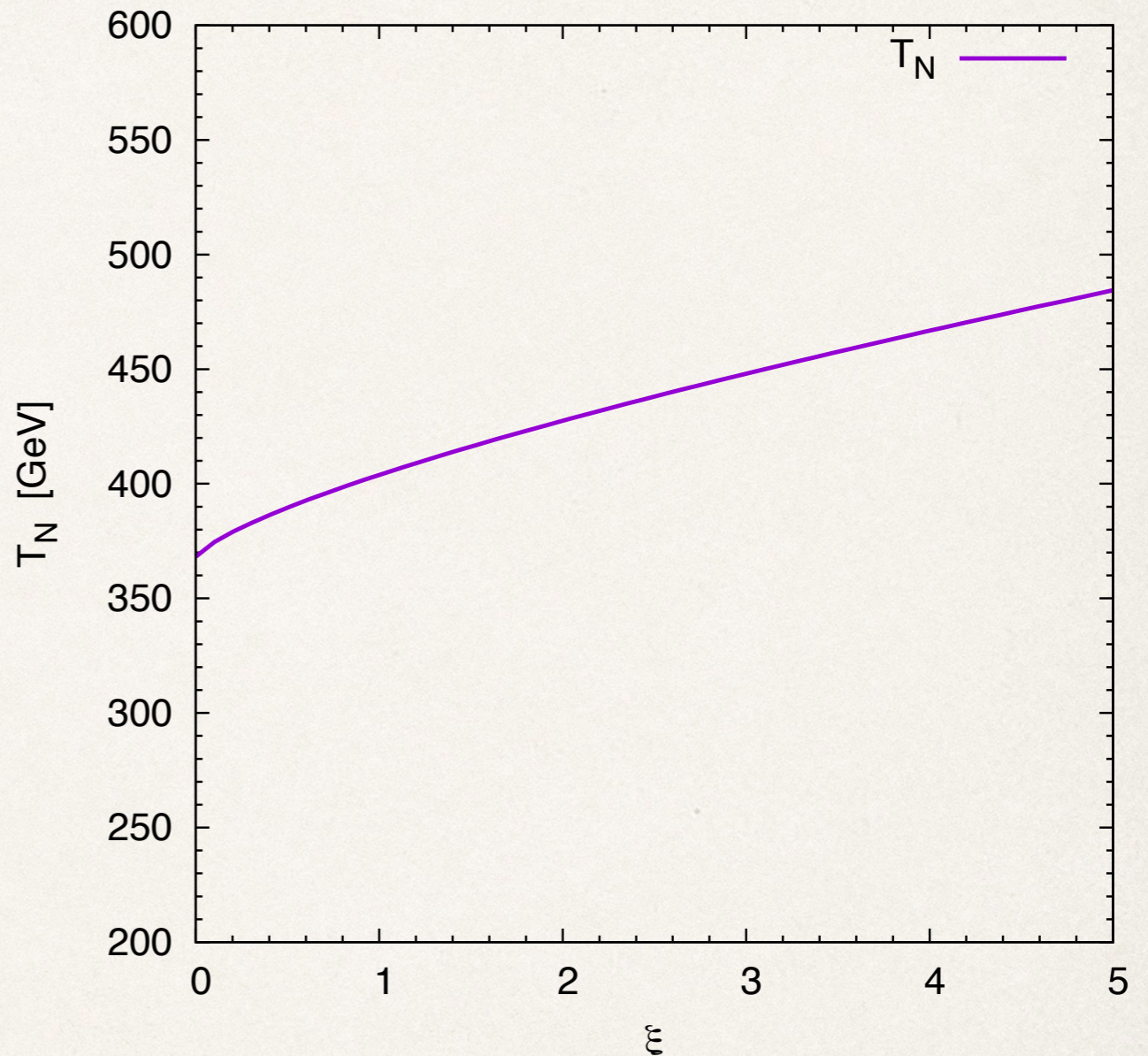|	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
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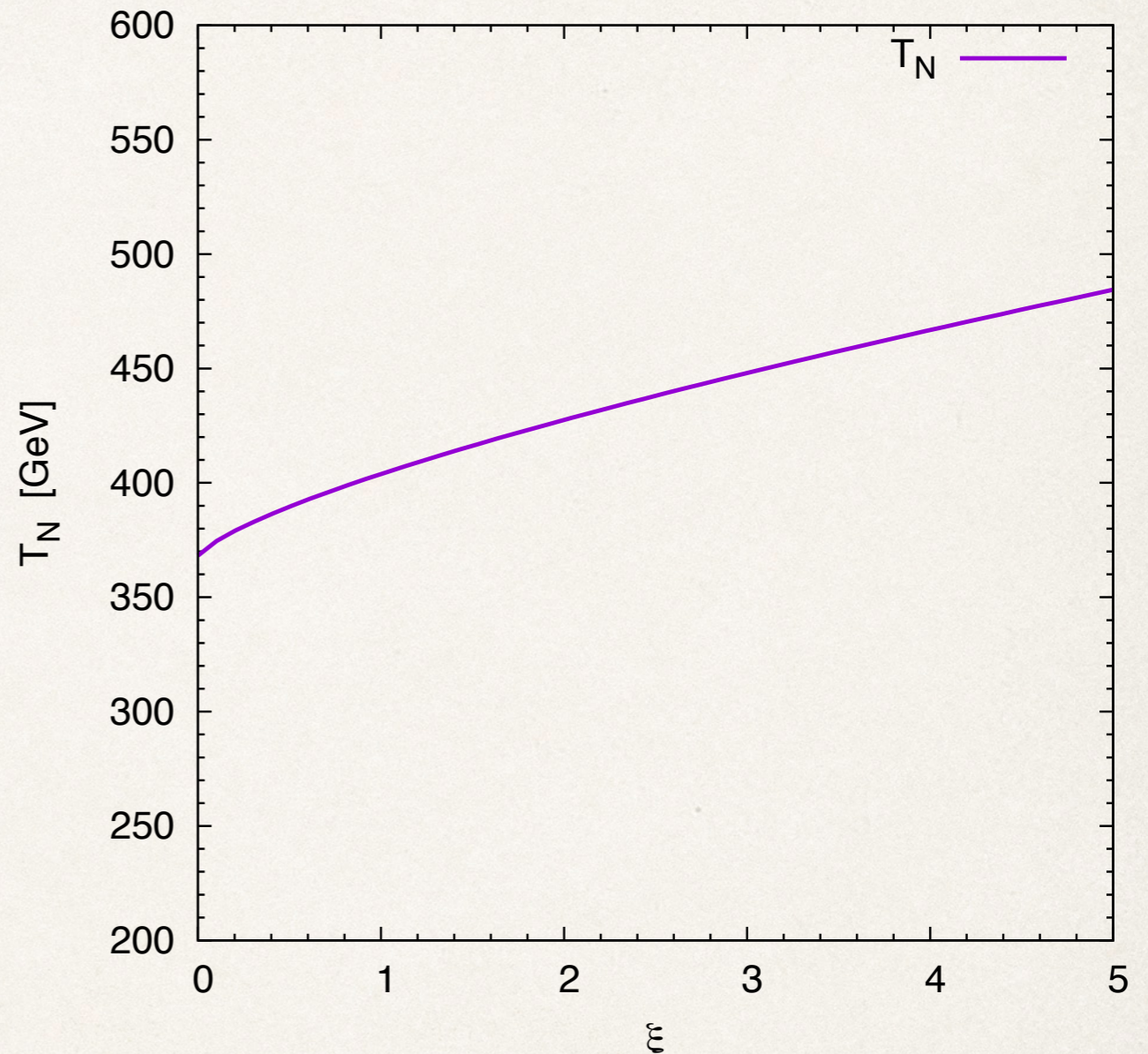|	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
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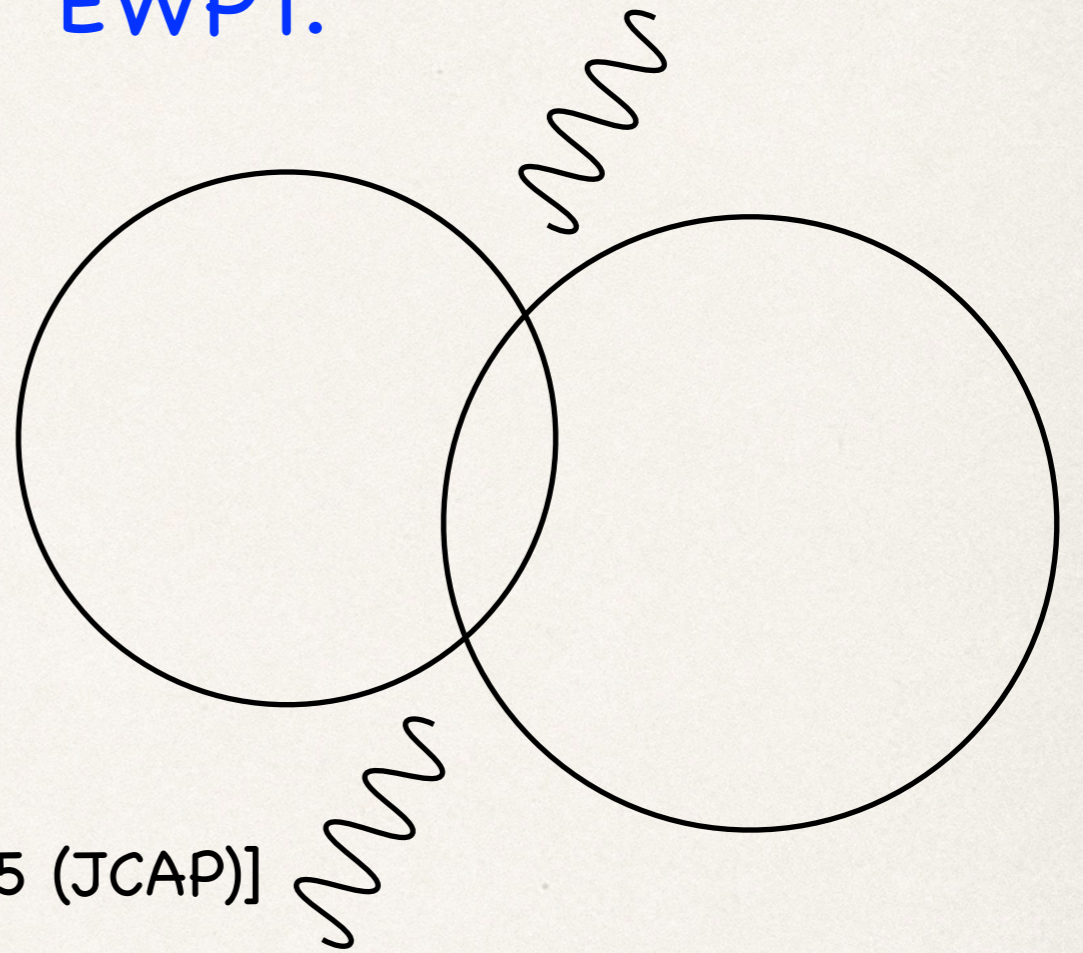**c.f.** $\dfrac{v_S(T_C)}{T_C} = \dfrac{4.851}{1.321} = 3.67$

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Gravitational Waves from 1st-order EWPT

GWs are induced by the 1st-order EWPT.

Sources of GW

(1) Bubble collisions,
(2) Sound waves,
(3) Turbulence

See, e.g.,
Refs. [C.Caprini et al, 1512.06239(JCAP), 1910.13125 (JCAP)]

2 important parameters: [Grojean, Servant, hep-ph/0607107(PRD)]

latent heat (α) , duration of PT (β)

$$\alpha \equiv \frac{\epsilon(T_*)}{\rho_{\text{rad}}(T_*)} \ \text{ and } \ \beta \equiv H_* T_* \frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\bigg|_{T=T_*}, \quad \epsilon(T) = \Delta V_{\text{eff}} - T\frac{\partial \Delta V_{\text{eff}}}{\partial T} \ \text{ and } \ \rho_{\text{rad}}(T) = \frac{\pi^2}{30}g_*(T)T^4,$$

Gravitational Waves from 1ˢᵗ-order EWPT

$$\Omega_{\mathrm{GW}}h^2 = \Omega_{\mathrm{col}}h^2 + \Omega_{\mathrm{sw}}h^2 + \Omega_{\mathrm{turb}}h^2$$
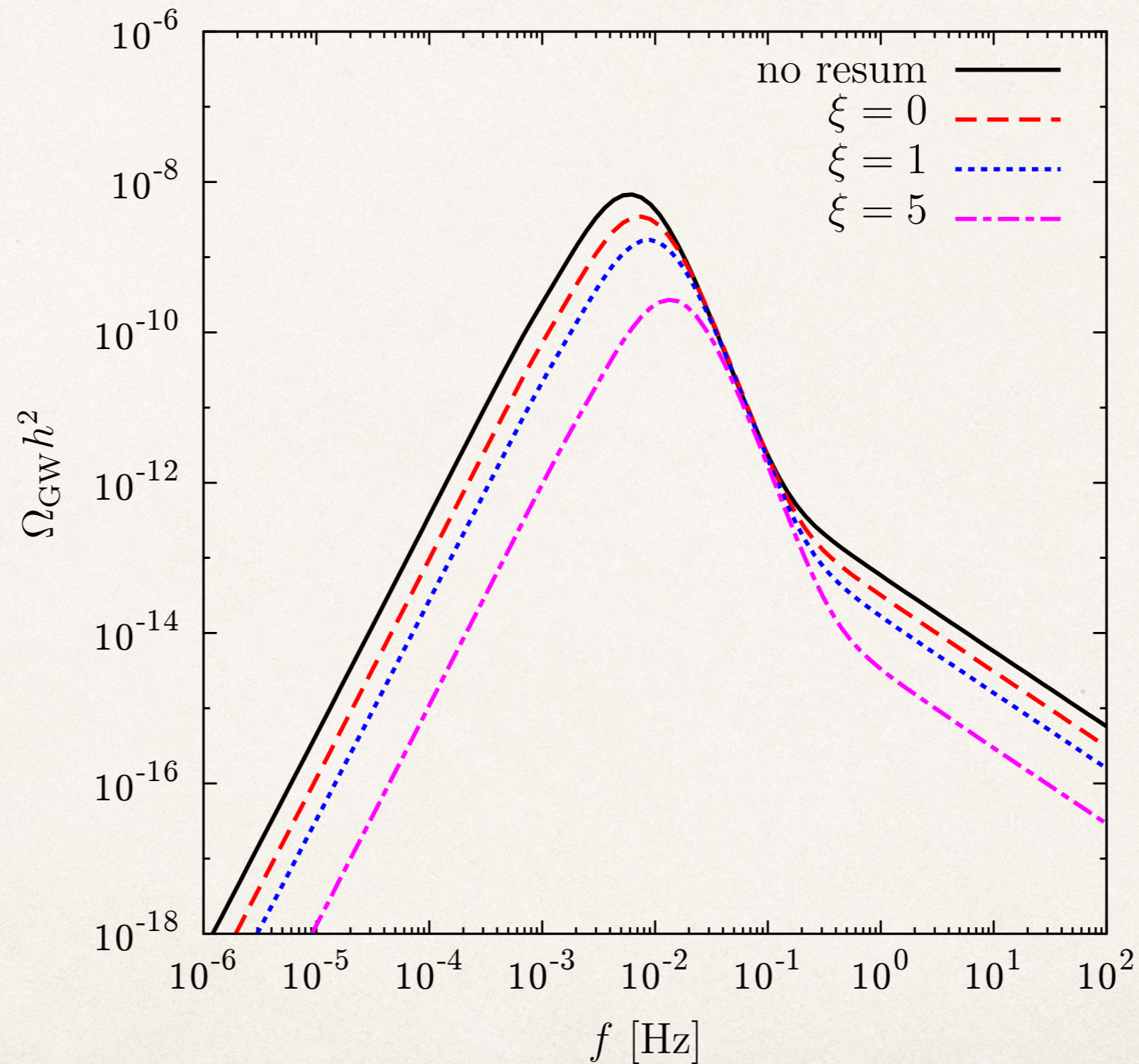
Dominant source is sound waves:

$$\Omega_{\mathrm{sw}}h^2(f) = 2.65 \times 10^{-6} \tilde{\beta}^{-1} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} v_w \left(\frac{f}{f_{\mathrm{sw}}}\right)^3 \left(\frac{7}{4 + 3(f/f_{\mathrm{sw}})^2}\right)^{7/2},$$

$$f_{\mathrm{sw}} = 1.9 \times 10^{-2} \,\mathrm{mHz}\, \frac{\tilde{\beta}}{v_w} \left(\frac{T_*}{100 \,\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}, \qquad \tilde{\beta} = \frac{\beta}{H_*}$$

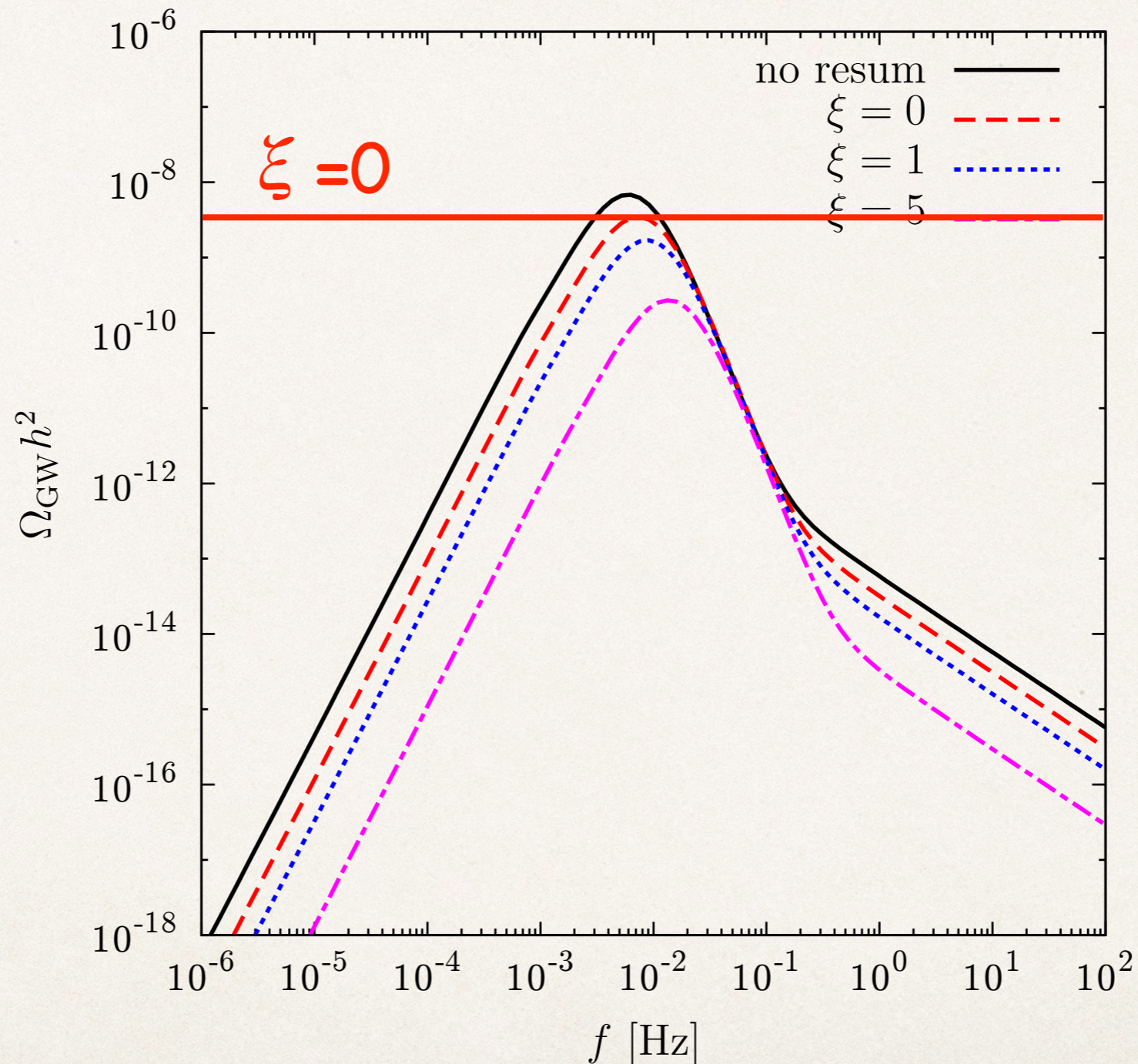$$\kappa_v \simeq \alpha/(0.73 + 0.083\sqrt{\alpha} + \alpha) \text{ for } v_w \simeq 1.$$

- α and β depend on ξ.
- How much ξ dependence can affect GW?

Impact of ξ on gravitational wave

ξ dependence of V_{eff} propagates to GW spectrum significantly!

Impact of ξ on gravitational wave

ξ dependence of V_eff propagates to GW spectrum significantly!

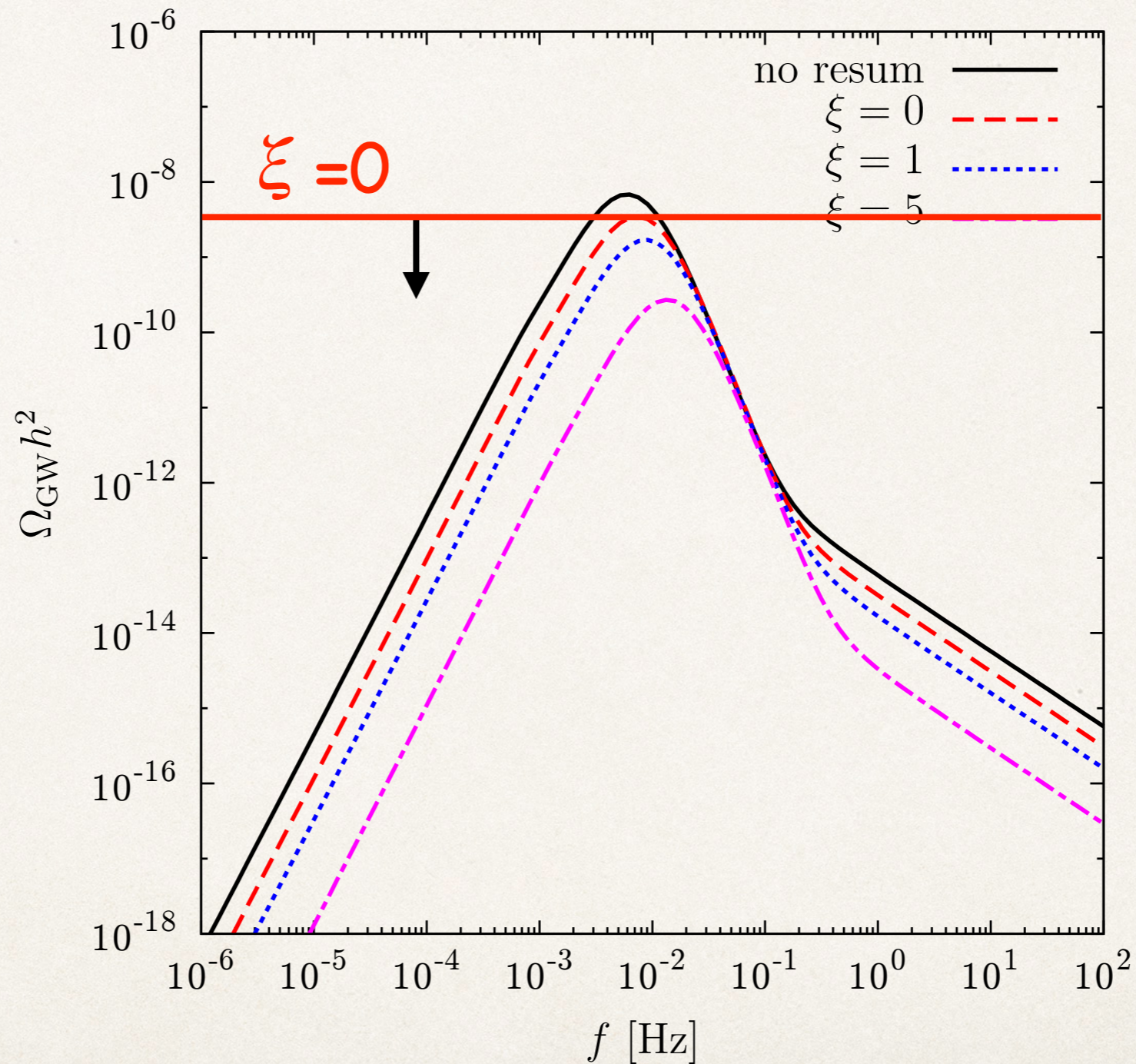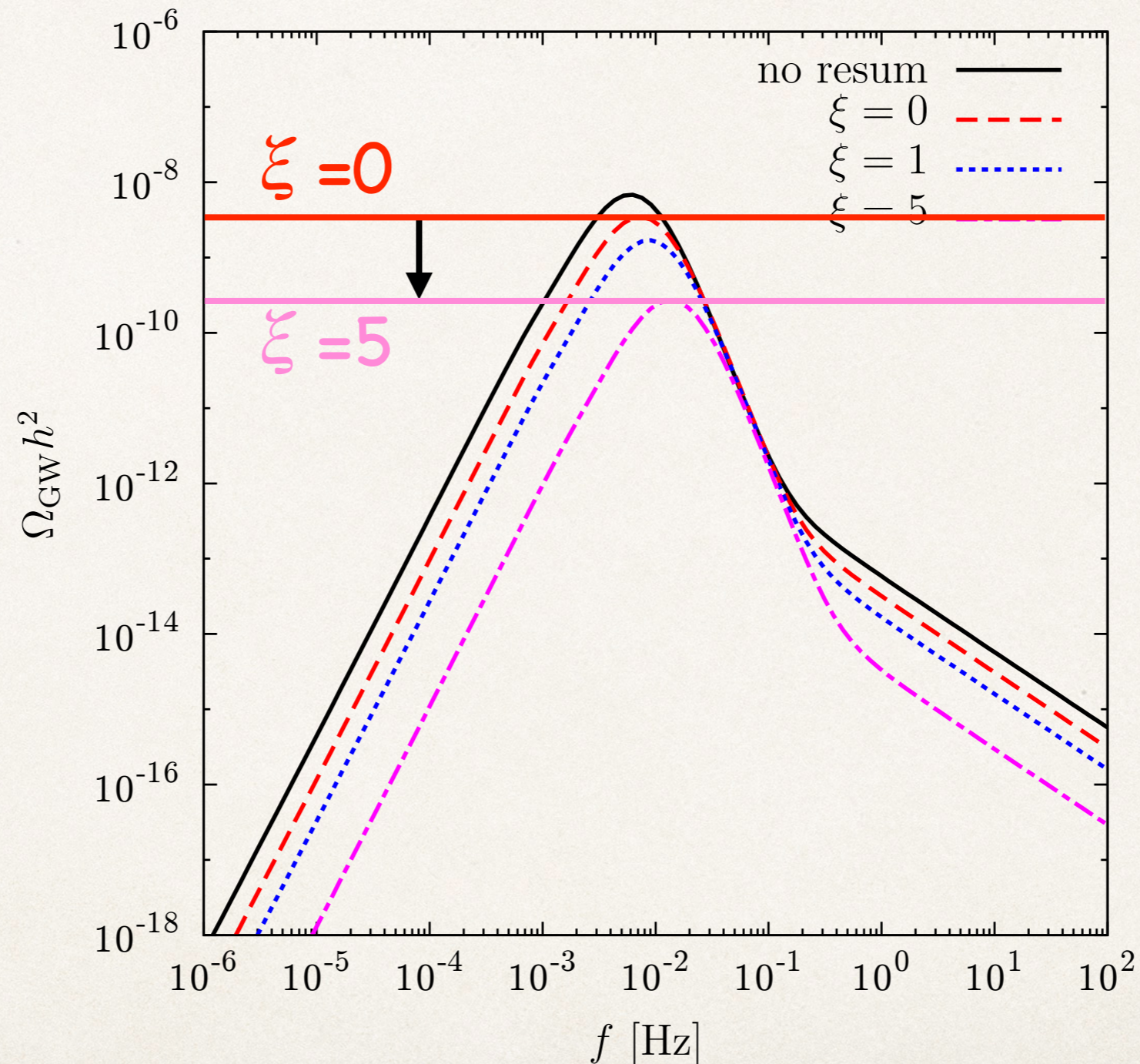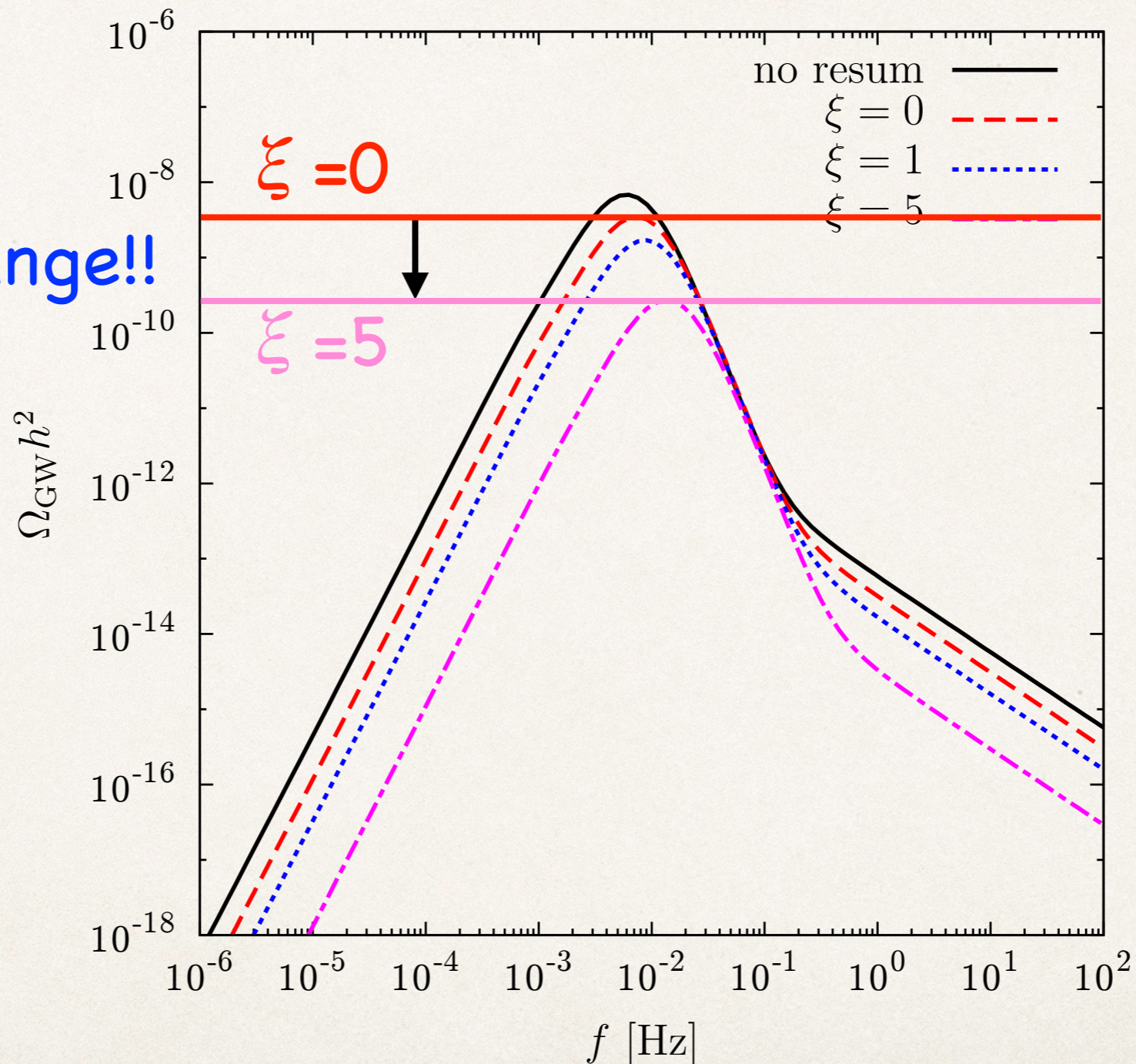# Impact of ξ on gravitational wave

ξ dependence of V$_{eff}$ propagates to GW spectrum significantly!

Impact of ξ on gravitational wave

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Impact of ξ on gravitational wave

~ 1 order change!!

ξ dependence of V$_{eff}$ propagates to GW spectrum significantly!

Summary

- We have evaluated the gauge fixing parameter (ξ) dependence on GW from the 1st-order phase transitions.

- Effective potential is ξ dependent.

- Such ξ dependence can propagate to nucleation temperature and eventually gravitational waves.
    Ω_{GW} can change $O(1)$ in magnitude varying ξ =0–5.

- Gauge-inv. method with consistent thermal resummation is necessary to get reliable results.

Backup

Thermal resummation

[Many refs: see, e.g., Parwani (92), Buchmüller et al (93), Chiku, Hatsuda (98), etc.]

Perturbative expansion gets worse at high T.

$\cdots$ #n sub-bubbles

$\sim g^2 T^2$

$\sim \dfrac{g^4 T^3}{m}\left(\dfrac{g^2 T^2}{m^2}\right)^{n-1}$

Dominant thermal terms are added and subtracted in the Lagrangian:

$$\mathcal{L}_B = \mathcal{L}_R + \mathcal{L}_{\mathrm{CT}} \to \left[\mathcal{L}_R + \Delta m_S^2 |S|^2 + \frac{1}{2}\Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial)Z'^\nu + \frac{1}{2}\Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial)Z'^\nu\right]$$

$$+ \left[\mathcal{L}_{\mathrm{CT}} - \Delta m_S^2 |S|^2 - \frac{1}{2}\Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial)Z'^\nu - \frac{1}{2}\Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial)Z'^\nu\right]$$

where
$$T_{00} = T_{0i} = T_{i0} = 0\,, \quad T_{ij} = g_{ij} - \frac{k_i k_j}{-\boldsymbol{k}^2}\,,$$

$$L_{\mu\nu} = P_{\mu\nu} - T_{\mu\nu}\,, \quad P_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\,,$$

[N.B.] Resummed Lagrangian preserves the gauge invariance.

Thermal resummation

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g^2 $g^2 T^2$ m $g^2 T^2$

Dominant thermal terms are added and subtracted in the Lagrangian: **new unperturbed part**

$$\mathcal{L}_B = \mathcal{L}_R + \mathcal{L}_{\mathrm{CT}} \to \left[\mathcal{L}_R + \Delta m_S^2 |S|^2 + \frac{1}{2}\Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial) Z'^\nu + \frac{1}{2}\Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial) Z'^\nu \right.$$

$$\left. + \left[\mathcal{L}_{\mathrm{CT}} - \Delta m_S^2 |S|^2 - \frac{1}{2}\Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial) Z'^\nu - \frac{1}{2}\Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial) Z'^\nu \right] \right.$$

where
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Thermal resummation

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Perturbative expansion gets worse at high T.

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Dominant thermal terms are added and subtracted in the Lagrangian:

new unperturbed part

$$\mathcal{L}_B = \mathcal{L}_R + \mathcal{L}_{\mathrm{CT}} \to \left[\mathcal{L}_R + \Delta m_S^2 |S|^2 + \frac{1}{2}\Delta m_L^2 Z'^{\mu} L_{\mu\nu}(i\partial) Z'^{\nu} + \frac{1}{2}\Delta m_T^2 Z'^{\mu} T_{\mu\nu}(i\partial) Z'^{\nu}\right]$$

$$+ \left[\mathcal{L}_{\mathrm{CT}} - \Delta m_S^2 |S|^2 - \frac{1}{2}\Delta m_L^2 Z'^{\mu} L_{\mu\nu}(i\partial) Z'^{\nu} - \frac{1}{2}\Delta m_T^2 Z'^{\mu} T_{\mu\nu}(i\partial) Z'^{\nu}\right]$$
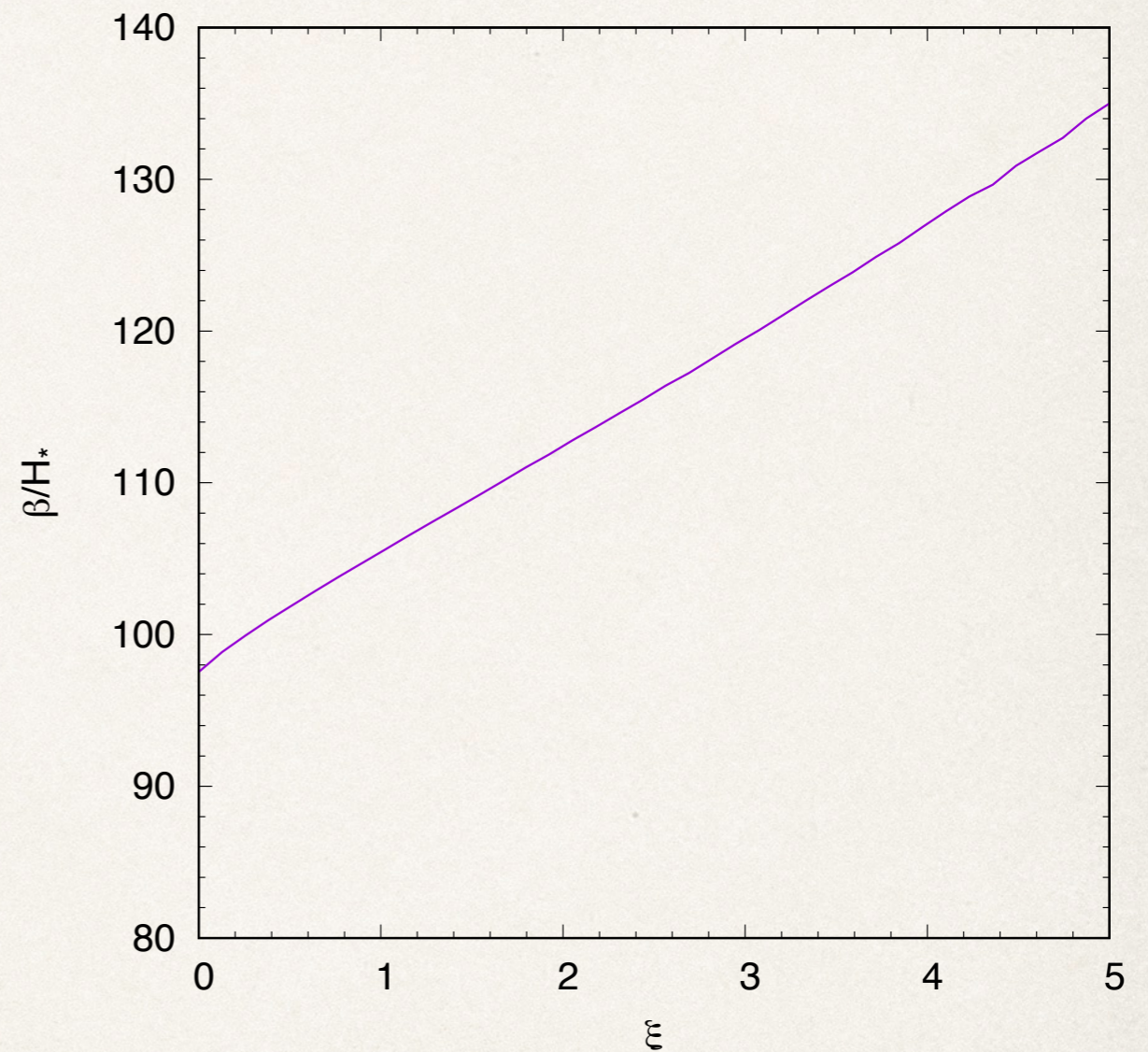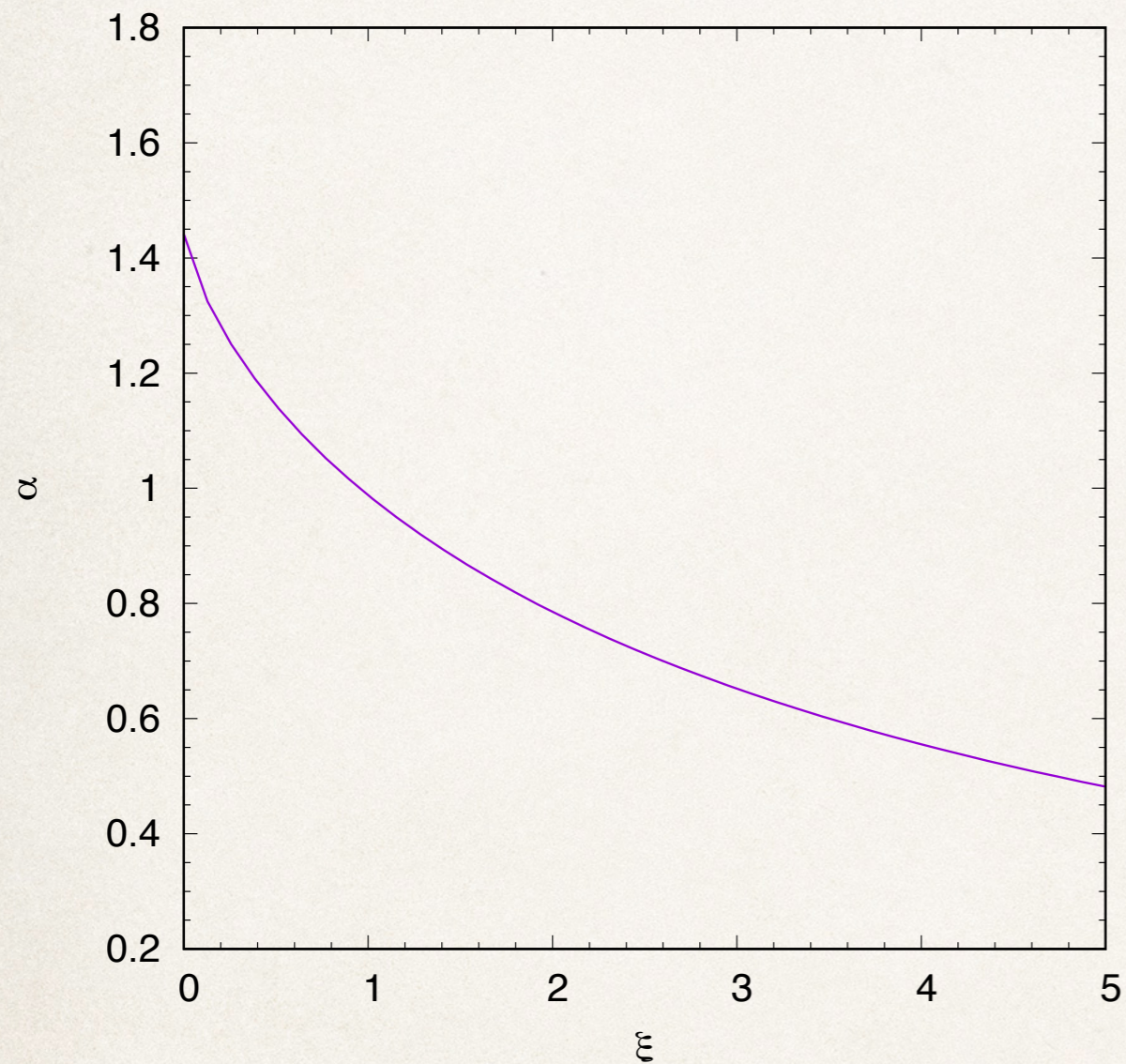
where
$$T_{00} = T_{0i} = T_{i0} = 0 , \quad T_{ij} = g_{ij} - \frac{k_i k_j}{-\boldsymbol{k}^2} , \qquad \text{new counterterm}$$

$$L_{\mu\nu} = P_{\mu\nu} - T_{\mu\nu} , \quad P_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} ,$$

[N.B.] Resummed Lagrangian preserves the gauge invariance.

Impacts of ξ on α on β

	no resum	$\xi = 0$	$\xi = 1$	$\xi = 5$
α	2.27	1.44	0.99	0.48
$\tilde{\beta}$	89.4	97.5	105.4	135.0

- α and β have significant dependences of ξ .

Where is Ton Duc Thang Univ.?

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Airport (SGN)

40min-1h by taxi

Ton Duc Thang U

I belong to Theoretical Particle Physics and Cosmology Research Group (TPPC) in Advanced Institute of Materials Science (AIMaS).