भारतीय प्रौद्योगिकी संस्थान गुवाहाटी
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Global Model Selection in b→$s\mu\mu$ decays:

Based on P.R.D 101 (2020) 5, 055025 & arXiv:2004.14687. In collaboration with Soumitra Nandi, Ipsita Ray, Srimoy Bhattacharya, and Sunando Patra

Why b→$s\mu\mu$?

- Transitions suppressed in the SM.
- Potentially sensitive to NP.
- Type of NP: Tree level ? loop level ?
- **Question**: NP or Hadronic uncertainties?
- Angular, Diff BF: Hadronic uncertainties.
- Ratios ($R_{K^{(*)}}$): Theoretically clean.
- "Optimized" observables (?)

Theory and operator basis

$$\mathcal{H}_{eff} = -\frac{4\,G_F}{\sqrt{2}}\left(\lambda_t \mathcal{H}_{eff}^{(t)} + \lambda_u \mathcal{H}_{eff}^{(u)}\right)$$

$$\begin{aligned}\mathcal{H}_{eff}^{(t)} &= C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{6} C_i \mathcal{O}_i + \\ &\sum_{i=7,8,9,10,P,S} \left(C_i \mathcal{O}_i + C_i' \mathcal{O}_i'\right), \\ \mathcal{H}_{eff}^{(u)} &= C_1(\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2(\mathcal{O}_2^c - \mathcal{O}_2^u)\,.\end{aligned}$$

$$\begin{aligned}\mathcal{O}_7 &= \frac{e}{g^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, & \mathcal{O}_7' &= \frac{e}{g^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}, \\ \mathcal{O}_9 &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \mu), & \mathcal{O}_9' &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu \mu), \\ \mathcal{O}_{10} &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \gamma_5 \mu), & \mathcal{O}_{10}' &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu \gamma_5 \mu), \\ \mathcal{O}_S &= \frac{e^2}{16\pi^2} m_b (\bar{s} P_R b)(\bar{\mu}\mu), & \mathcal{O}_S' &= \frac{e^2}{16\pi^2} m_b (\bar{s} P_L b)(\bar{\mu}\mu), \\ \mathcal{O}_P &= \frac{e^2}{16\pi^2} m_b (\bar{s} P_R b)(\bar{\mu}\gamma_5 \mu), & \mathcal{O}_P' &= \frac{e^2}{16\pi^2} m_b (\bar{s} P_L b)(\bar{\mu}\gamma_5 \mu)\,.\end{aligned}$$

b→$s\mu\mu$: Data

- **Angular observables (moments, Likelihood):** $B^0 \to K^{*0}\mu^+\mu^-$: JHEP 02, 104 (2016), P.R.L 125 (2020) 1(LHCb), 011802 and JHEP 10, 047 (2018) (ATLAS). (P_4', P_5'): P.R.L. 118, 111801 (2017)(Belle) , $A_I(B \to K^*)$: JHEP 06, 133 (2014)(LHCb). $B^+ \to K^+\mu^+\mu^-$ (A_{FB}, F_H): PRD 98, 112011, (2018)(CMS). $B_s \to \phi\mu^+\mu^-$: JHEP 09, 179 (2015)(LHCb).

- **Differential branching fractions:** $B^0 \to K^{*0}\mu^+\mu^-$: JHEP 11, 047 (2016), $B^+ \to K^{*+}\mu^+\mu^-$:JHEP 06, 133 (2014)(LHCb). $B^+ \to K^+\mu^+\mu^-$ and $B^0 \to K^0\mu^+\mu^-$: JHEP 06, 133 (2014)(LHCb), 1908.01848(Belle). $A_I(B \to K)$: JHEP 06, 133 (2014)(LHCb), 1908.01848(Belle).

- $\boldsymbol{R_{K^*}}$**, low and central bin:** Old result: JHEP 08, 055 (2017)(LHCb). Recent measurements: arXiv:1904.02440 (Belle). $\boldsymbol{R_K}$: Old result: PRL 113 (2014), 151601(LHCb). Updated result: PRL 122 (2019) 19, 191801(LHCb). Also arXiv: 1908.01848(Belle).

- **BR($\boldsymbol{B_s \to \mu^+\mu^-}$):** HFLAV (Average of CMS, ATLAS and LHCb).

- **Radiative modes:** BR($B \to X_s\gamma$): Eur. Phys. J. C77, 201 (2017), BR($B_s \to \phi\gamma$): Nucl. Phys. B867, 1 (2013)(LHCb), BR($B^{+,0} \to K^*\gamma$): 1412.7515(HFLAV)

The different datasets

Data displayed in the previous slide combined into five datasets;

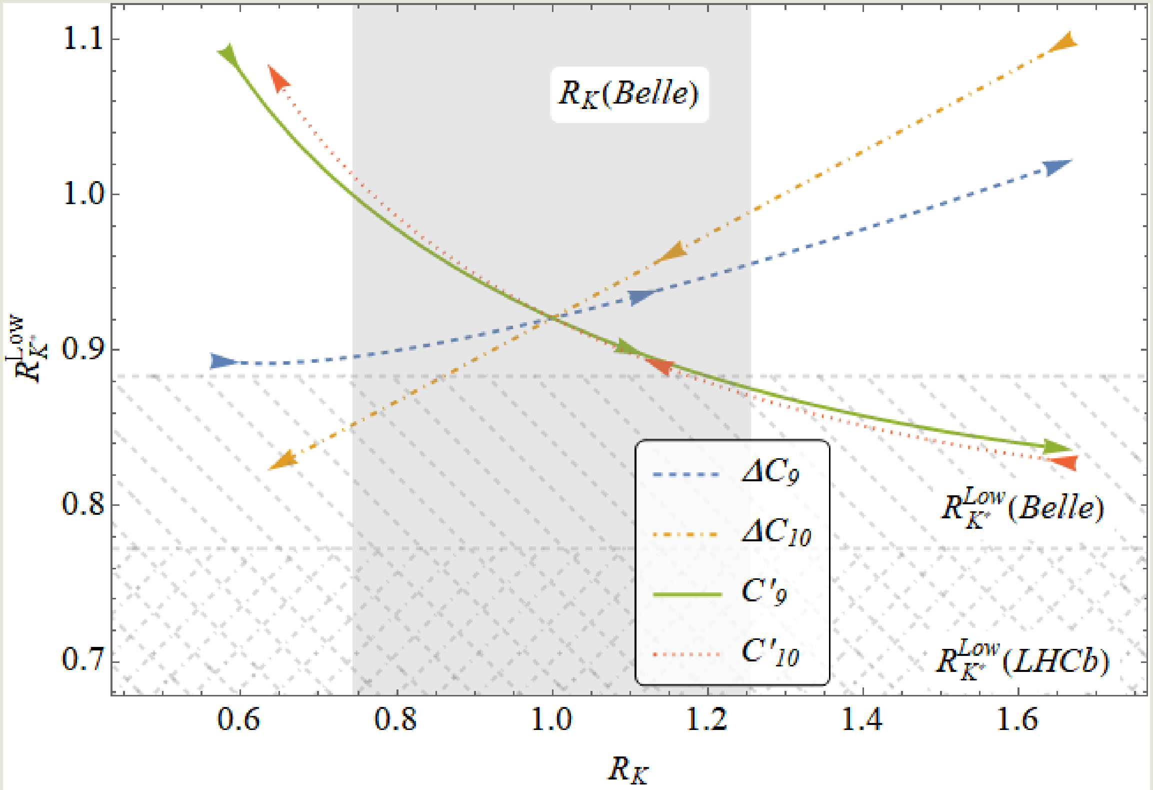
1. **Likelihood New dataset (214)**
2. **Moment New dataset (258)**
3. **Likelihood Old dataset (211)**
4. **Moment Old dataset (255)**
5. **Likelihood 2020 dataset (224) (complex case)**

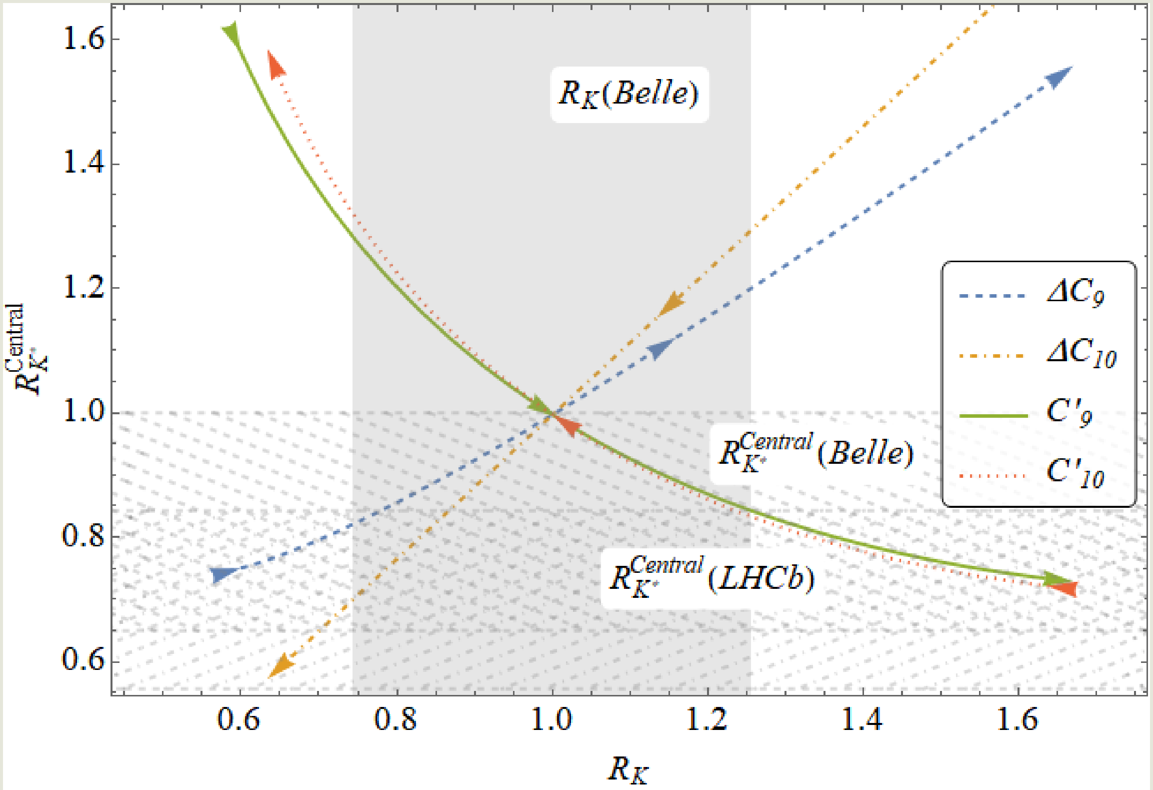
(**Old**: Old R_{K^*}(LHCb,2017), Old R_K(LHCb, 2014).

New: Old R_{K^*}(LHCb,2017)+New R_{K^*}(Belle,2019),

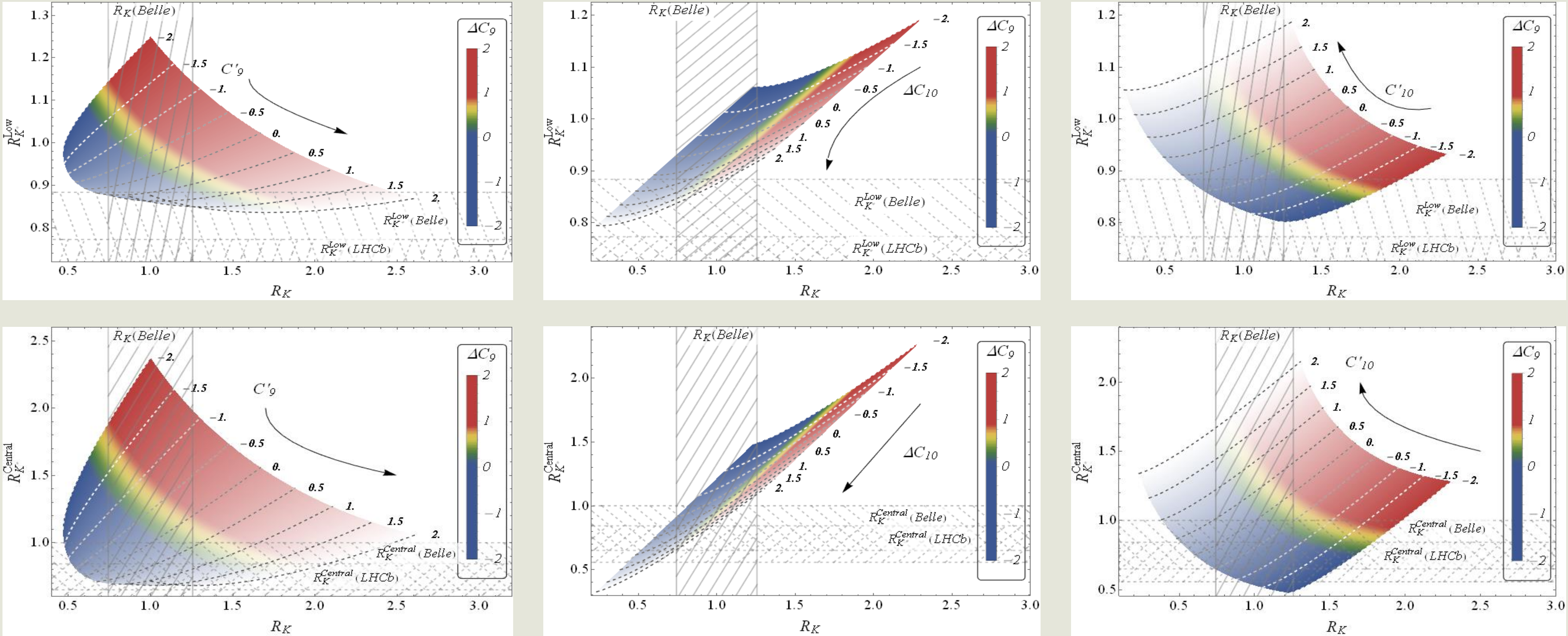
Old R_K(LHCb, 2014) $\rightarrow$New R_K(LHCb,Belle,2019).

Lessons from R_K R_{K^*}: 1 operator scenarios

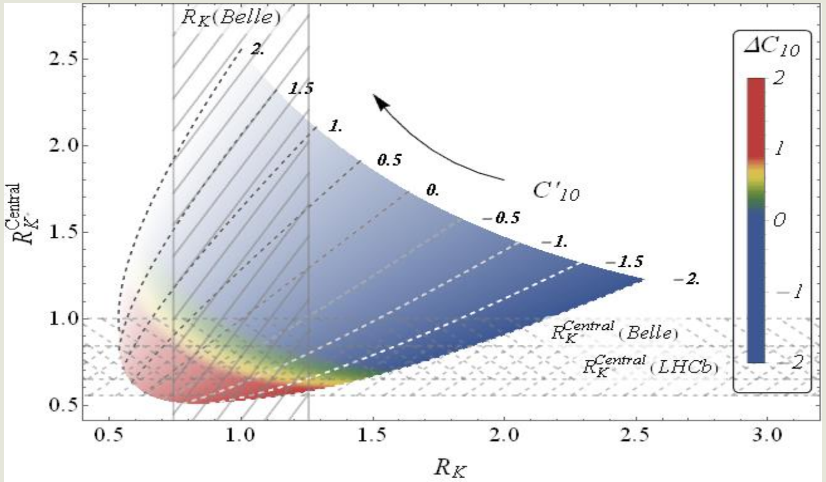


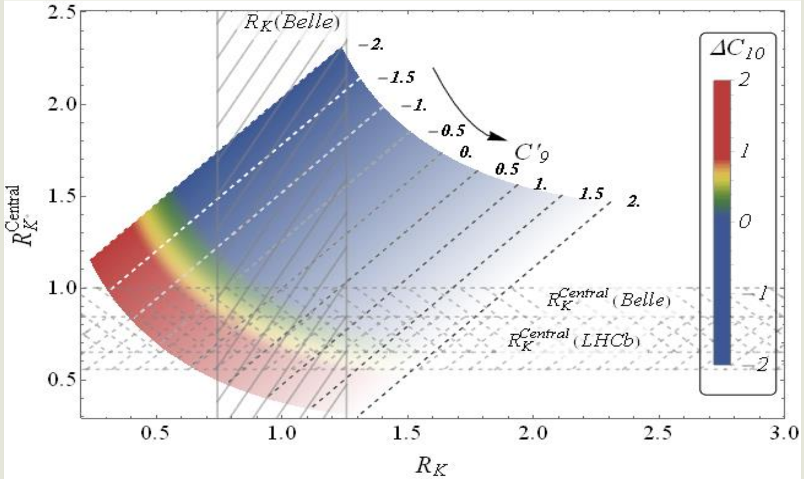


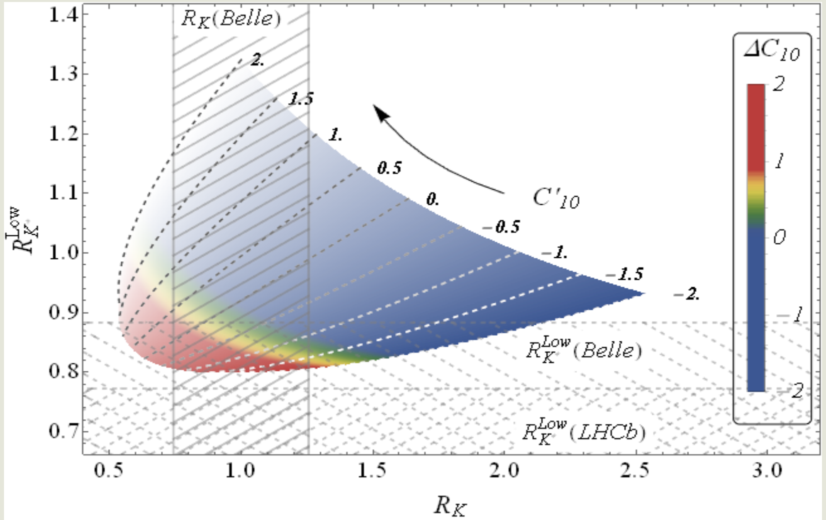
Lessons from R_K R_{K^*}: 2 operator scenarios

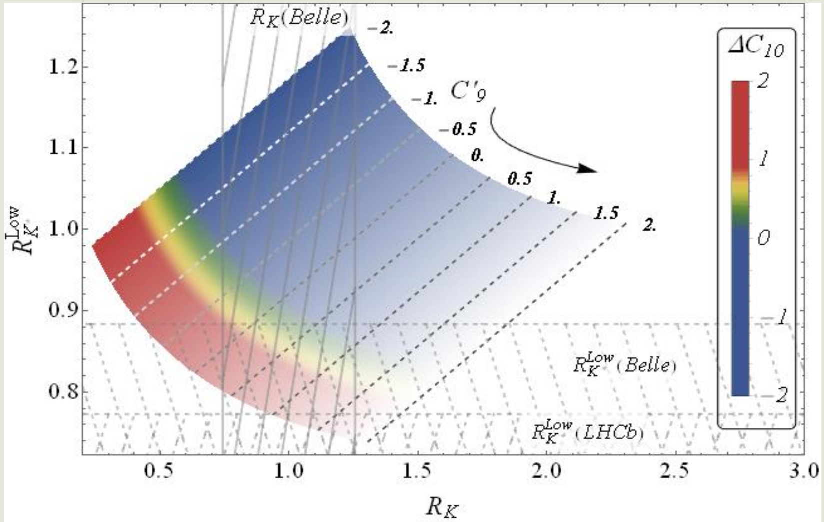


Lessons from R_K R_{K^*}: 2 operator scenarios









Comparison: 1 operator scenarios

Dataset	χ^2/DOF	p-val(%)	Value	χ^2/DOF	p-val(%)	Value
		C_7'			ΔC_9	
Likelihood 2020	317.42/214	5.52×10^{-6}	$Re(C_7') \rightarrow$ -0.040(13) $Im(C_7') \rightarrow$ 0.050(71)	234.22/214	16.35	$Re(\Delta C_9) \rightarrow$ -1.16(11) $Im(\Delta C_9) \rightarrow$ 1.39(34)
Likelihood 2016	343.89/253	1.22×10^{-4}	$Re(C_7') \rightarrow$ -0.038(16) $Im(C_7') \rightarrow$ -0.0052(253)	266.21/253	27.20	$Re(\Delta C_9) \rightarrow$ -1.27(14) $Im(\Delta C_9) \rightarrow$ -1.39(39)
Moments 2016	344.63/278	0.39	$Re(C_7') \rightarrow$ -0.033(16) $Im(C_7') \rightarrow$ -0.0059(305)	288.41/278	32.11	$Re(\Delta C_9) \rightarrow$ -1.31(18) $Im(\Delta C_9) \rightarrow$ 1.26(45)
		C_9'			ΔC_{10}	
Likelihood 2020	326.1/214	1.20×10^{-6}	$Re(C_9') \rightarrow$ -0.13(15) $Im(C_9') \rightarrow$ -0.80(49)	313.55/214	1.06×10^{-5}	$Re(\Delta C_{10}) \rightarrow$ 0.66(18) $Im(\Delta C_{10}) \rightarrow$ -1.88(28)
Likelihood 2016	348.25/253	6.55×10^{-5}	$Re(C_9') \rightarrow$ -0.19(15) $Im(C_9') \rightarrow$ -0.25(78)	342.12/253	1.58×10^{-4}	$Re(\Delta C_{10}) \rightarrow$ 0.38(14) $Im(\Delta C_{10}) \rightarrow$ 0.23(65)
Moments 2016	348.29/278	0.26	$Re(C_9') \rightarrow$ -0.099(148) $Im(C_9') \rightarrow$ -0.093(438)	334.22/278	1.16	$Re(\Delta C_{10}) \rightarrow$ 0.53(15) $Im(\Delta C_{10}) \rightarrow$ -0.15(81)
		C_{10}'			C_S	
Likelihood 2020	315.16/214	8.11×10^{-6}	$Re(C_{10}') \rightarrow$ 0.37(11) $Im(C_{10}') \rightarrow$ -0.11(108)	327.53/214	9.24×10^{-7}	$Re(C_S) \rightarrow$ -0.045(40) $Im(C_S) \rightarrow$ -0.0019(3709)
Likelihood 2016	339.14/253	2.38×10^{-4}	$Re(C_{10}') \rightarrow$ 0.37(11) $Im(C_{10}') \rightarrow$ 0.11(29)	349.64/253	5.34×10^{-5}	$Re(C_S) \rightarrow$ -0.044(54) $Im(C_S) \rightarrow$ 0.0073(2424)
Moments 2016	342.23/278	0.51	$Re(C_{10}') \rightarrow$ 0.31(12) $Im(C_{10}') \rightarrow$ -0.0023(3204)	348.52/278	0.26	$Re(C_S) \rightarrow$ -0.035(173) $Im(C_S) \rightarrow$ 0.022(273)
		C_P			C_S'	
Likelihood 2020	327.52/214	9.26×10^{-7}	$Re(C_P) \rightarrow$ -0.0080(124) $Im(C_P) \rightarrow$ -0.0013(2282)	327.54/214	9.23×10^{-7}	$Re(C_S') \rightarrow$ -0.045(39) $Im(C_S') \rightarrow$ 0.0012(4660)
Likelihood 2016	349.7/253	5.34×10^{-5}	$Re(C_P) \rightarrow$ -0.0069(441) $Im(C_P) \rightarrow$ -0.0095(5930)	349.69/253	5.3×10^{-5}	$Re(C_S') \rightarrow$ 0.033(302) $Im(C_S') \rightarrow$ -0.028(355)
Moments 2016	348.45/278	0.26	$Re(C_P) \rightarrow$ 0.23(66) $Im(C_P) \rightarrow$ -0.089(740)	348.52/278	0.26	$Re(C_S') \rightarrow$ -0.030(253) $Im(C_S') \rightarrow$ -0.029(261)
		C_P'				
Likelihood 2020	327.49/214	9.32×10^{-7}	$Re(C_P') \rightarrow$ 0.0083(122) $Im(C_P') \rightarrow$ 0.00046(17156)			
Likelihood 2016	349.67/253	5.31×10^{-5}	$Re(C_P') \rightarrow$ 0.0076(124) $Im(C_P') \rightarrow$ 0.00097(23752)			
Moments 2016	348.53/278	0.26	$Re(C_P') \rightarrow$ 0.0066(128) $Im(C_P') \rightarrow$ 0.0015(2876)			

TABLE I: Comparitive study of the results obtained from the analysis of different data set. The results are presented only for complex the Wilson coefficients (WC).

The curious case of ΔC_9

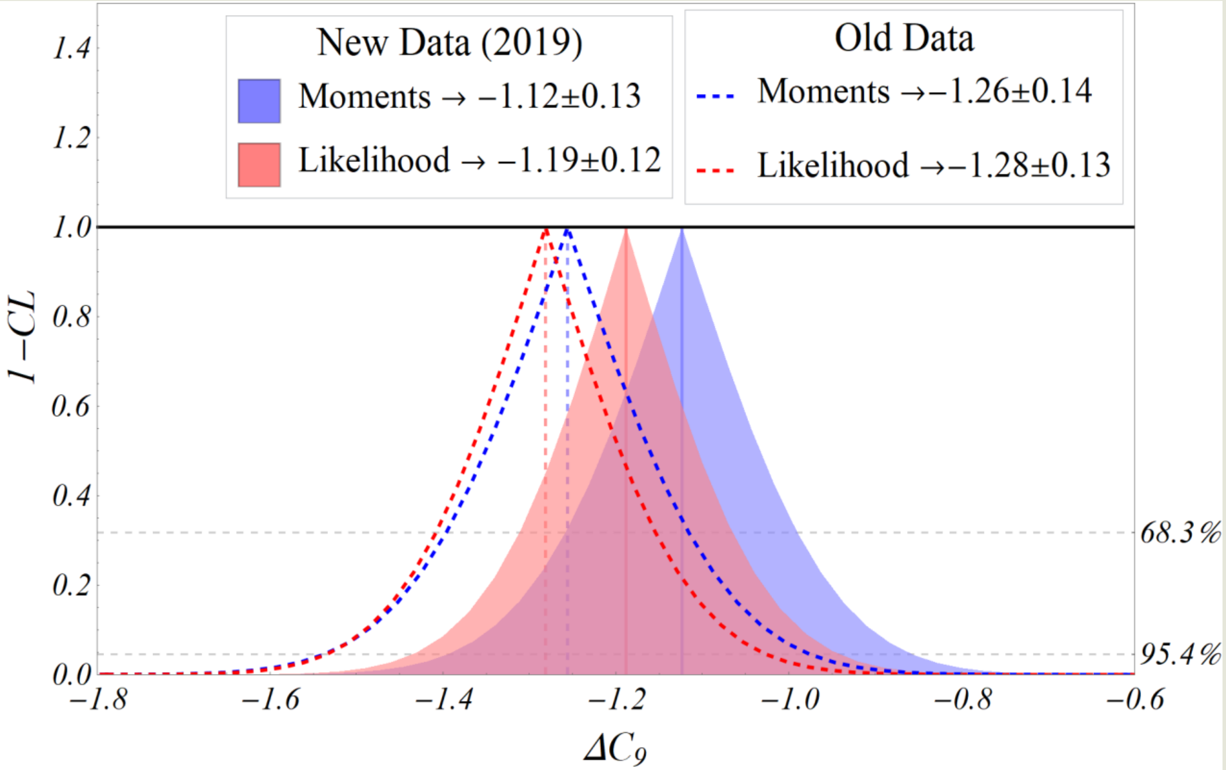
- Both for purely real and complex cases, the only one operator scenario that yields an acceptable fit for the data is ΔC_9.
- Real: 1 parameter, Complex: 2 parameters.
- General notion: Real part consistent with -1.
- Imaginary part small (~0)

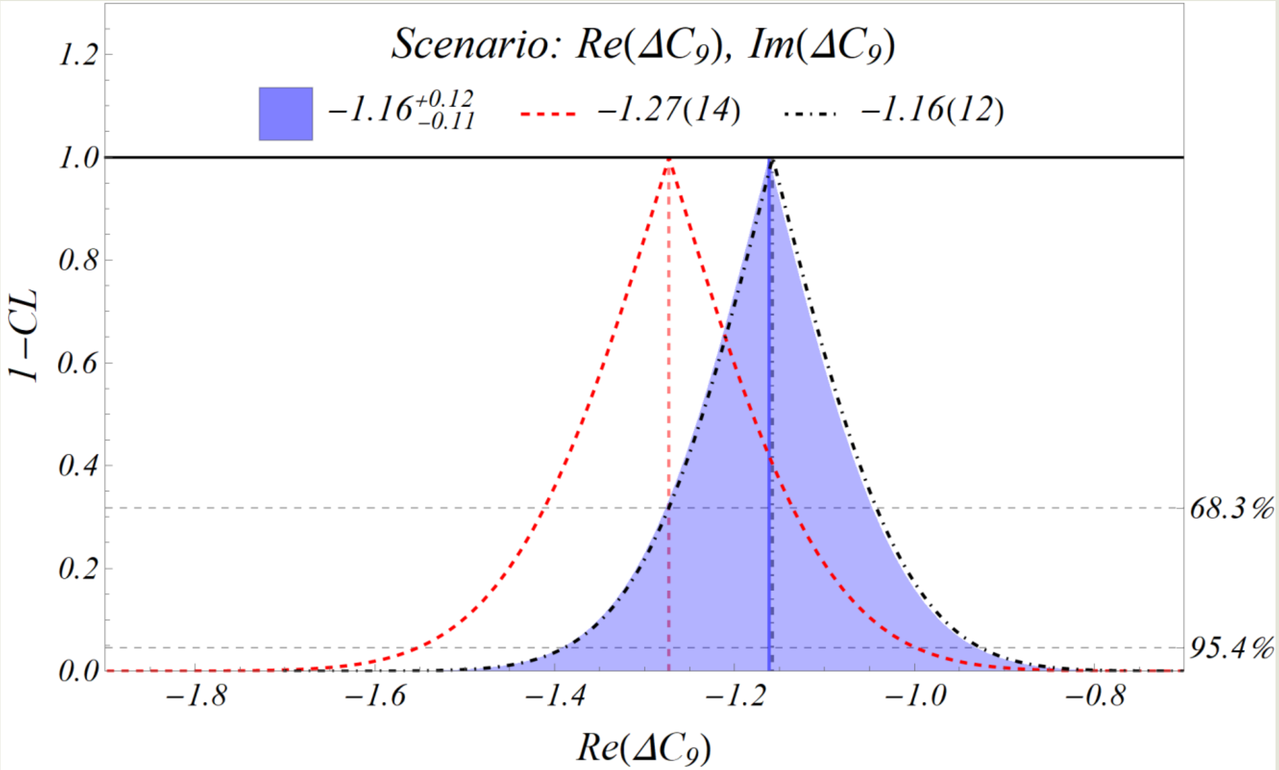
But…

With CP-asymmetric observables in $B_s \to \phi\mu\mu$		
χ^2_{Min}/DOF	p-value (%)	parameter spaces
238.93/215	12.59	$Re(\Delta C_9) \to -1.10(11)$
234.22/214	16.35	$Re(\Delta C_9) \to -1.16^{+0.12}_{-0.11}$ $Im(\Delta C_9) \to 1.39^{+0.31}_{-0.39}$

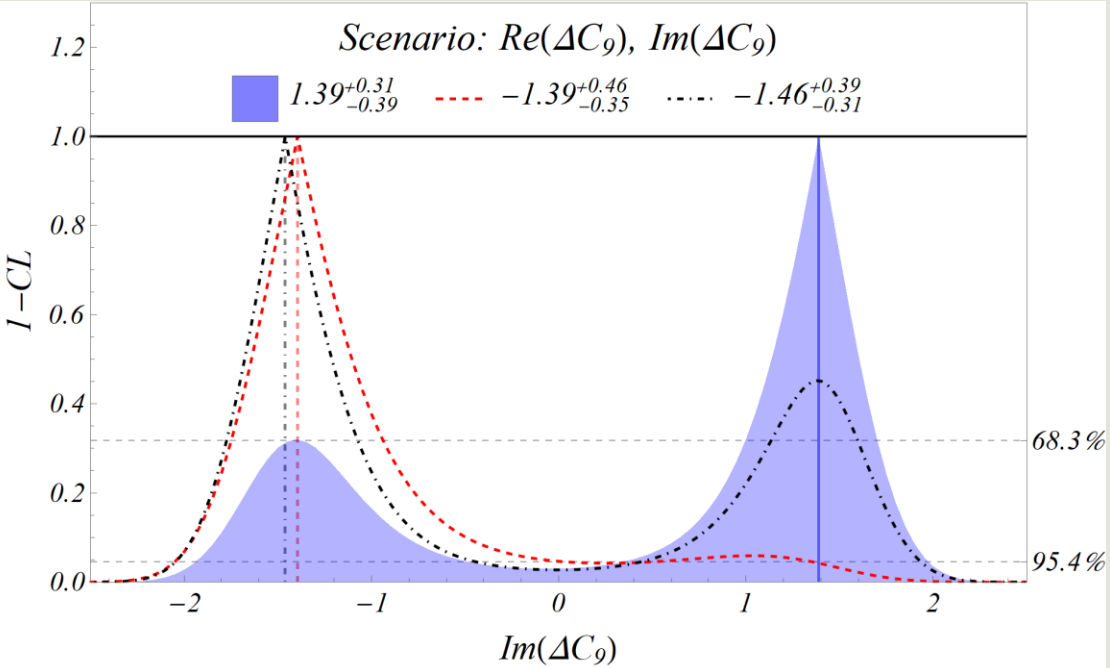
Without CP-asymmetric observables in $B_s \to \phi\mu\mu$		
χ^2_{Min}/DOF	p-value (%)	parameter spaces
230.80/203	8.79	$Re(\Delta C_9) \to -1.10(11)$
225.93/202	11.9	$Re(\Delta C_9) \to -1.16(12)$ $Im(\Delta C_9) \to -1.46^{+0.39}_{-0.31}$

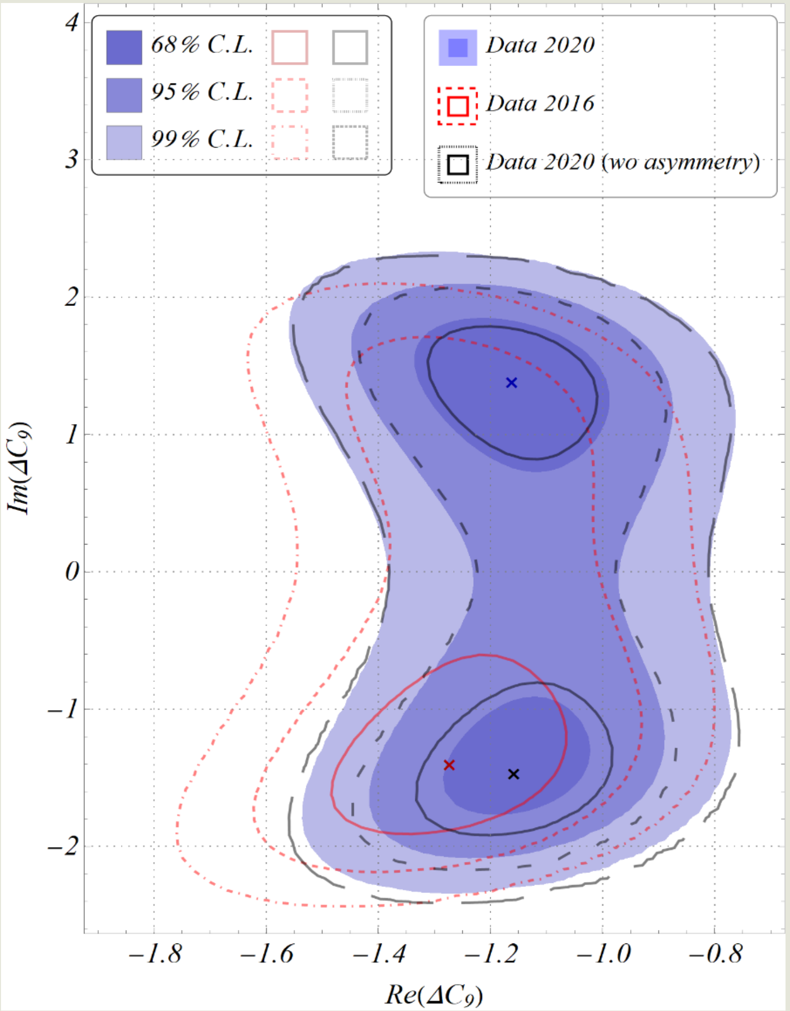
The curious case of ΔC_9





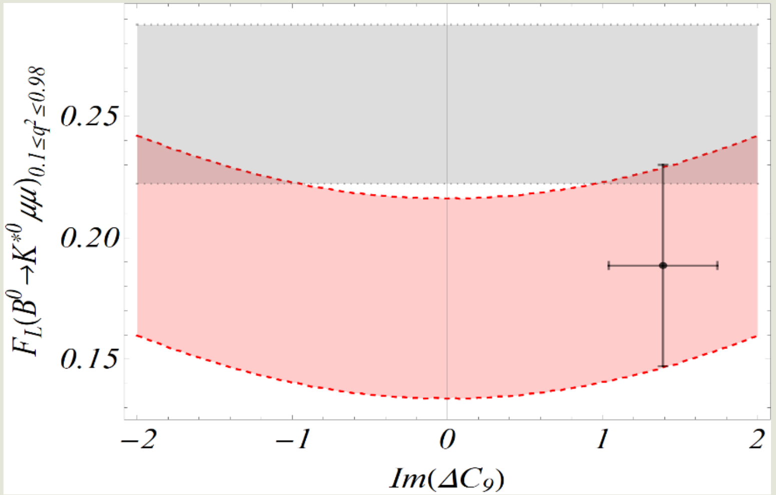
The curious case of ΔC_9

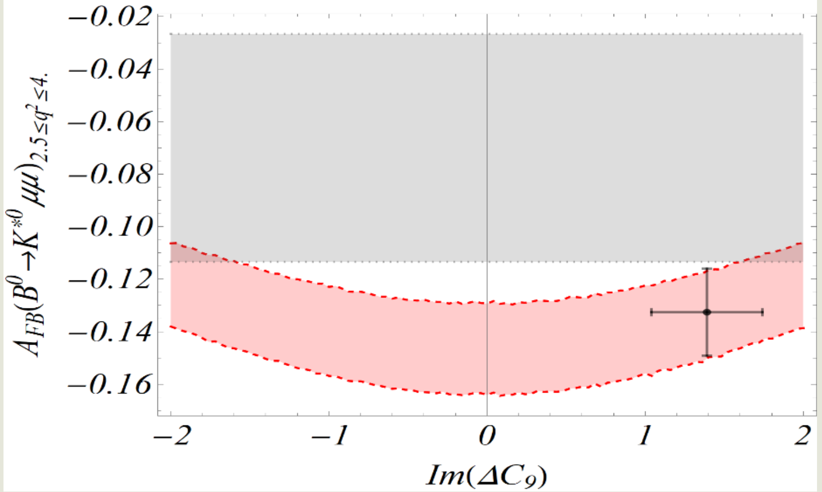


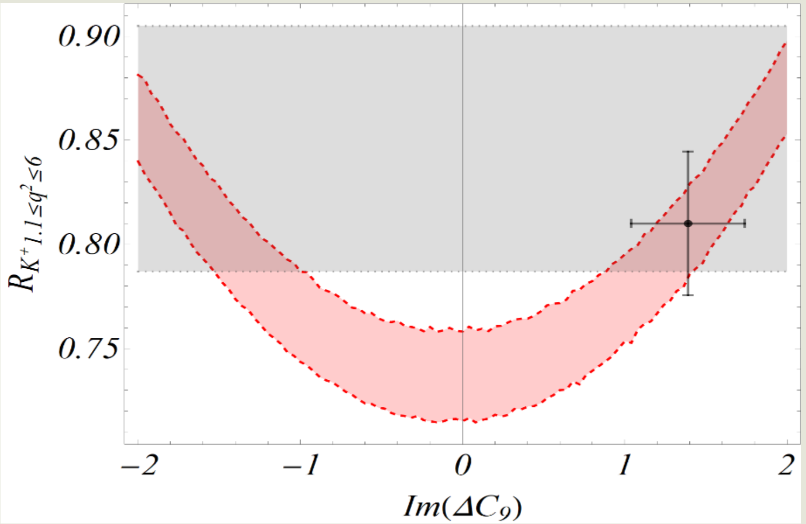


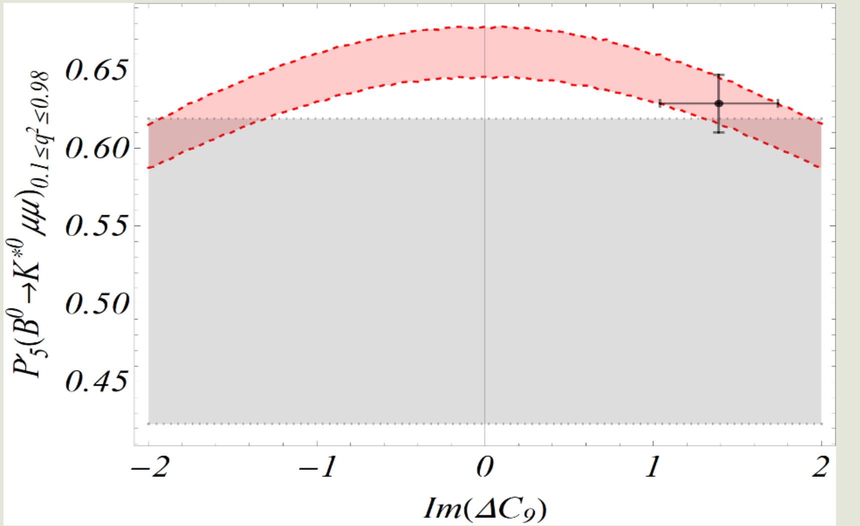
What is the reason for this imaginary part being inconsistent with 0 at 1σ?

The curious case of ΔC_9





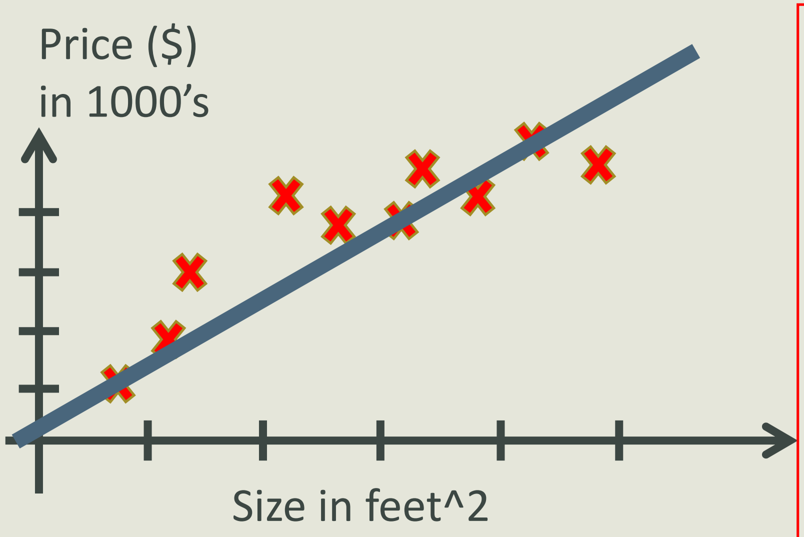




b→$s\mu\mu$: How?

- "…a clever choice of observables could drastically reduce the sensitivity to hadronic inputs and enhance the sensitivity to New Physics" (JHEP 12 (2014) 125, (SDG, JM etal)).
- "…even optimized observables are affected by sizable uncertainties, since hadronic contributions generated by current-current operators with charm are difficult to estimate, especially for $q^2 \sim 4m_c^2 \simeq 6.8$ GeV^2." (JHEP 06 (2016) 116, (LS, AP etal)).
- Plethora of data. **Data-Driven Analysis:** Let data decide the model. Works in principle for both NP and Hadronic (form factor) models. (For form factor "model selection": look into JHEP 06, 165 (2020) ,(SJ,SN,SP) & *JHEP* 08 (2020) 08, 006, (SI, RW).

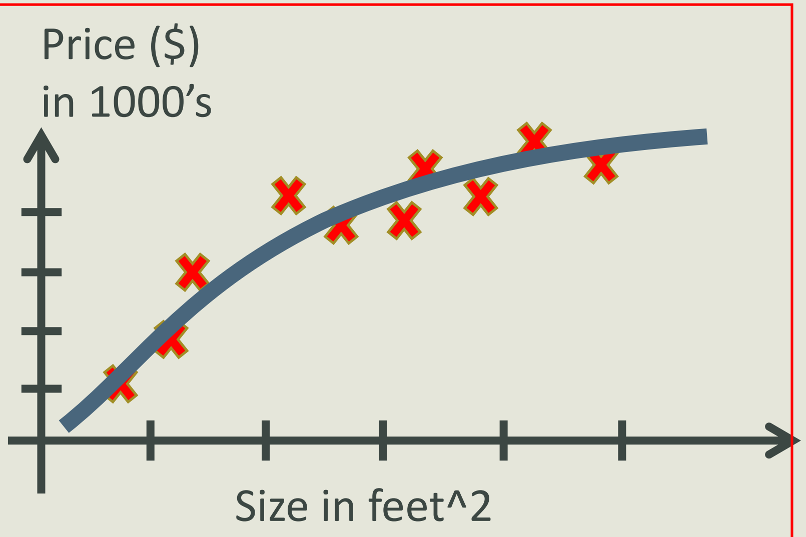
The problem of model selection



$$h_\theta(x) = \theta_0 + \theta_1 x$$

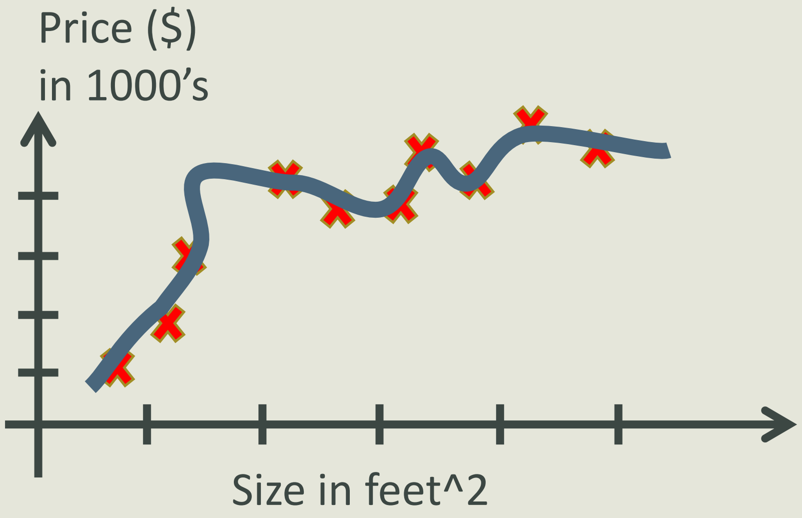
Underfitting

High bias



$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2$$

Just right



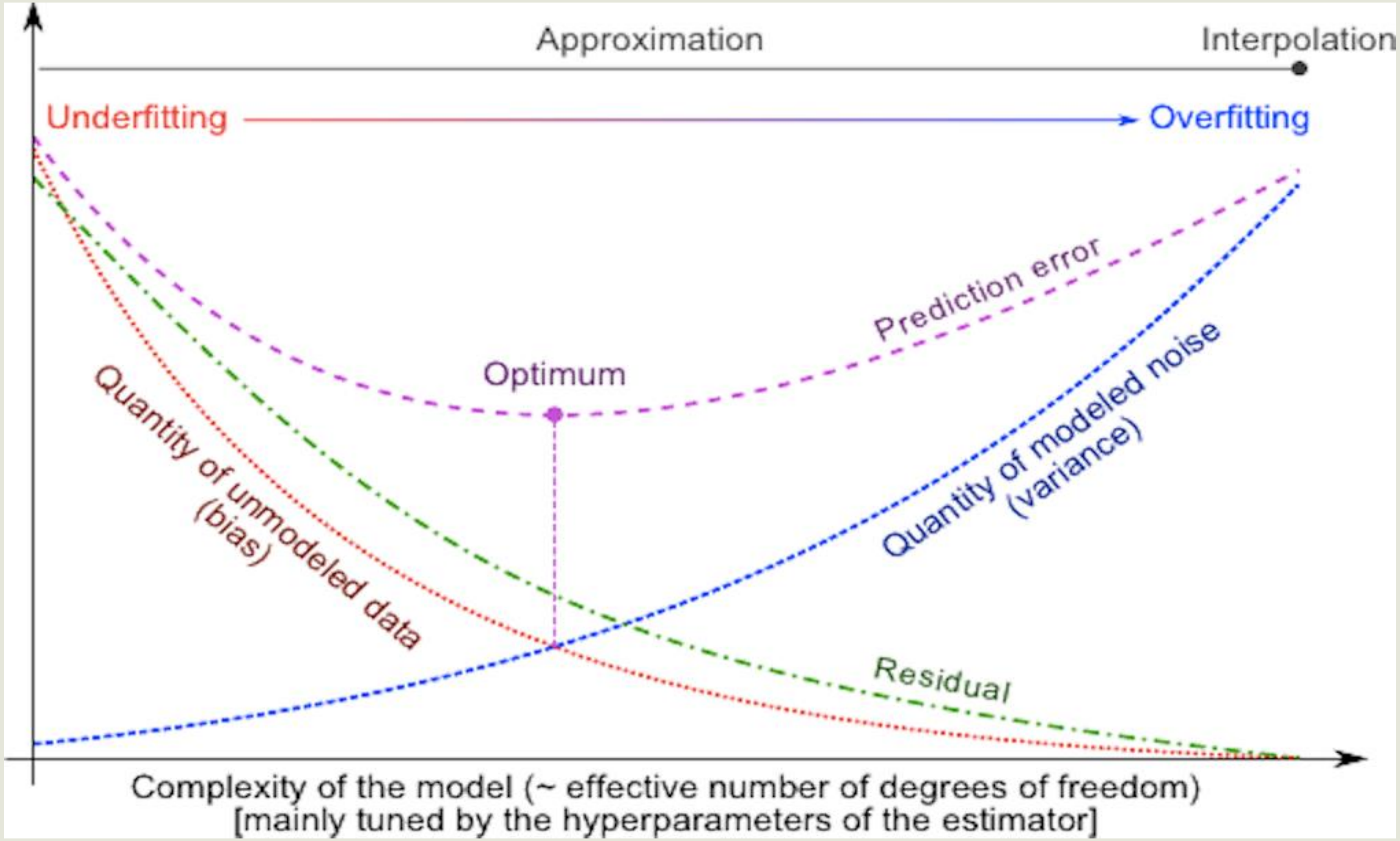
$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4 + \cdots$$

Overfitting

High variance

Slide credit: Andrew Ng

The problem of Model selection



The problem of Model selection

- **The general problem of model selection:** A model can almost never represent a certain observation exactly. Simplistic model → too few parameters → unrealistically simple assumptions → high bias → poor prediction. Large no. of parameters → fit noise as well → miss important trends.
- **Motivation:** "All models are wrong but some are useful." (George E.P. Box, Journal of the American Statistical Association, Volume 71, 1976-issue 356).

Model Selection: How?

Cross-Validation *does this…*

- One of the most powerful, reliable but computationally expensive methods.
- Most straightforward (most expensive) →**L**eave **O**ne **O**ut **C**ross **V**alidation:
    1. N data points and a set of models.
    2. Remove n-th data point from sample.
    3. Fit model to remaining N-1 points.
    4. Compute Loss for the n-th left out data point.
    5. Repeat.
    6. Calculate Mean Loss (MSE) for the model from step 4.
- Draws on predictive error ⇒ can detect under and overfitting.
- Drawback: **Small data-set → becomes unstable**

Model Selection: How?

AIC_c previously used in Eur.Phys.J. C79 (2019) no.1, 21

- $AIC = \chi^2_{min} + 2K$ **Akaike (1974) & Takeuchi (1976)**

$n \gg K$

- $AIC_c = \chi^2_{min} + 2K + \frac{2K(K+1)}{n-K-1}$ **Sugiura (1978)**

$\frac{n}{K} < 40$, n = sample size and K = no. of parameters

- Model Selection: $\Delta AIC_c^i = AIC_c^i - AIC_c^{min}, w_i^{\Delta AIC_c} = \frac{e^{\left(-\frac{\Delta AIC_c^i}{2}\right)}}{\sum_{r=1}^{R} e^{\left(-\frac{\Delta AIC_c^i}{2}\right)}}$

- Drawback: **Depend on MLE estimate and don't account for uncertainty in data. Too Simple models selected.**

So a comparative global b→$s\mu^+\mu^-$?

- 9 C_W's (Wilson Coefficients) →

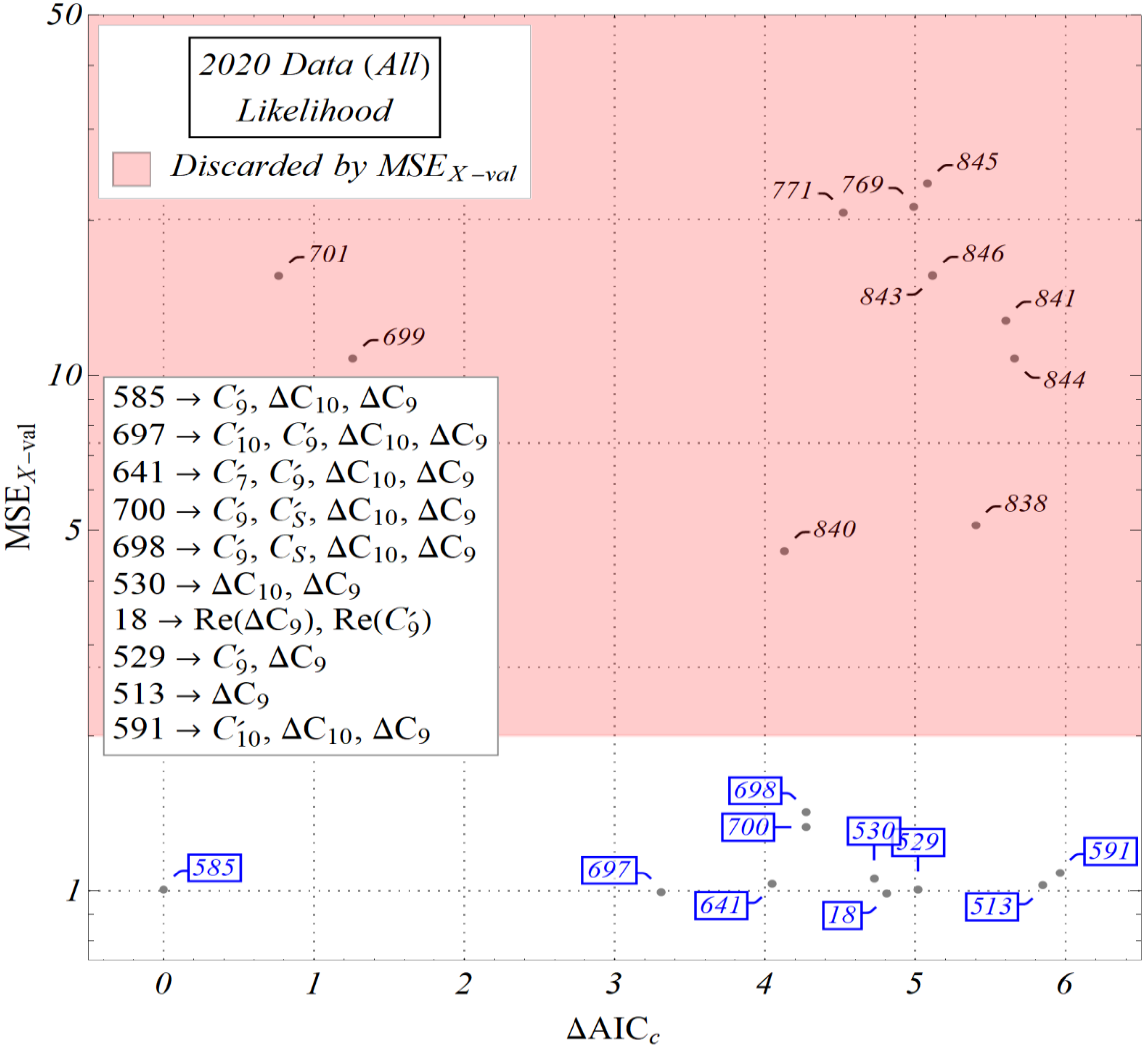
  $$C_7', \Delta C_9, C_9', \Delta C_{10}, C_{10}', C_S, C_S', C_P, C_P'$$

- All real → 511 combinations, 9 parameters .
- Complex →1022 (Real+Real&Imaginary), 18 parameters.
- $>$ 200 obs. ⇒ Cross Validation can be done
- 2 types of Angular obs. →
  1. Unbinned Max. Likelihood (total >210)
  2. Principal Moments (total >250)
- Neither $w_i^{\Delta AIC_c}$ nor MSE tells the whole truth.
- Why not use both $w_i^{\Delta AIC_c}$ and MSE?

Methodology

- **Define models:** Real → 511 models, Complex →1022
- **Optimize:** Frequentist χ^2 optimization on all models. 5 types of fit corresponding to the five datasets .
- All optimizations done with a Mathematica package OpTex (S.Patra).
- **Post Process:** Find outliers (“Pulls”) & “influential data” (“Cook’s distances”). Normality check for pull distribution.
- **Fisher Matrix:** Gaussian parameter-profile likelihoods → HESSE errors. Profile Likelihood curve: 1σ CL of profile likelihoods of the said parameter.

Results: Model selection (Likelihood 2020)



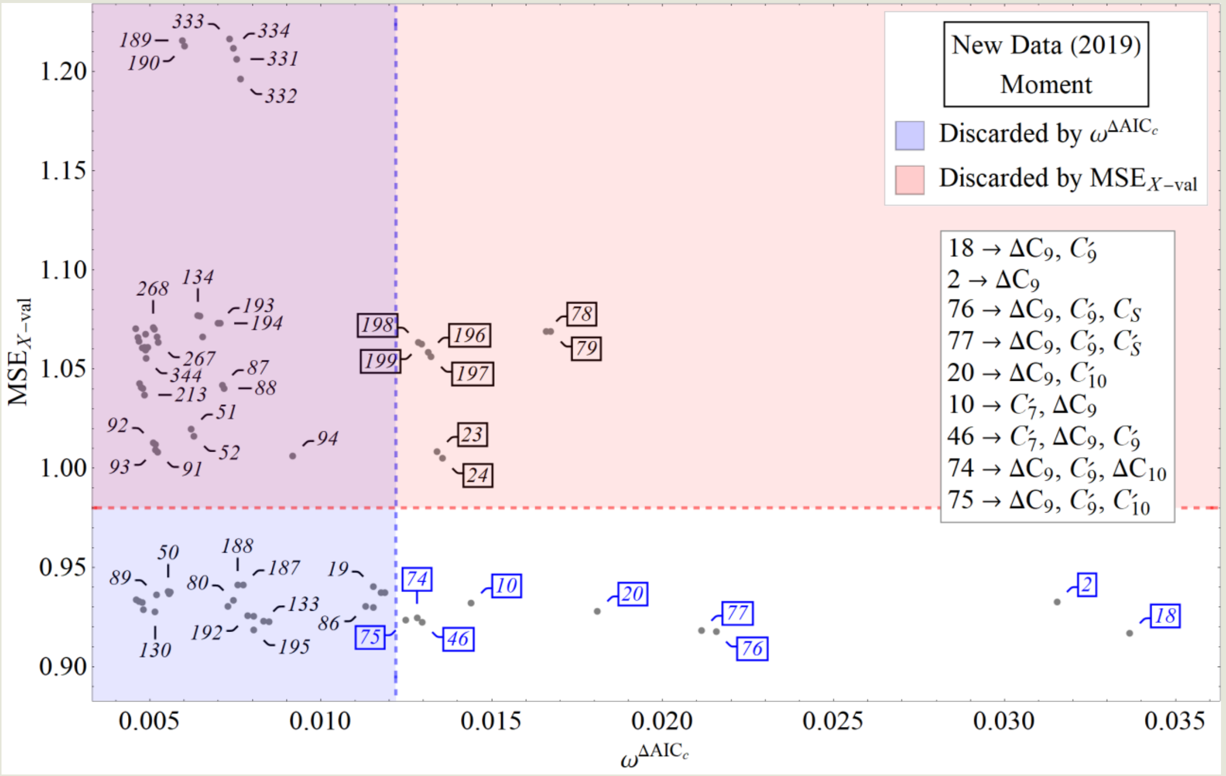
Model	ΔAICc	MSE_{X-Val}	χ^2_{Min}/DOF	p-value(%)	Result
585.	0.	1.05	220.04/210	30.34	Re(ΔC_9) → -1.45±0.24, Im(ΔC_9) → 2.23±0.32 Re(C_9') → 0.54±0.23, Im(C_9') → 0.15±0.23, Re(ΔC_{10}) → 0.58±0.21, Im(ΔC_{10}) → -0.51±0.41
697.	3.33	1.02	219.07/208	28.56	Re(ΔC_9) → -1.5±0.24, Im(ΔC_9) → 2.11±0.45, Re(C_9') → 0.53±0 .23, Im(C_9') → 0.26±0 .43, Re(ΔC_{10}) → 0.58±0.21, Im(ΔC_{10}) → -0.5±0.37 Re(C_{10}') → -0.072±0.179, Im(C_{10}') → 0.79±0.83
641.	4.07	1.06	219.81/208	27.39	Re(C_7') → -0.0058±0.0138, Im(C_7') → -0.0075±0.0447, Re(ΔC_9) → -1.44±0.24, Im(ΔC_9) → 2.24±0.33, Re(C_9') → 0.55±0.37, Im(C_9') → 0.17±0.43, Re(ΔC_{10}) → 0.6±0.22, Im(ΔC_{10}) → -0.53±0.41
530.	4.72	1.09	228.97/212	20.18	Re(ΔC_9) → -1.43±0.26, Im(ΔC_9) → 2.14±0.38, Re(ΔC_{10}) → 0.38±0.22, Im(ΔC_{10}) → -0.54±0.5
18.	4.79	1.01	233.18/214	17.55	Re(ΔC_9) → -1.122±0.099, Re(C_9') → 0.46±0.19
529.	5.01	1.03	229.26/212	19.8	Re(ΔC_9) → -1.16±0.11, Im(ΔC_9) → -0.093±0.41, Re(C_9') → 0.39±0.23, Im(C_9') → -1.35±0.36
513.	5.84	1.05	234.22/214	16.35	Re(ΔC_9) → -1.16±0.11, Im(ΔC_9) → 1.39±0.34
591.	5.97	1.12	226.01/210	21.35	Re(ΔC_9) → -1.49±0.42, Im(ΔC_9) → 2.21±0.38, Re(ΔC_{10}) → 0.43±0.25, Im(ΔC_{10}) → -0.45±0.9, Re(C_{10}') → -0.22±0.2, Im(C_{10}') → 0.25±2.07

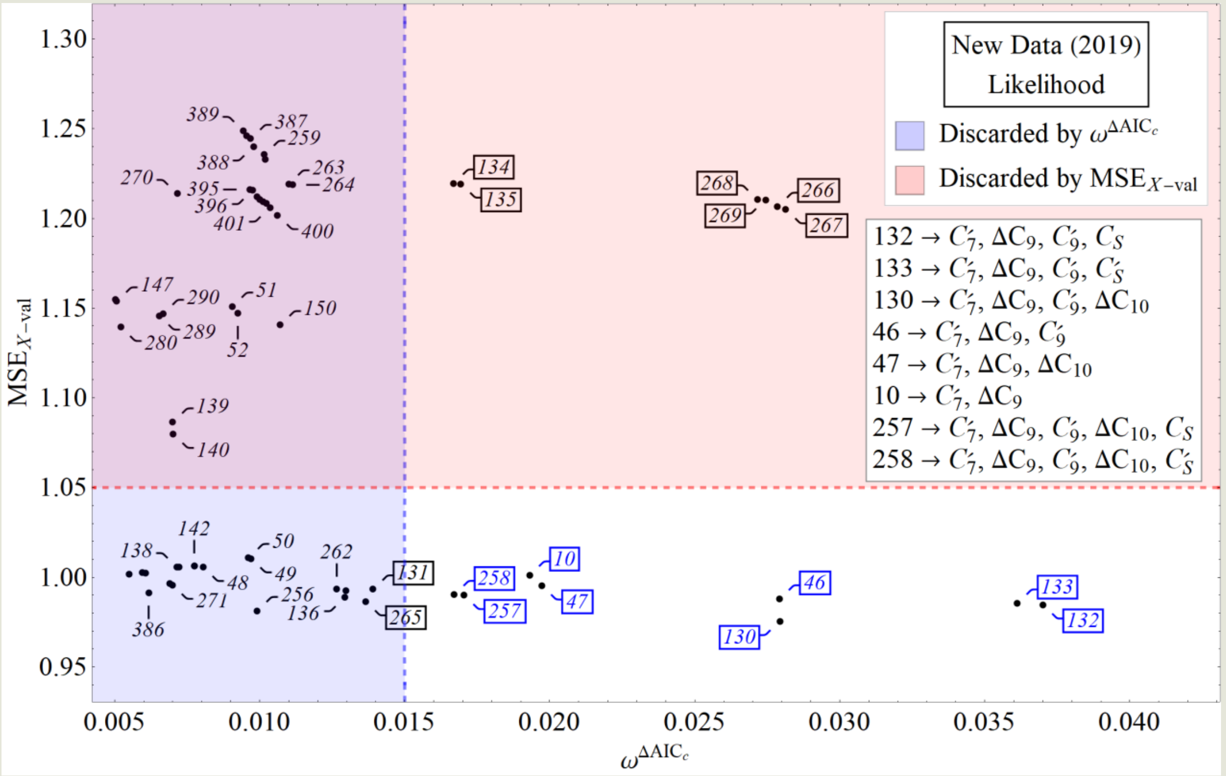
TABLE II: The selected models which pass the criterion of $\Delta AIC_c \leq 6$ and $MSE_{X\text{-val}} < 2$.

Models	Best fit values with uncertainties
585	$Re(\Delta C_9) \to -1.23 \pm 0.18,\ Im(\Delta C_9) \to 2.01 \pm 0.31,\ Re(C_9') \to 0.64 \pm 0.23,\ Im(C_9') \to 0.034 \pm 0.225,$ $Re(\Delta C_{10}) \to 0.51 \pm 0.21,\ Im(\Delta C_{10}) \to -0.18 \pm 0.25$
697	$Re(\Delta C_9) \to -1.17 \pm 0.21,\ Im(\Delta C_9) \to 2.12 \pm 0.33,\ Re(C_9') \to 0.58 \pm 0.23,\ Im(C_9') \to -0.049 \pm 0.359,$ $Re(\Delta C_{10}) \to 0.54 \pm 0.21,\ Im(\Delta C_{10}) \to -0.061 \pm 0.318,\ Re(C_{10}') \to -0.026 \pm 0.177,\ Im(C_{10}') \to 0.29 \pm 0.32$
641	$Re(C_7') \to -0.0042 \pm 0.0137,\ Im(C_7') \to 0.0036 \pm 0.0265,\ Re(\Delta C_9) \to -1.23 \pm 0.2,\ Im(\Delta C_9) \to 2.00 \pm 0.32,$ $Re(C_9') \to 0.7 \pm 0.3,\ Im(C_9') \to -0.018 \pm 0.309,\ Re(\Delta C_{10}) \to 0.52 \pm 0.21,\ Im(\Delta C_{10}) \to -0.21 \pm 0.29$
530	$Re(\Delta C_9) \to -1.2 \pm 0.19,\ Im(\Delta C_9) \to 1.8 \pm 0.35,\ Re(\Delta C_{10}) \to 0.24 \pm 0.2,\ Im(\Delta C_{10}) \to -0.22 \pm 0.29$
529	$Re(\Delta C_9) \to -1.14 \pm 0.11,\ Im(\Delta C_9) \to -0.14 \pm 0.39,\ Re(C_9') \to 0.38 \pm 0.23,\ Im(C_9') \to -1.19 \pm 0.35$
513	$Re(\Delta C_9) \to -1.14 \pm 0.11,\ Im(\Delta C_9) \to 1.14 \pm 0.33$
591	$Re(\Delta C_9) \to -1.07 \pm 0.25,\ Im(\Delta C_9) \to 1.98 \pm 0.33,\ Re(\Delta C_{10}) \to 0.4 \pm 0.22,\ Im(\Delta C_{10}) \to 0.29 \pm 0.46,$ $Re(\Delta C_{10}') \to -0.2 \pm 0.12,\ Im(\Delta C_{10}') \to 0.63 \pm 0.45$

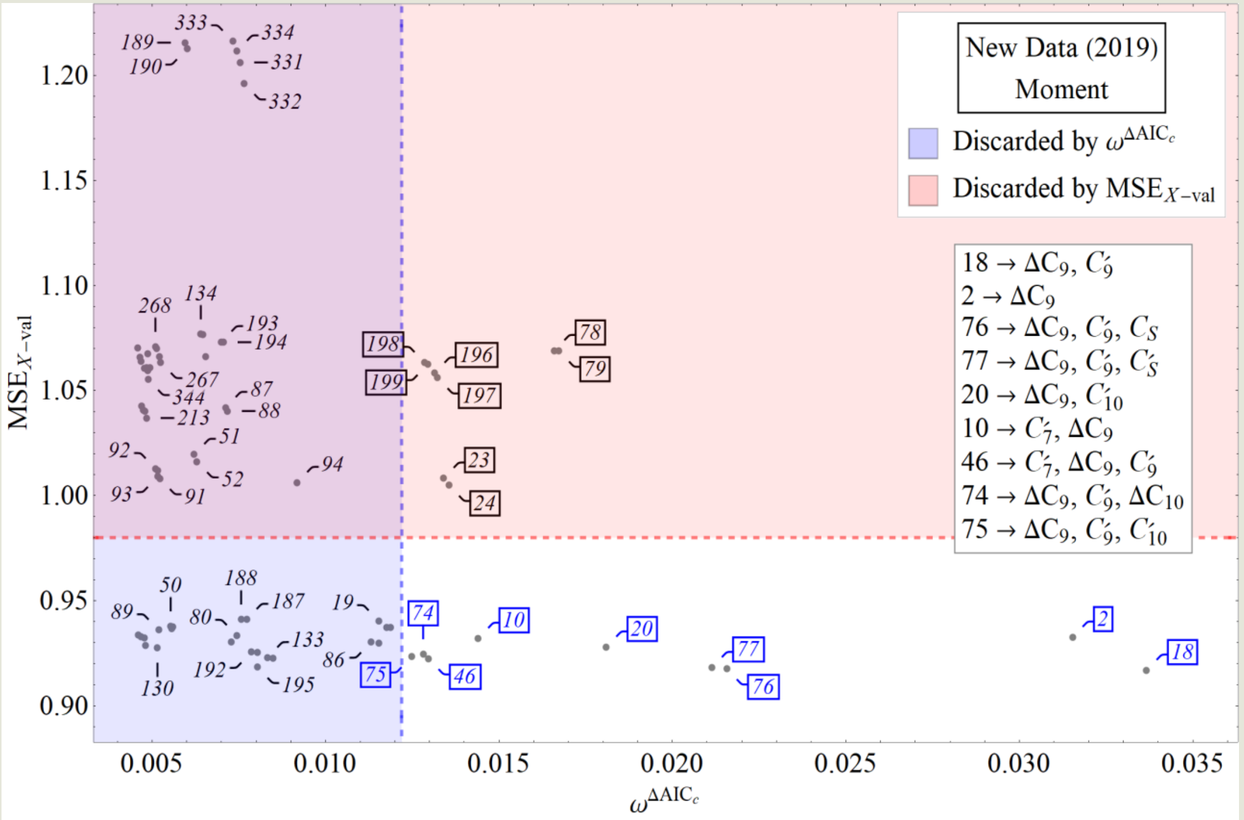
TABLE III: Modified parameter spaces of the selected models (only with imaginary WCs) in table II after incorporating the data on CP-asymmetric observables from LHCb [1].

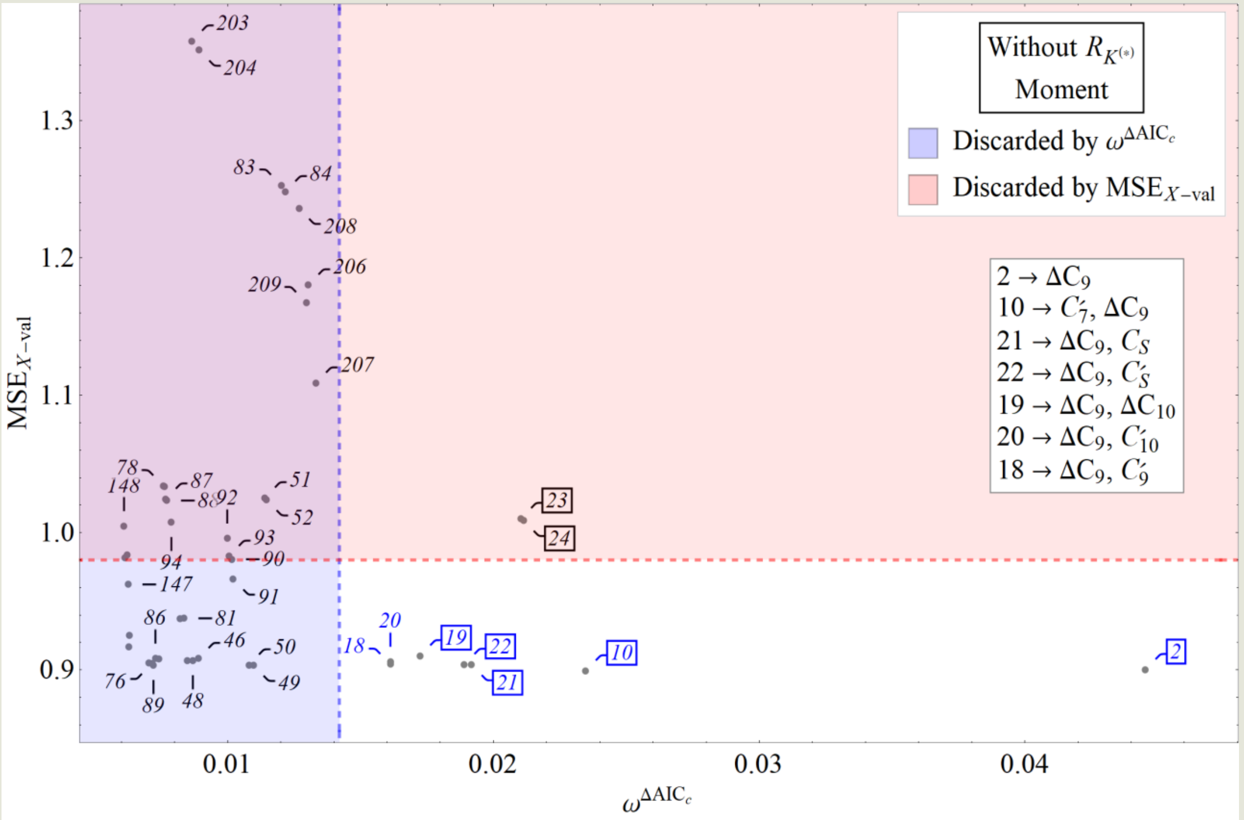
Results: Model Selection (New data)



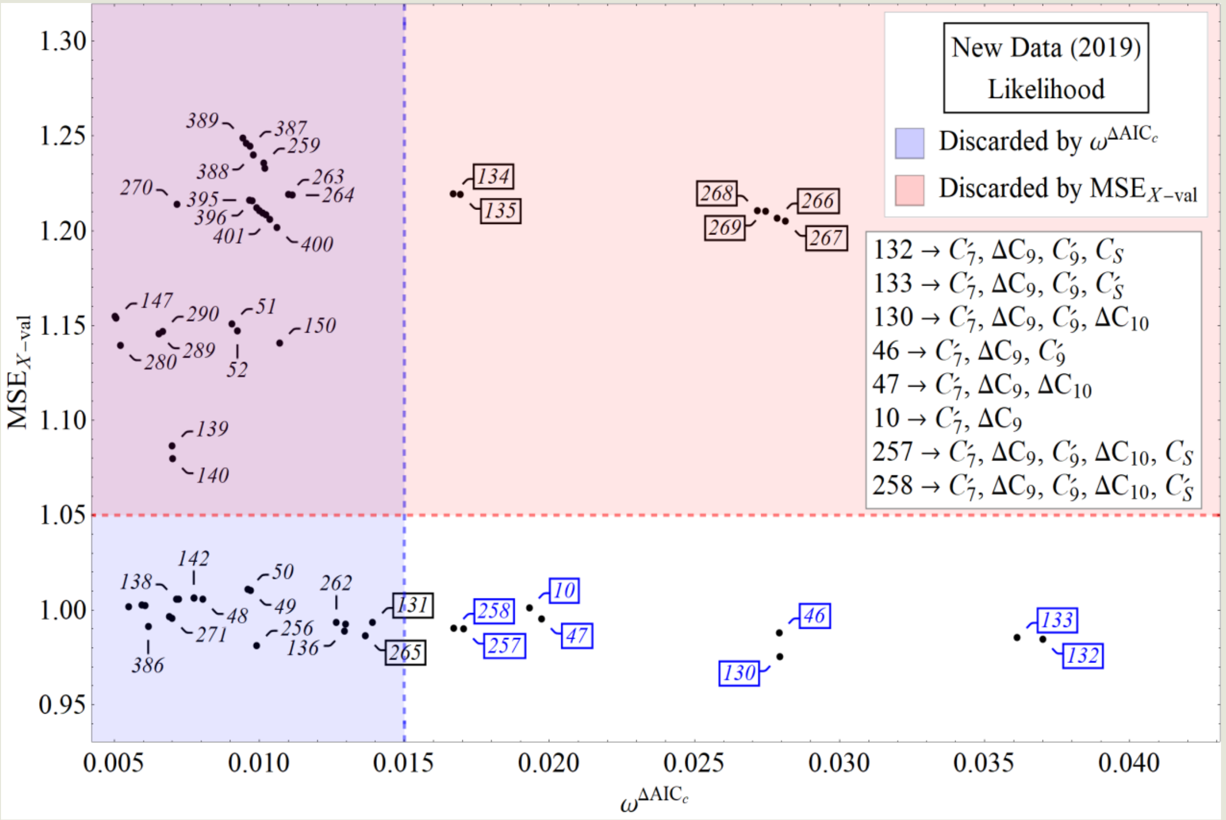


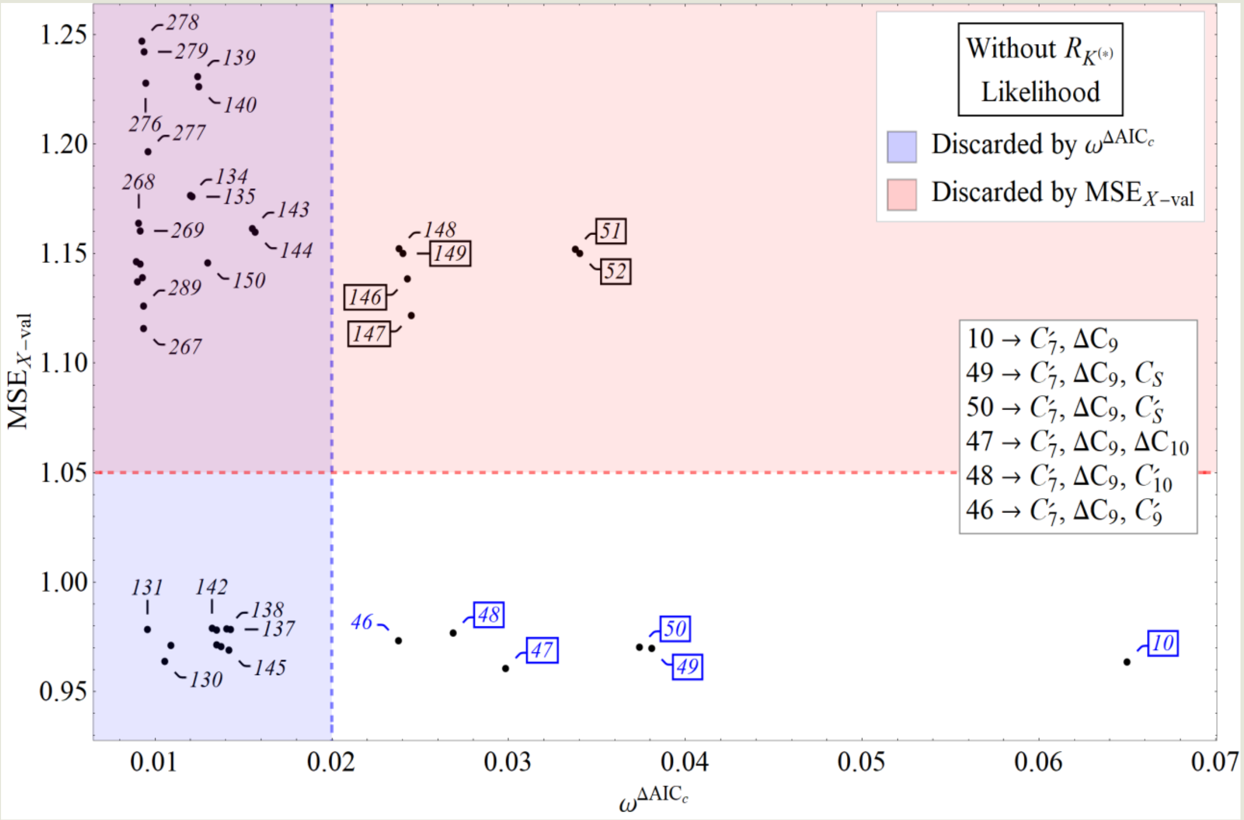
The impact of $R_{K^{(*)}}$ (Moments)



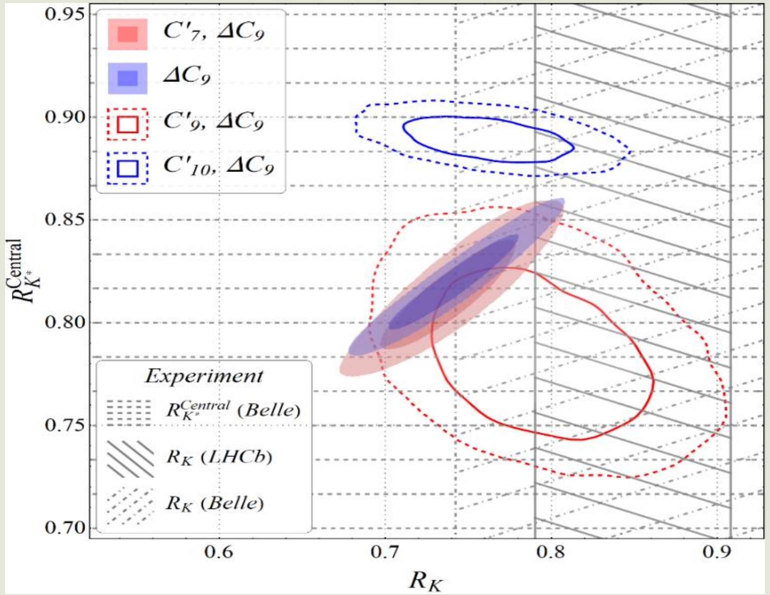


The impact of $R_{K^{(*)}}$ (Likelihood)



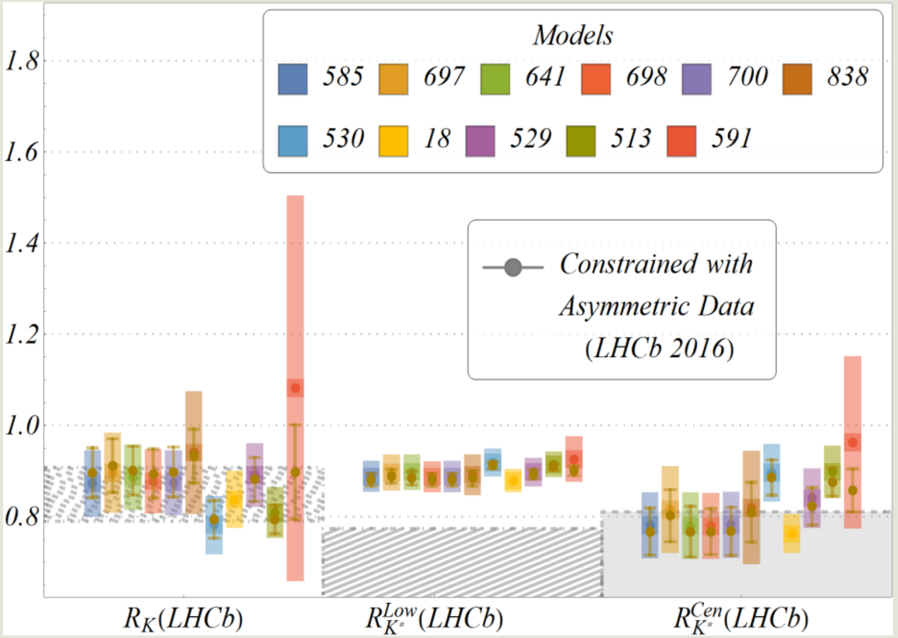


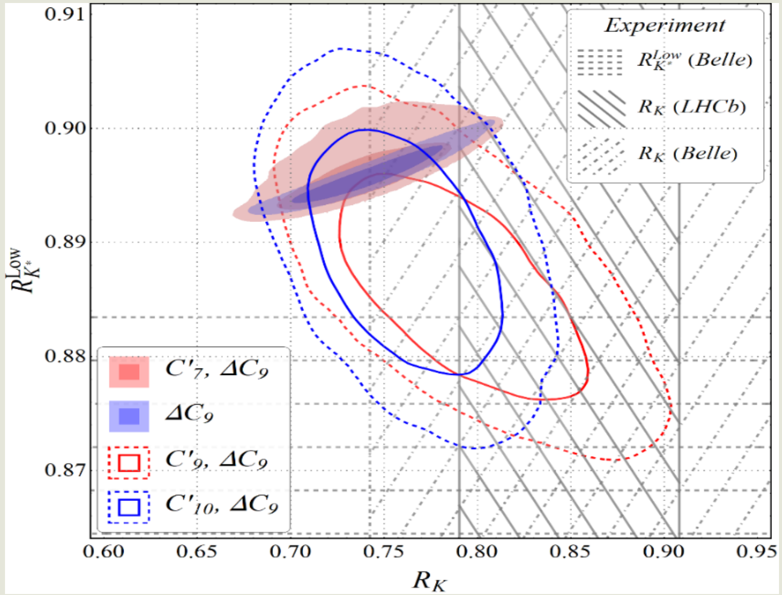
$R_{K^{(*)}}$ predictions



Real

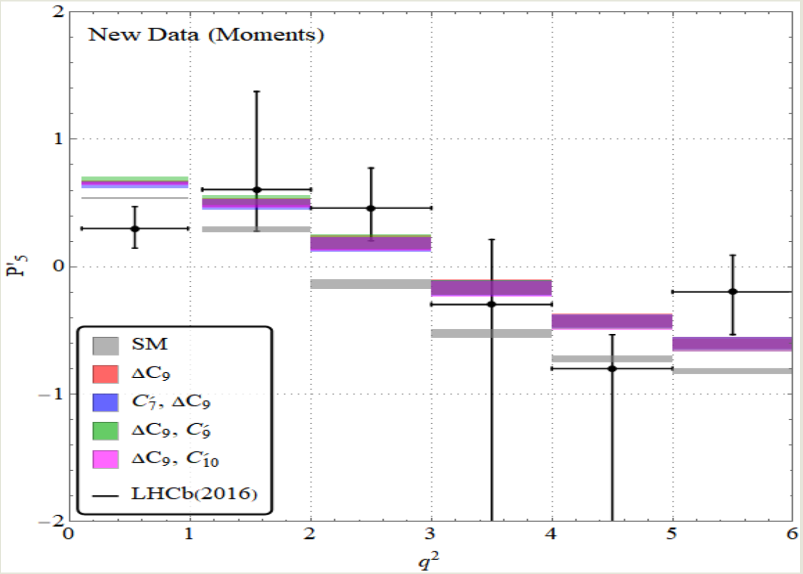
Complex





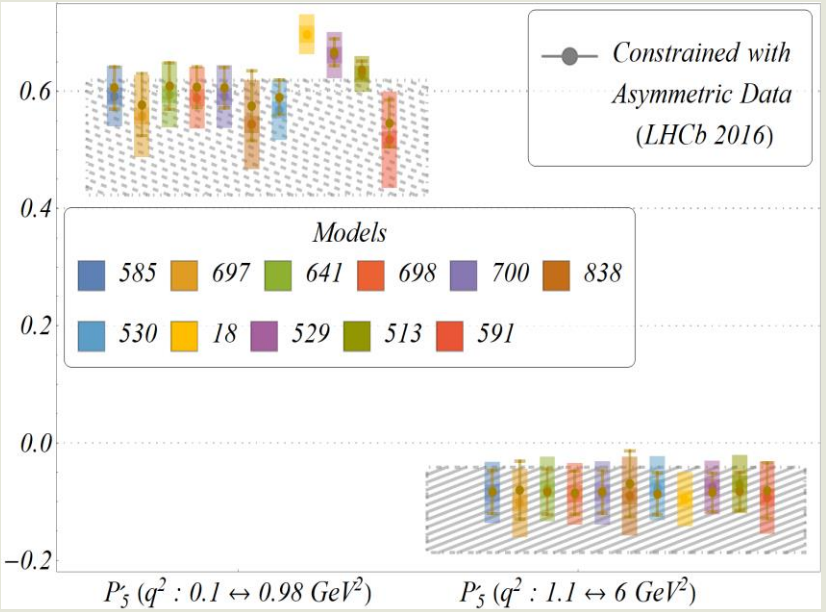
Real

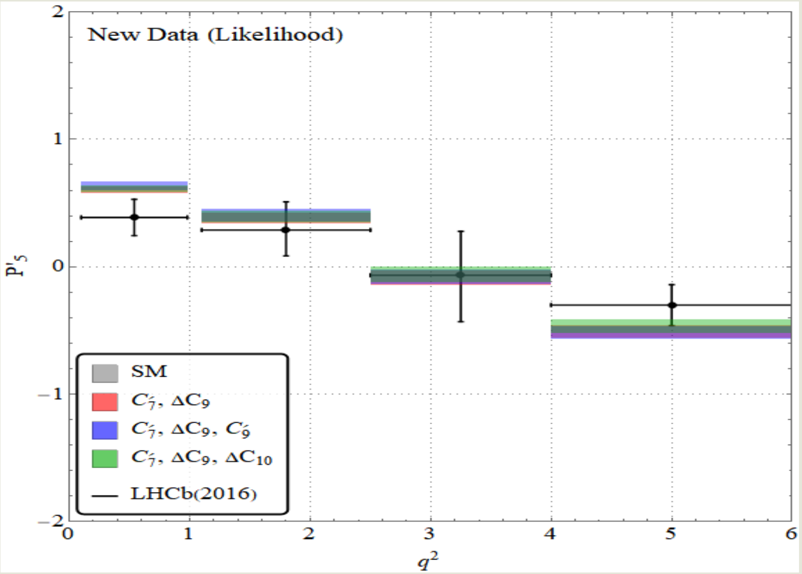
P'_5 predictions



Real

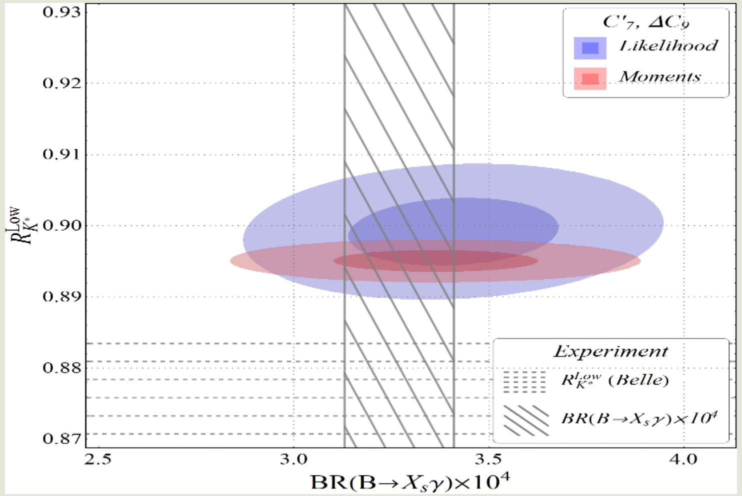
Complex

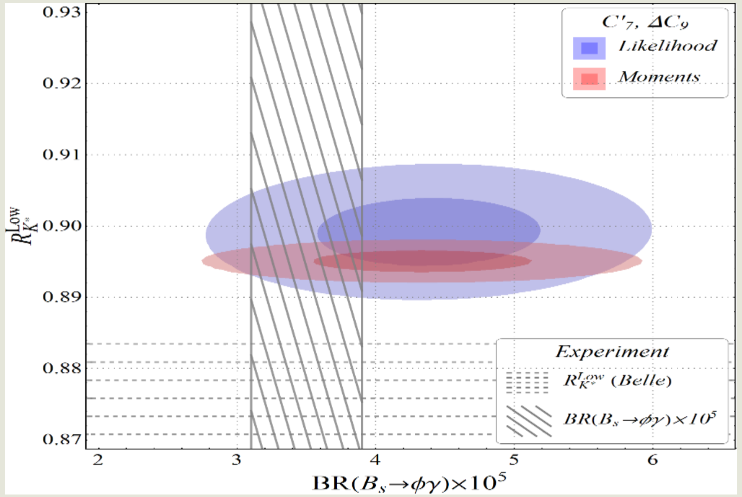


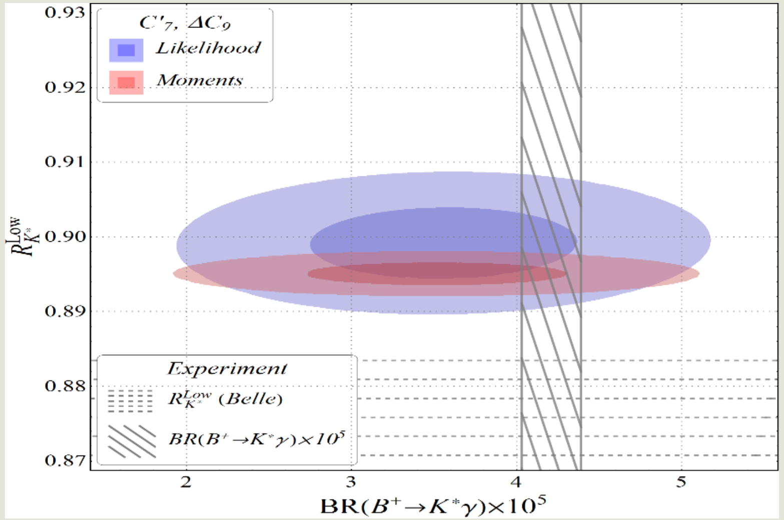


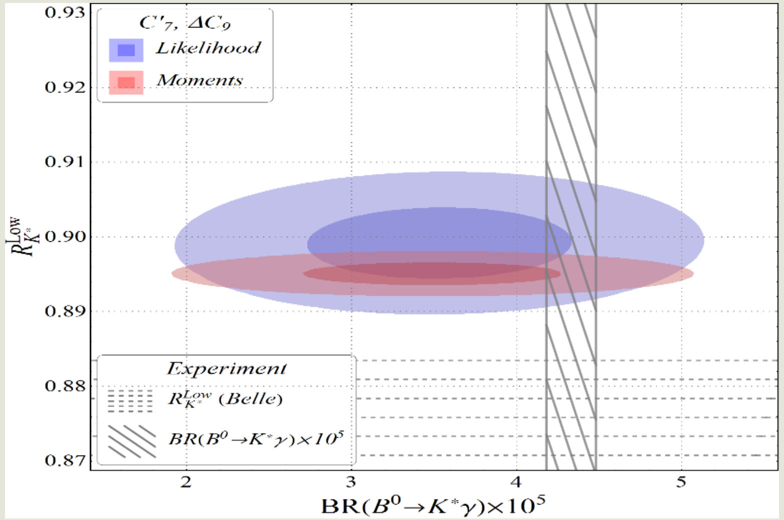
Real

$R_{K^*}^{low}$ and Radiative constraints

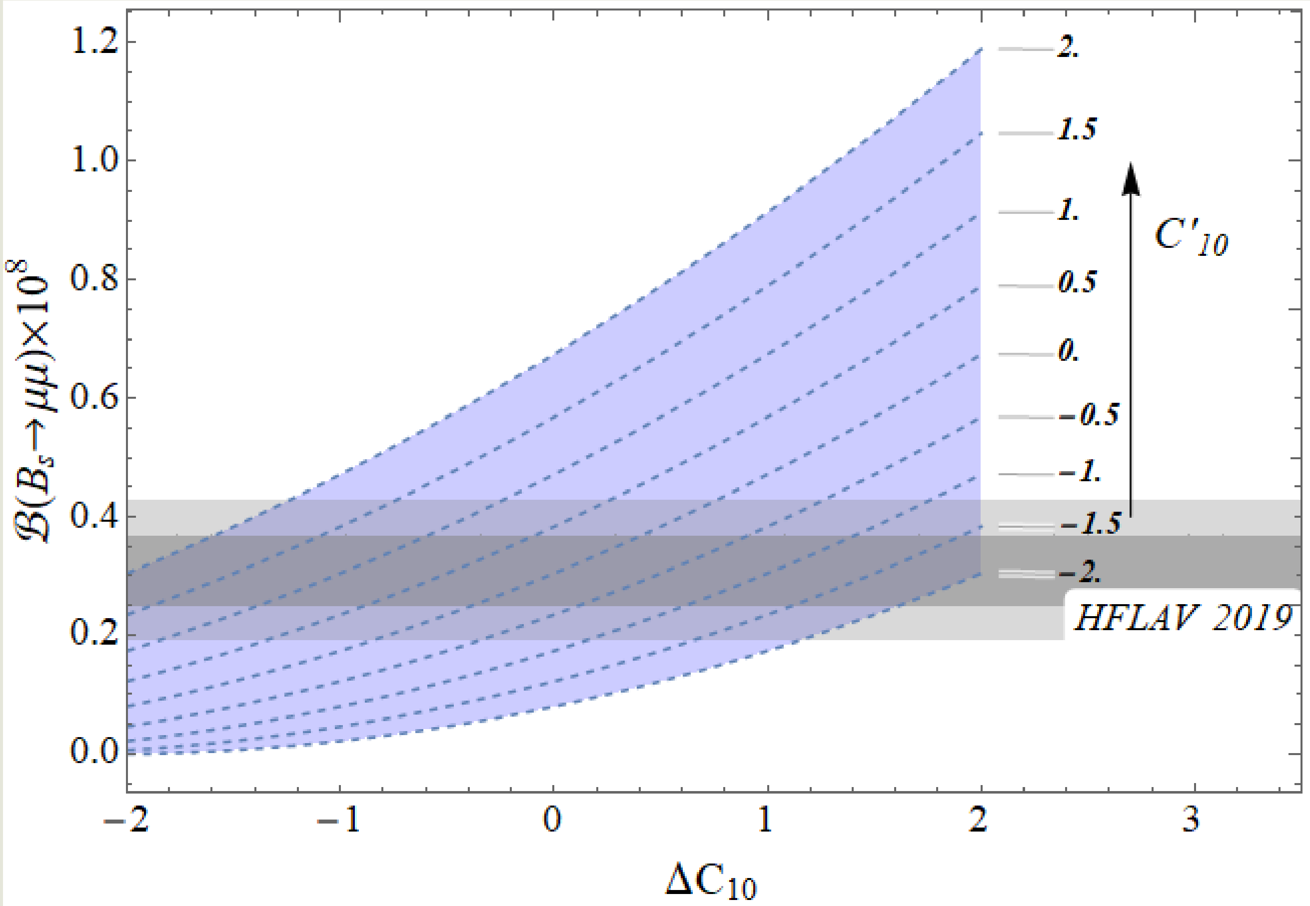




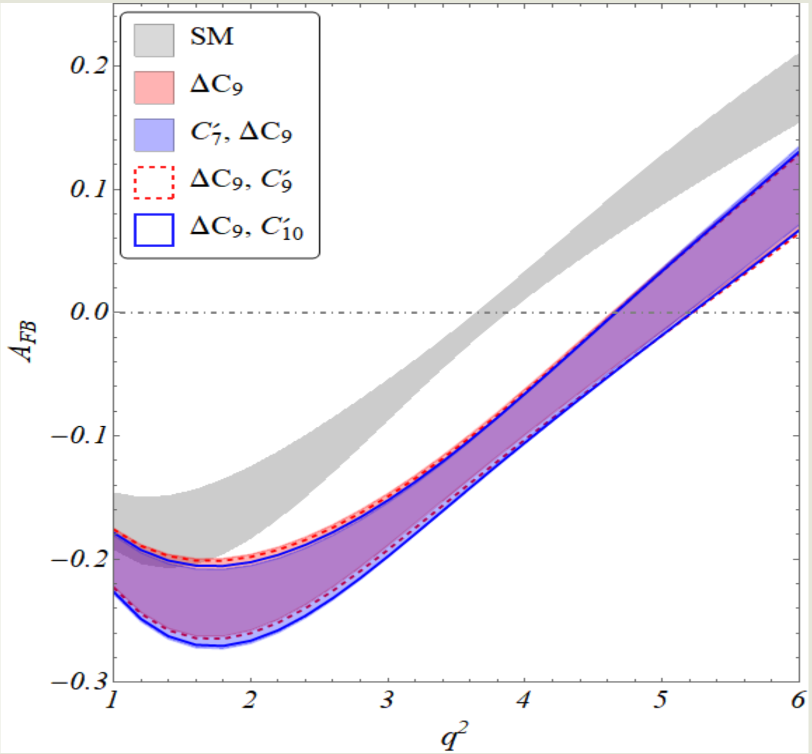


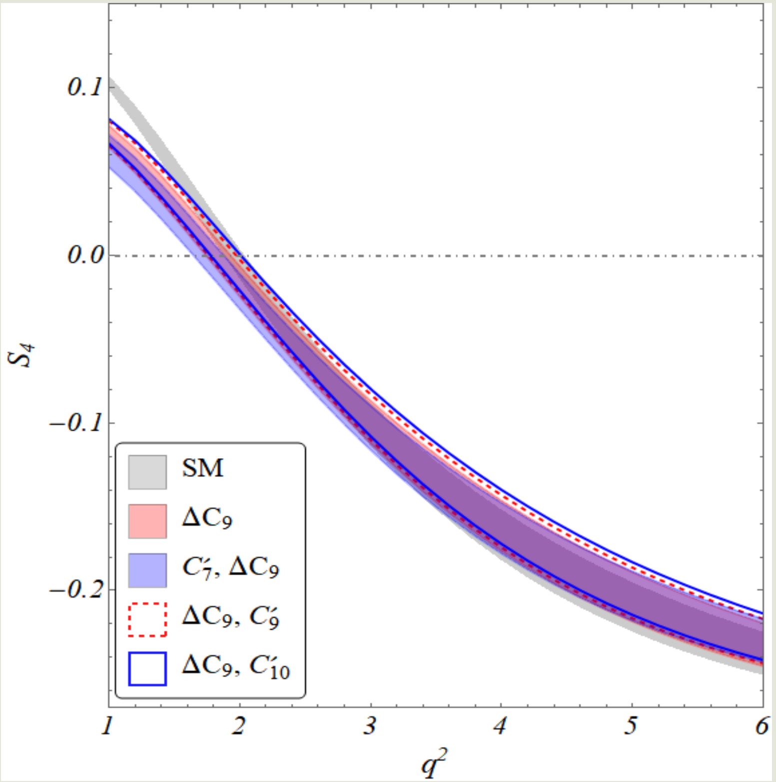


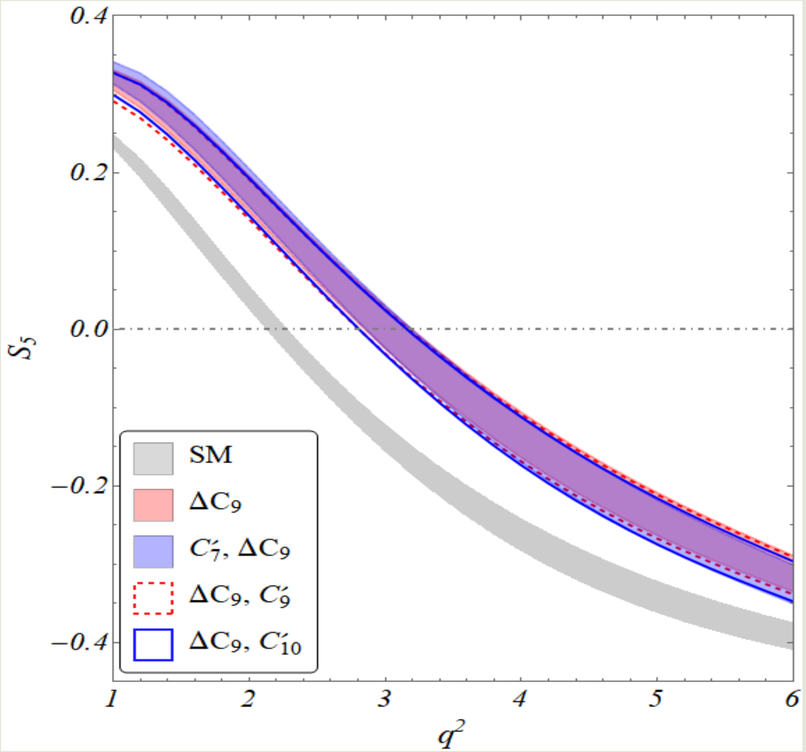
Constraints from $B_s \to \mu\mu$



q^2 distributions for observables







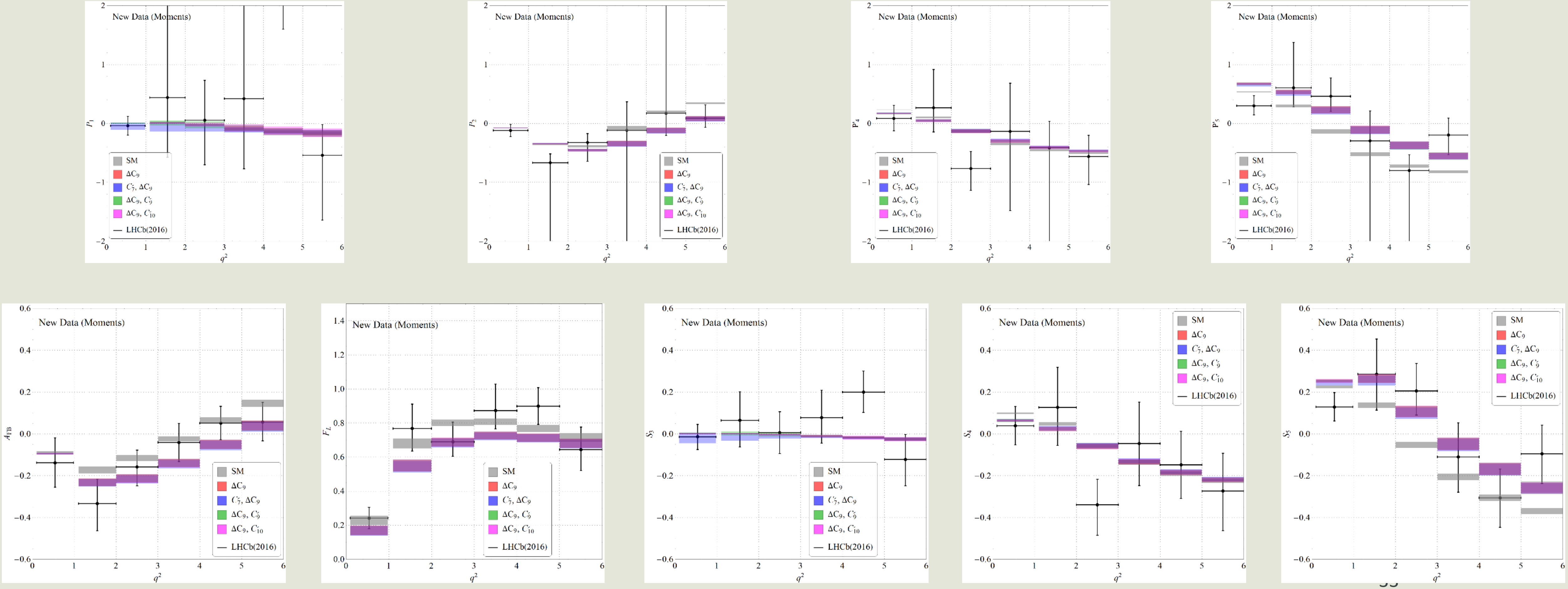
Discussions and Future Prospects

- No NP in electron modes.
- Correlations between SM and NP parameters neglected. Only analysis by 2006.03489 (FM etal., GAMBIT) involving "$\mathrm{Re}(C_{7,9,10})$" only.
- Involved analysis involving imaginary parts.
- Comparing distributions rather than MLE estimates: **Bayesian model selection.**
- Data points with large uncertainties: Bayesian might miss portions of parameter space. (shown for $b \to c\tau\nu$ in talk by S.P. and 2008.04316 (SN, SP etal.)): **Neural Network.**

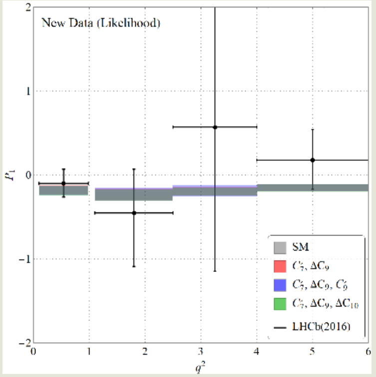
We're on it. Stay Tuned!

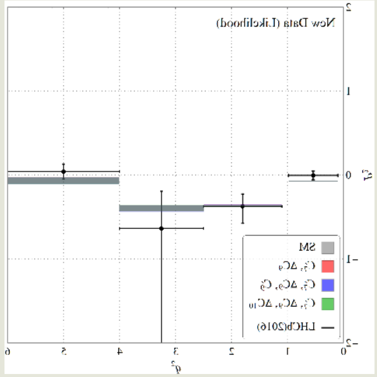
THANK YOU!

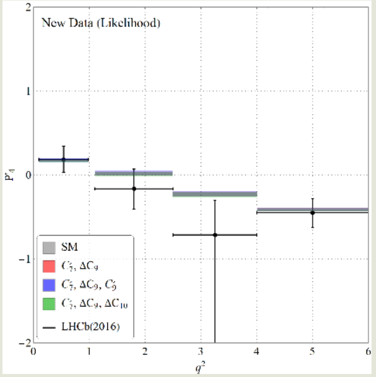
backup: q^2 *distributions for observables* (w.o. LFUV) (moments)

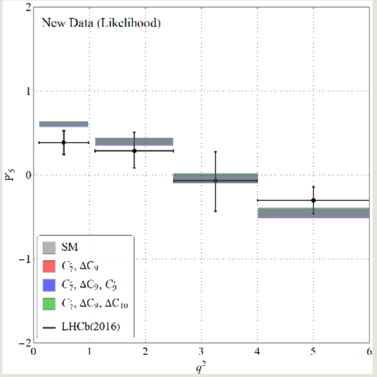


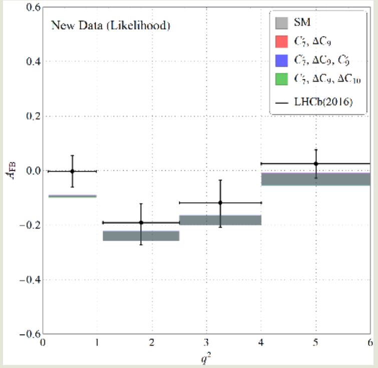
backup: q^2 *distributions for observables* (w.o. LFUV) (moments)

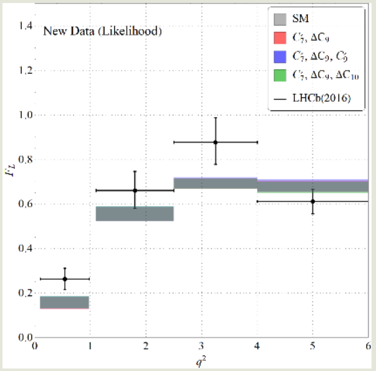


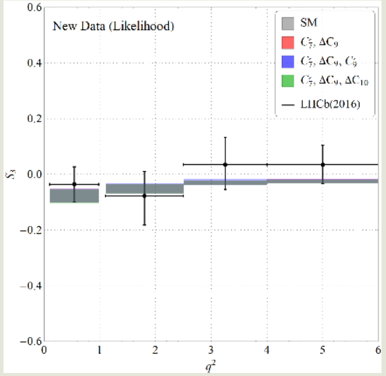


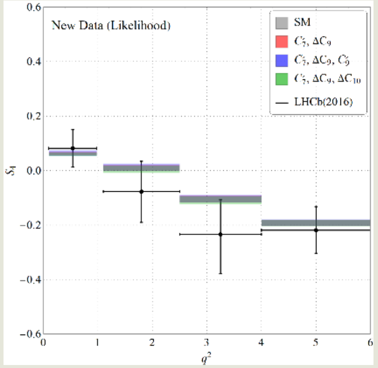


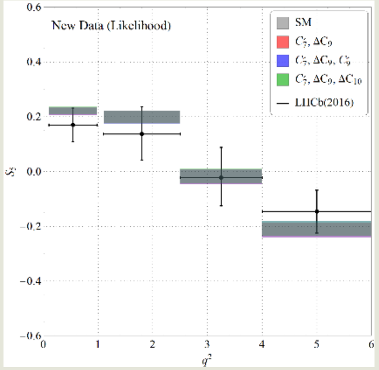




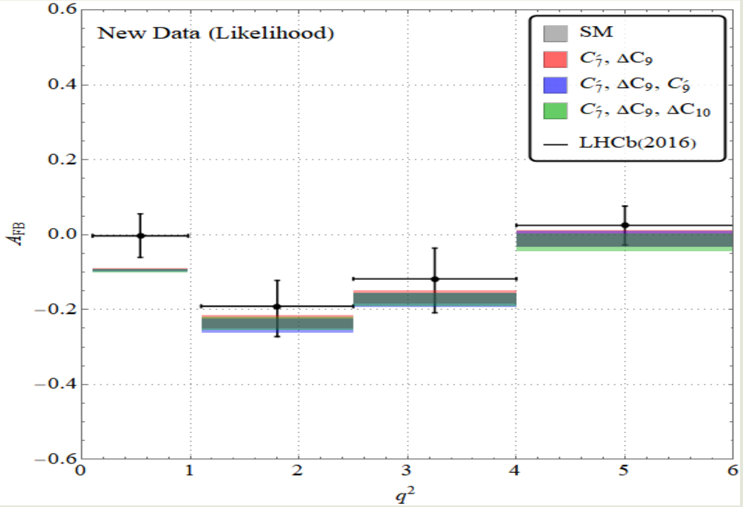


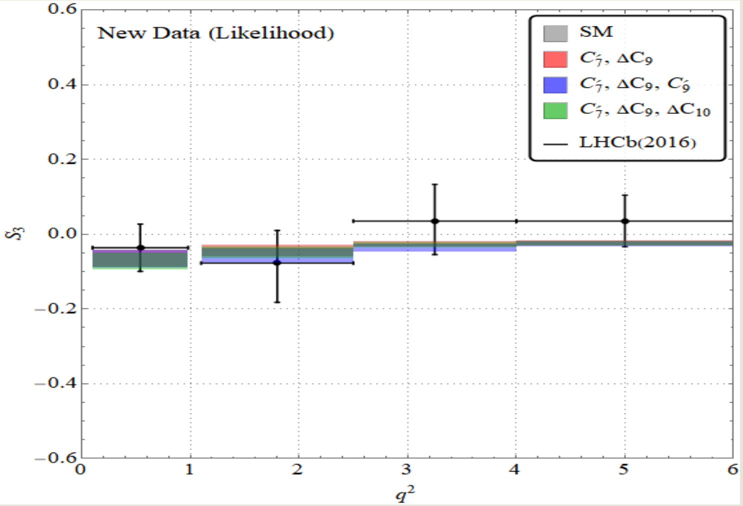


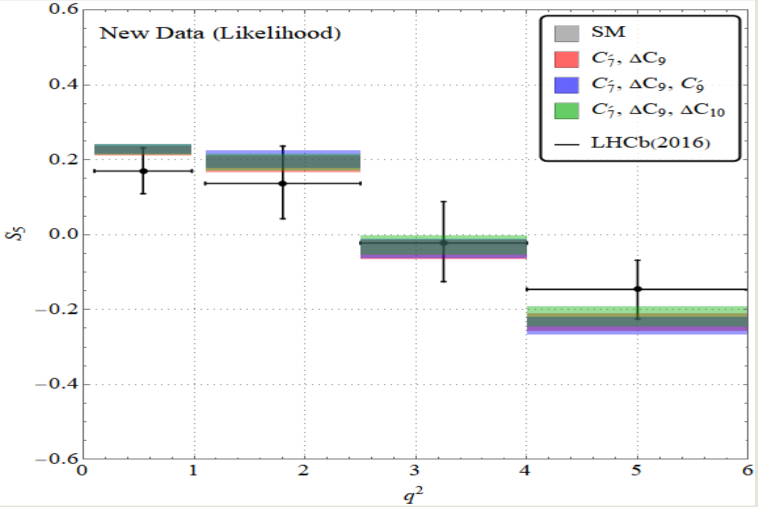


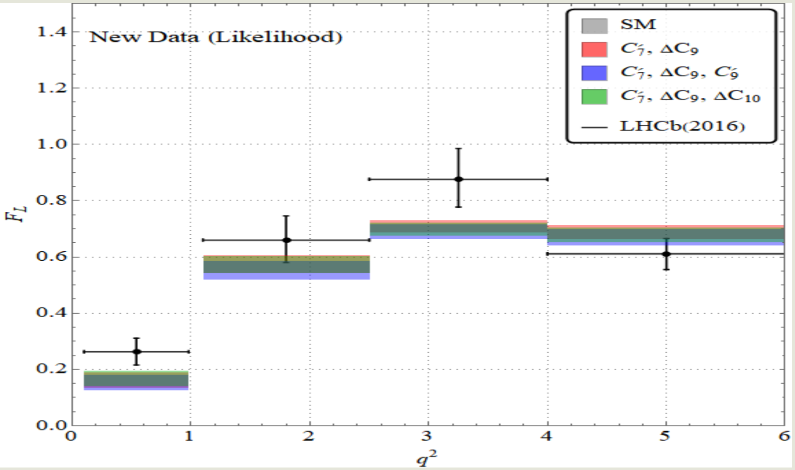


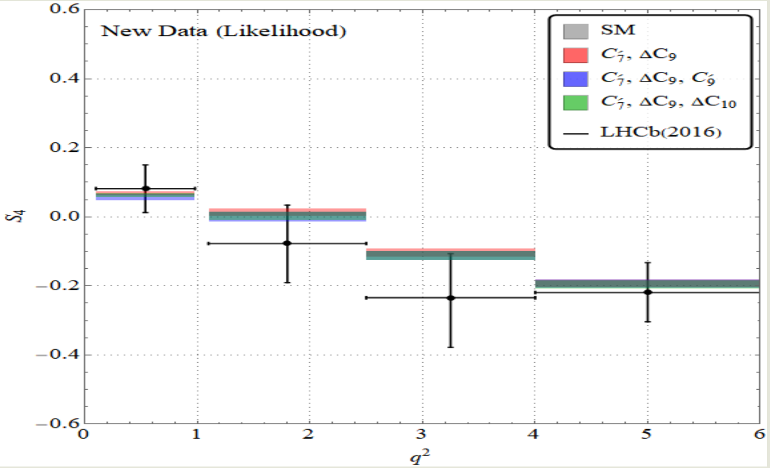
Results: Comparison: CP averaged (Likelihood)



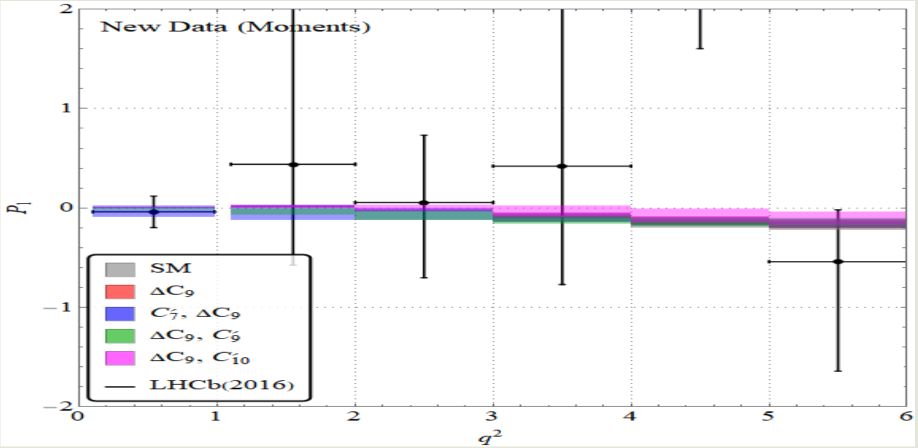


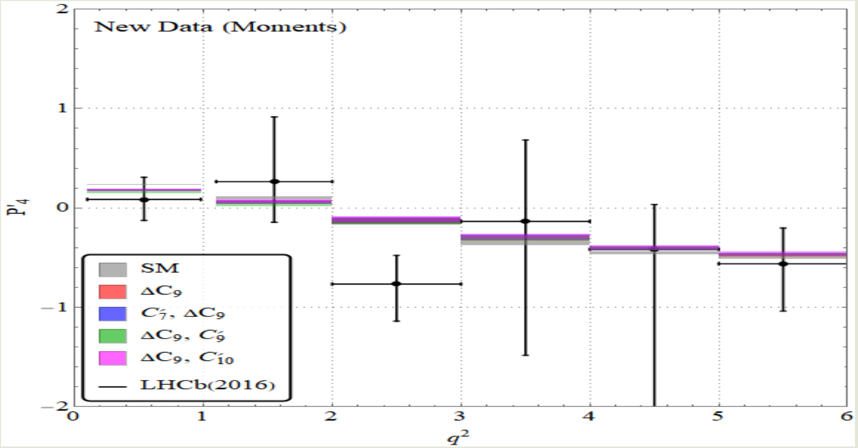


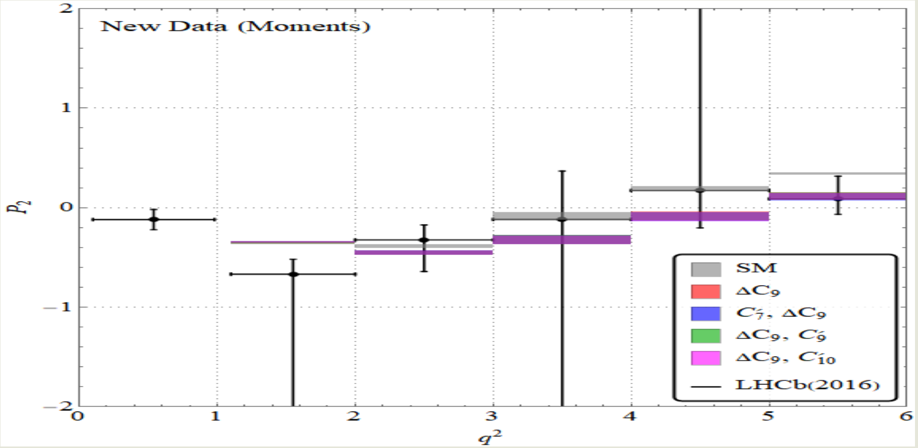


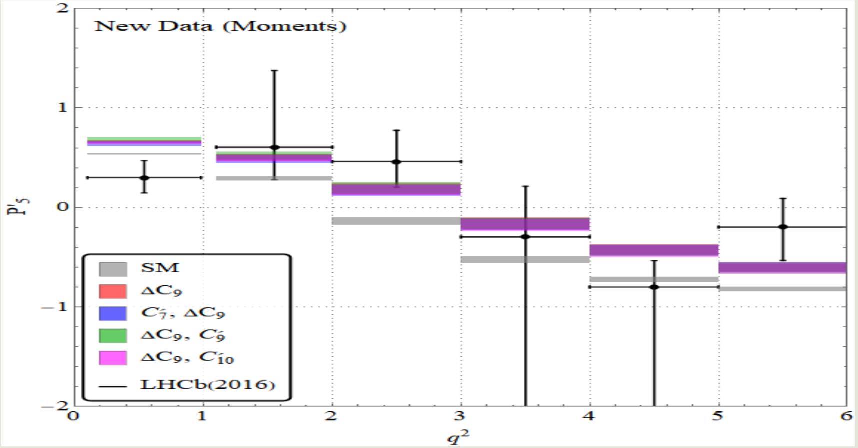


Results: Comparison: Optimized (Moment)

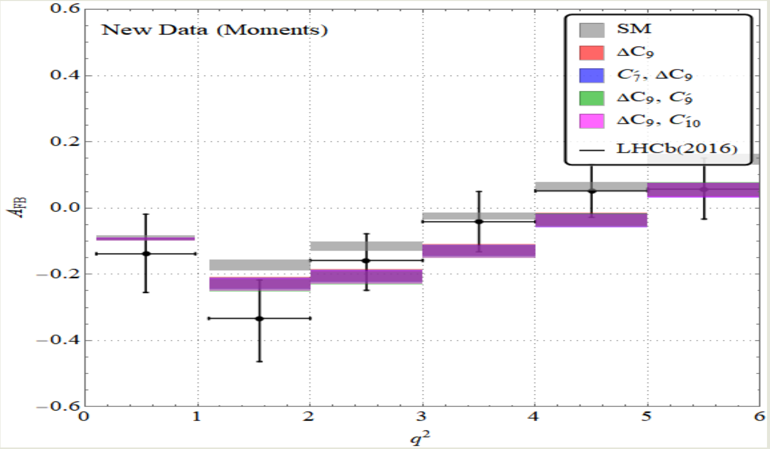


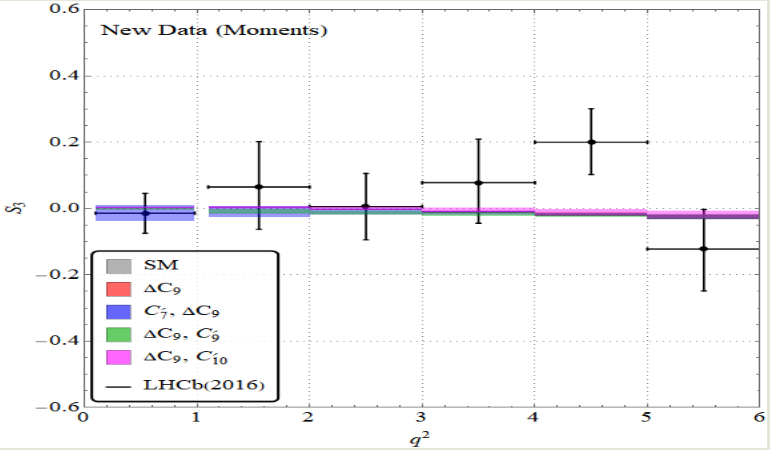


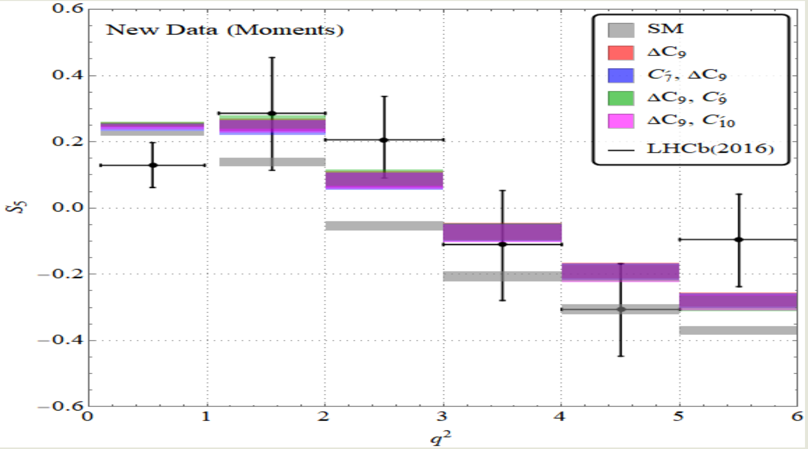


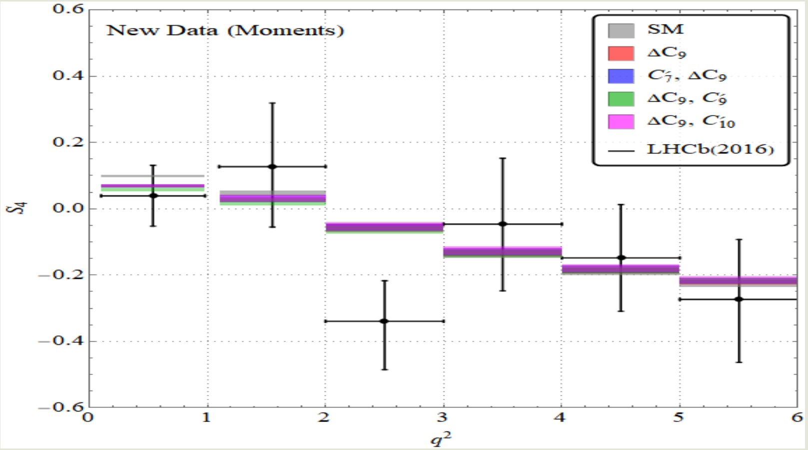


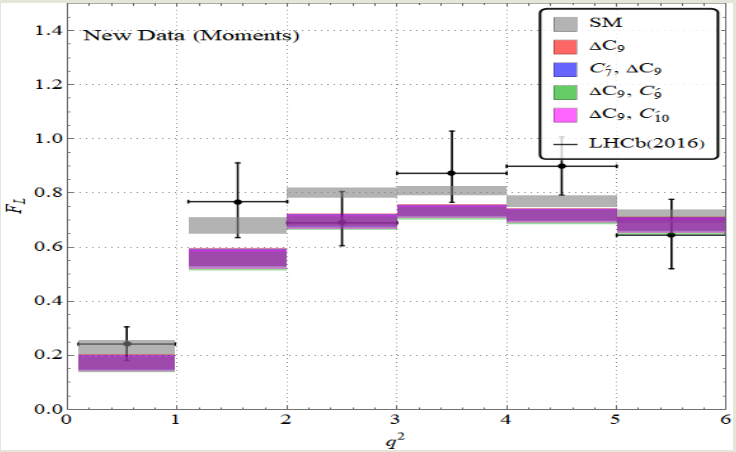
Backup: **Results: Comparison: CP averaged (Moments)**



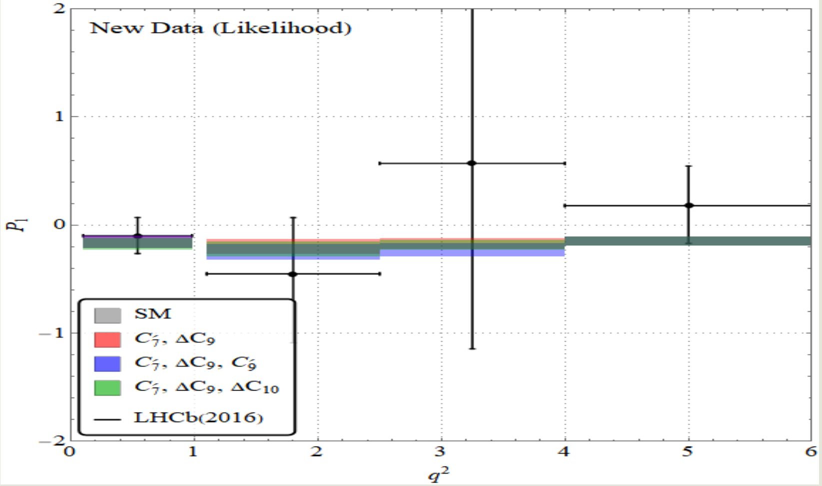


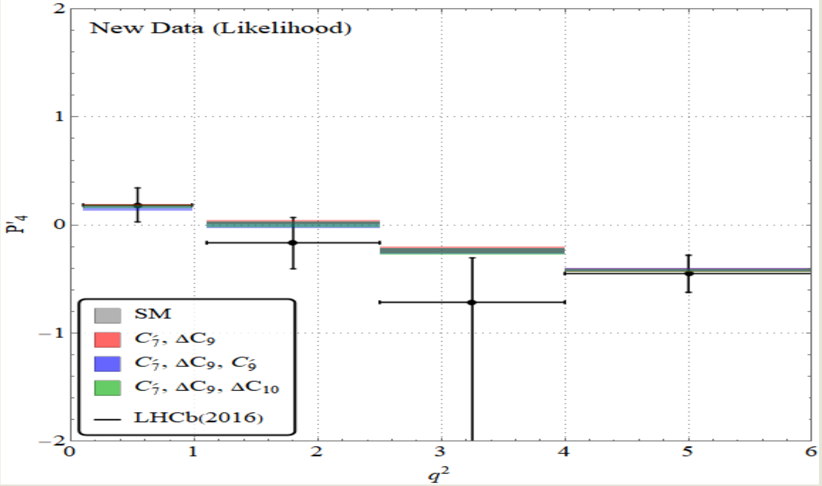


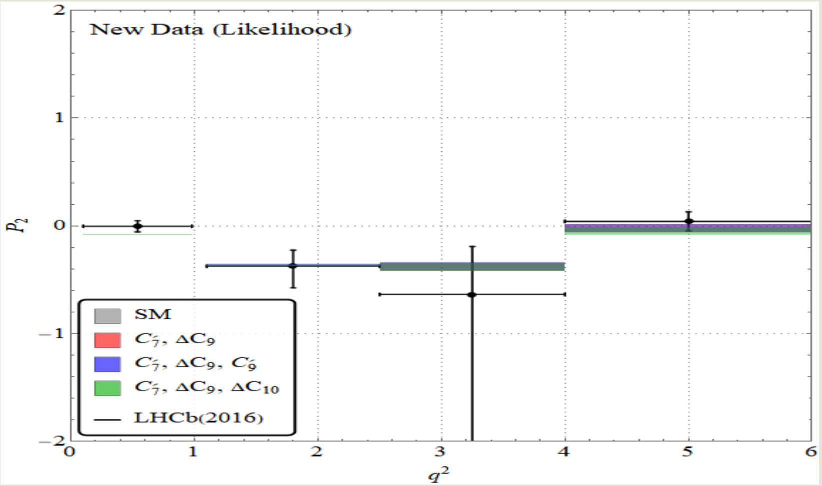


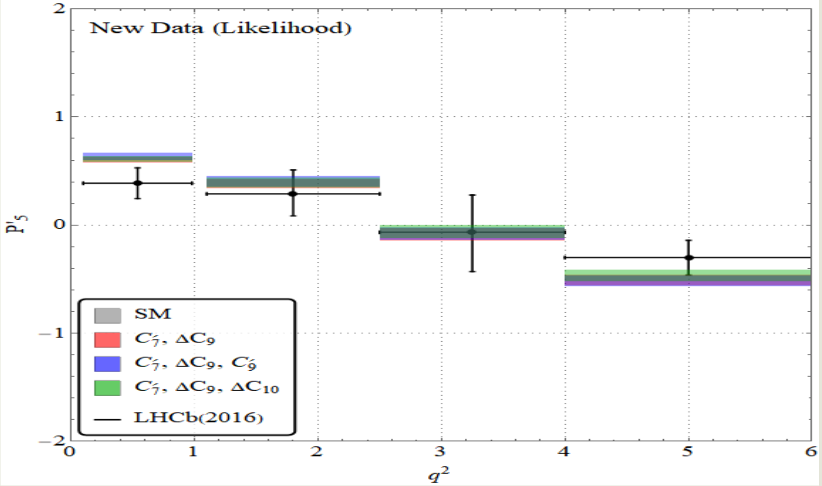


Backup: **Results: Comparison: Optimized (Likelihood)**

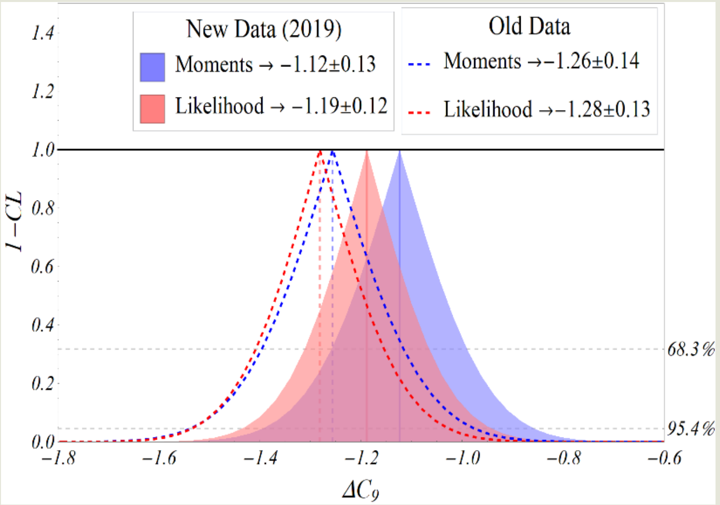


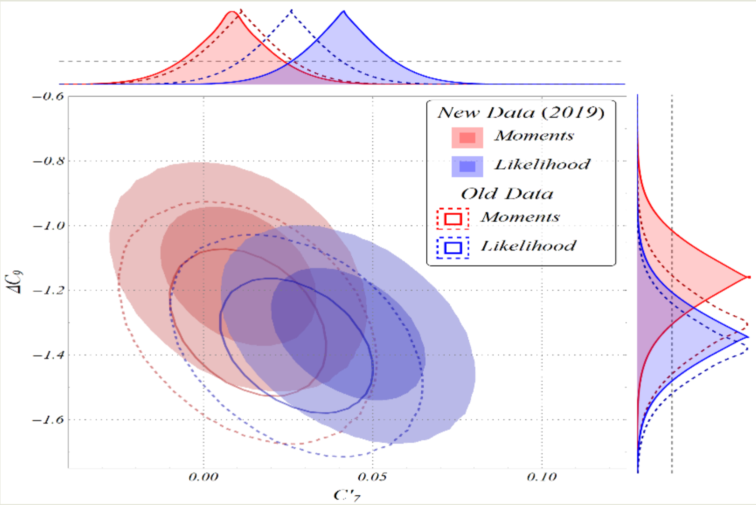


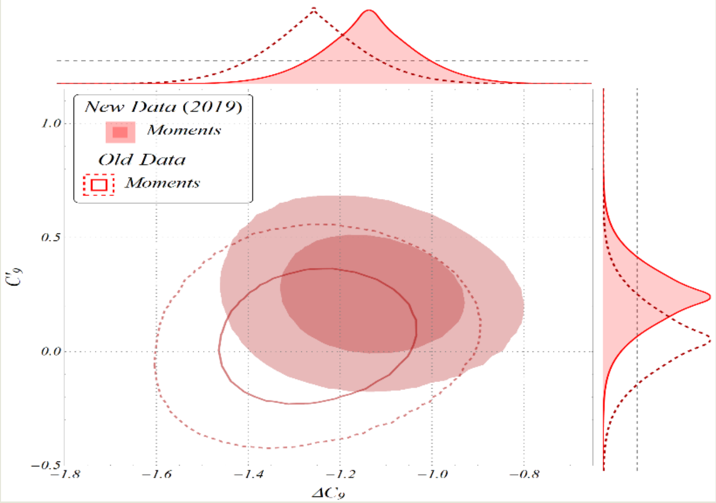


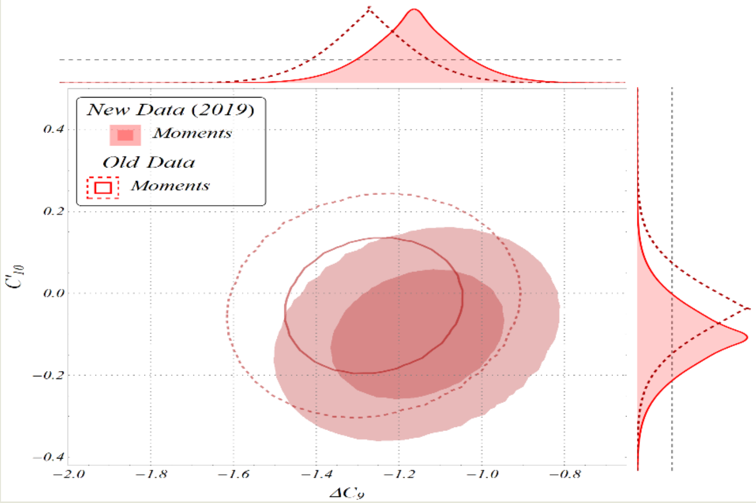


Backup: **Allowed parameter spaces (1 & 2)**

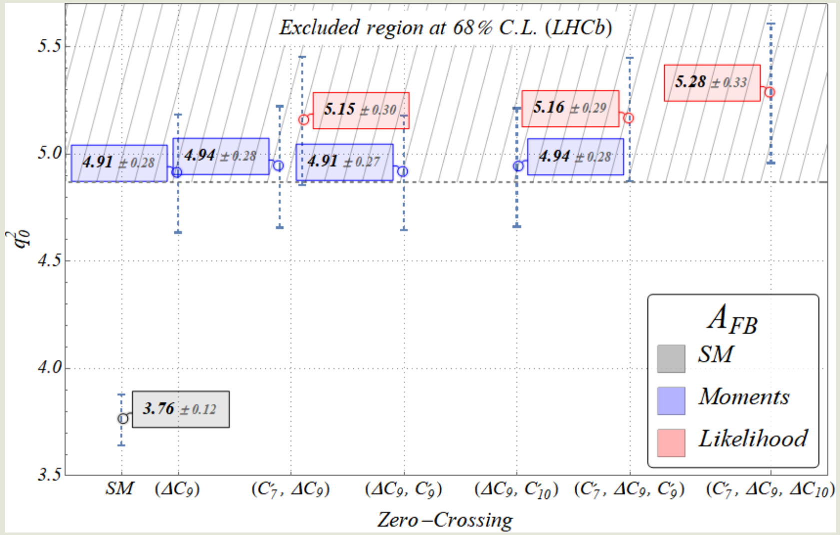


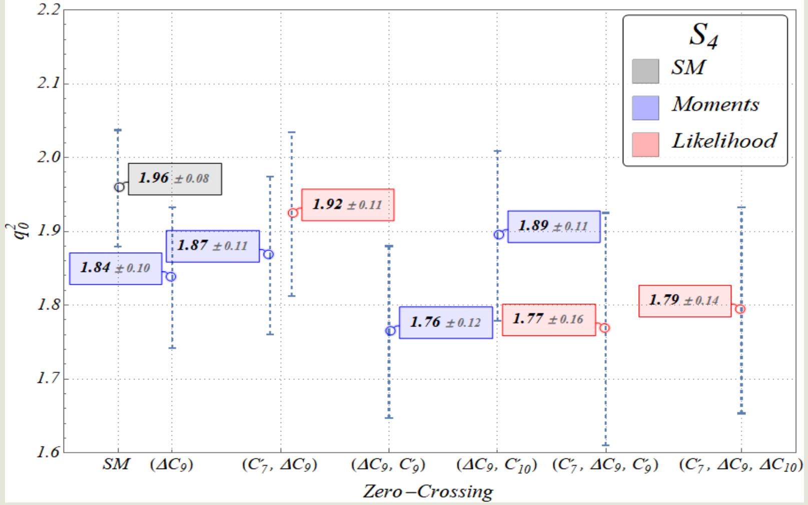


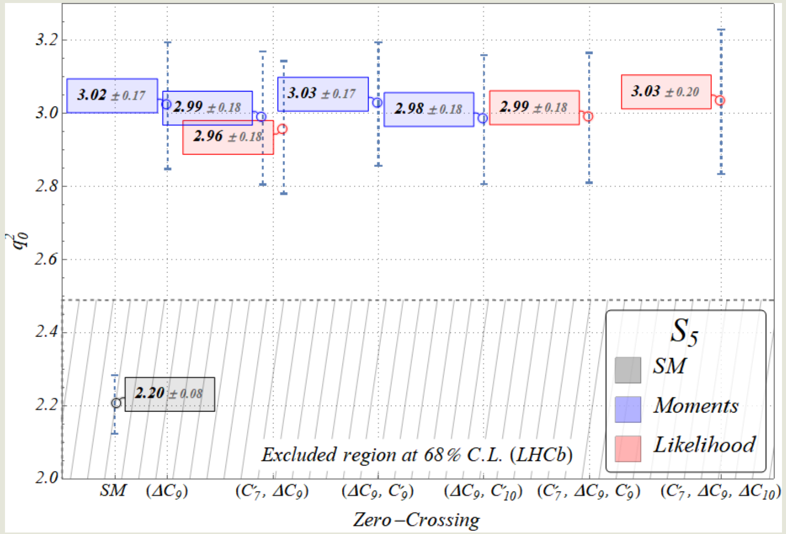




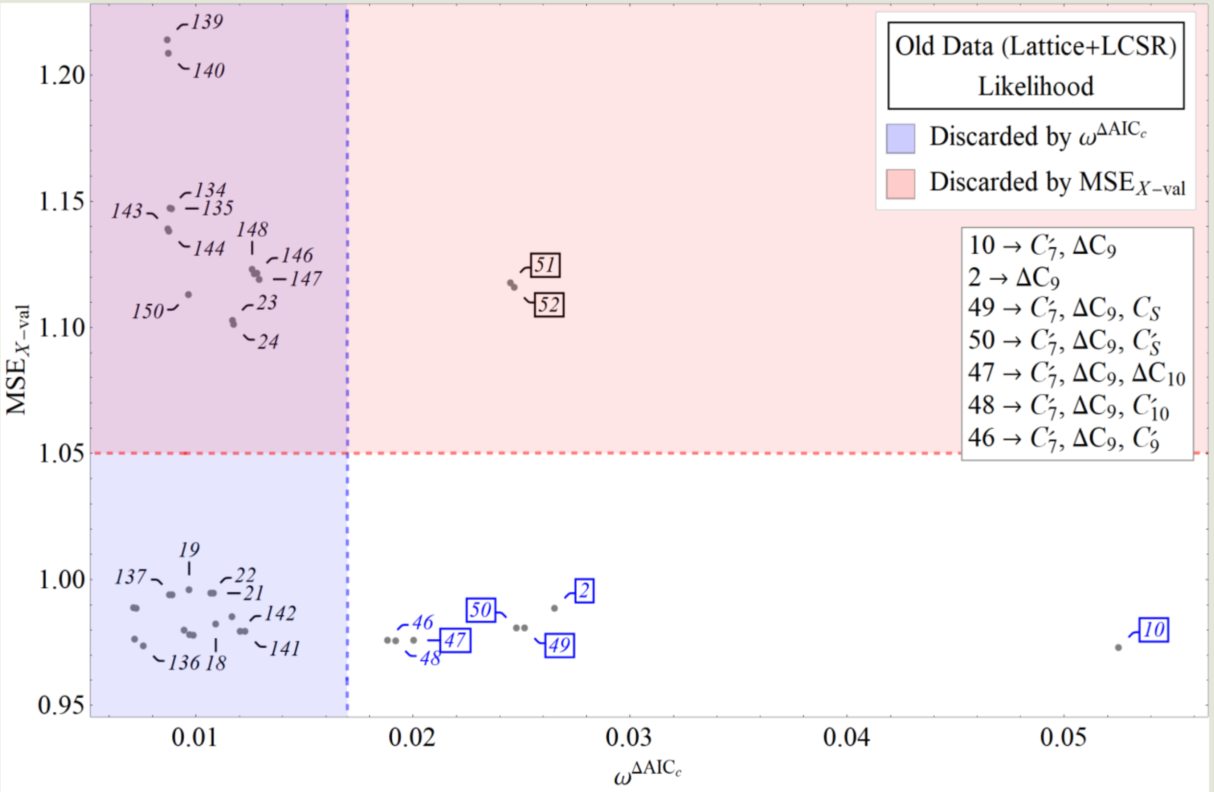
Backup: **Zero crossing values**

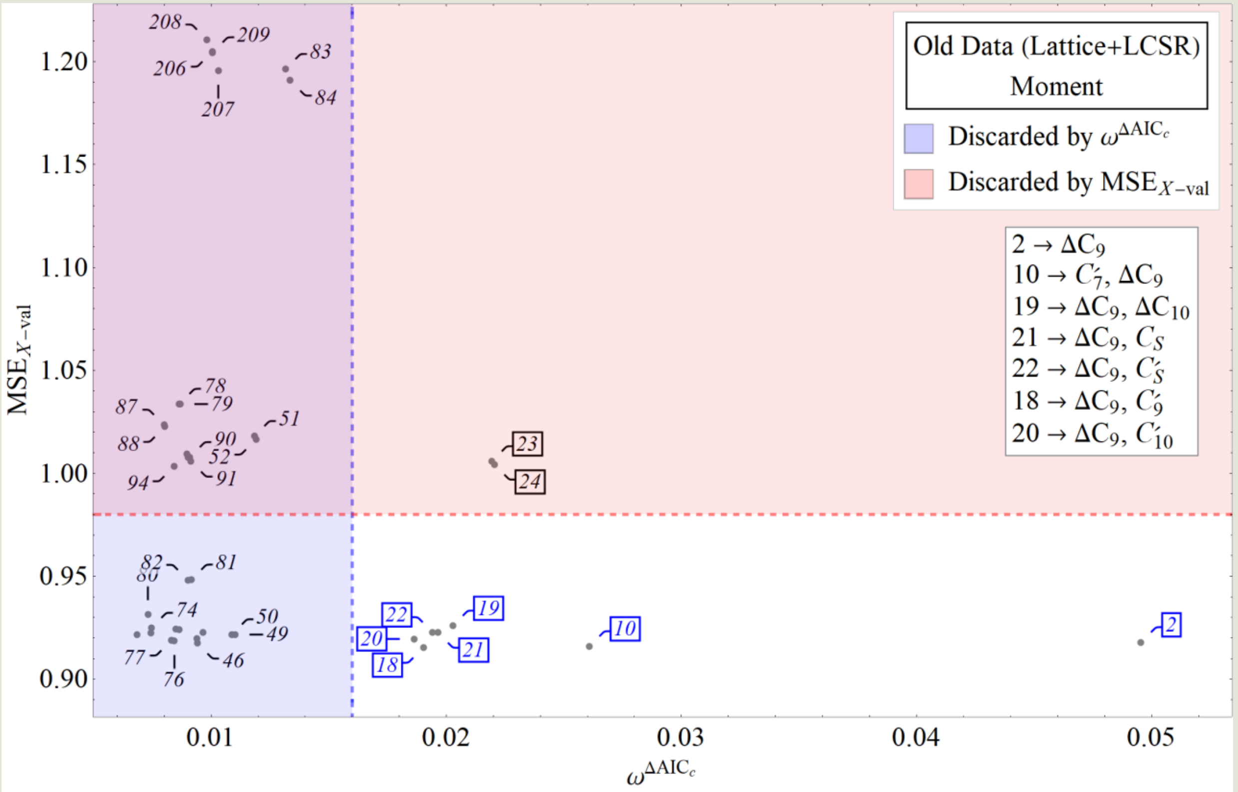






backup: Results: Model Selection (Old data)





backup: Results: Model Selection (Old data)

TABLE II: Fit-qualities, model selection criteria, parameter estimates and effects on radiative decays for the 'best' selected models with the 'Old' data-set, with the 'Moments' estimate of the angular observables. Selected models are obtained from fig. 1a. Last four columns showcase the deviations (in units of 'σ') between the experimental value of the radiative decays and the corresponding value obtained with the fit results.

Model Index	$\chi^2_{\rm Min}/$ DOF	p-val (%)	$\omega^{\Delta \rm AIC_c}$ (%)	$\rm MSE_{X-val}$	Parameter Values	Deviation in σ $\rm B\to X_s\gamma$	$B^+ \to K^*\gamma$	$\Delta \rm B^0 \to K^*\gamma$	$\Delta \rm B_s \to \phi\gamma$
2	245.67/254	63.5	5.	0.918	$\Delta C_9 \to$ -1.26±0.14	–	–	–	–
10	244.92/253	63.1	2.6	0.916	$C_7' \to$ 0.013±0.015 $\Delta C_9 \to$ -1.3±0.15	0.32	−0.87	−1.06	1.22
19	245.42/253	62.2	2.	0.926	$\Delta C_9 \to$ -1.22±0.16 $\Delta C_{10} \to$ 0.061±0.123	–	–	–	–
21	245.48/253	62.1	2.	0.923	$\Delta C_9 \to$ -1.27±0.15 $C_S \to$ -0.021±0.026	–	–	–	–
22	245.51/253	62.	1.9	0.923	$\Delta C_9 \to$ -1.27±0.15 $C_S' \to$ 0.02±0.026	–	–	–	–
18	245.55/253	62.	1.9	0.915	$\Delta C_9 \to$ -1.25±0.14 $C_9' \to$ 0.067±0.195	–	–	–	–
20	245.59/253	61.9	1.9	0.92	$\Delta C_9 \to$ -1.26±0.14 $C_{10}' \to$ -0.03±0.109	–	–	–	–

TABLE III: Same as table II, but with the 'Likelihood' estimate of the angular observables. Selected models are obtained from fig. 1b.

Model Index	$\chi^2_{\rm Min}/$ DOF	p-val (%)	$\omega^{\Delta \rm AIC_c}$ (%)	$\rm MSE_{X-val}$	Parameter Values	Deviation in σ $\rm B\to X_s\gamma$	$B^+ \to K^*\gamma$	$\Delta \rm B^0 \to K^*\gamma$	$\Delta \rm B_s \to \phi\gamma$
10	213.78/209	39.6	5.3	0.973	$C_7' \to$ 0.028±0.015 $\Delta C_9 \to$ -1.37±0.14	0.37	−0.85	−1.04	1.24
2	217.19/210	35.2	2.7	0.989	$\Delta C_9 \to$ -1.28±0.13	–	–	–	–
49	213.2/208	38.8	2.5	0.981	$C_7' \to$ 0.029±0.015 $\Delta C_9 \to$ -1.4±0.14 $C_S \to$ -0.028±0.019	0.38	−0.85	−1.04	1.25
50	213.23/208	38.7	2.5	0.981	$C_7' \to$ 0.029±0.015 $\Delta C_9 \to$ -1.4±0.14 $C_S' \to$ 0.028±0.019	0.38	−0.85	−1.04	1.25
47	213.65/208	37.9	2.	0.976	$C_7' \to$ 0.029±0.015 $\Delta C_9 \to$ -1.39±0.15 $\Delta C_{10} \to$ -0.042±0.117	0.38	−0.85	−1.04	1.25
48	213.74/208	37.8	1.9	0.976	$C_7' \to$ 0.027±0.015 $\Delta C_9 \to$ -1.37±0.14 $C_{10}' \to$ -0.024±0.111	0.36	−0.85	−1.04	1.24
46	213.77/208	37.7	1.9	0.976	$C_7' \to$ 0.027±0.015 $\Delta C_9 \to$ -1.37±0.14 $C_9' \to$ 0.017±0.217	0.37	−0.85	−1.04	1.24

backup:Results: Model Selection (New data)

TABLE IV: Fit-qualities, model selection criteria, parameter estimates and effects on radiative decays for the 'best' selected models with the 'New' data-set, with the 'Moments' estimate of the angular observables. Selected models are obtained from fig. 2a. Last four columns showcase the deviations (in units of 'σ') between the experimental value of the radiative decays and the corresponding value obtained with the fit results.

Model Index	$\chi^2_{\rm Min}$/ DOF	p-val (%)	$\omega^{\Delta{\rm AIC}_c}$ (%)	MSE$_{X-{\rm val}}$	Parameter Values	Deviation in σ			
						B→$X_s\gamma$	$B^+ \to K^*\gamma$	ΔB$^0 \to K^*\gamma$	ΔB$_s \to \phi\gamma$
18	250.28/256	58.9	3.4	0.917	$\Delta C_9 \to$ -1.13±0.13 $C_9' \to$ 0.25±0.17	–	–	–	–
2	252.44/257	56.9	3.2	0.933	$\Delta C_9 \to$ -1.12±0.13	–	–	–	–
76	249.12/255	59.2	2.2	0.918	$\Delta C_9 \to$ -1.18±0.14 $C_9' \to$ 0.34±0.19 $C_S \to$ -0.035±0.016	–	–	–	–
77	249.16/255	59.1	2.1	0.918	$\Delta C_9 \to$ -1.18±0.14 $C_9' \to$ 0.34±0.19 $C_S' \to$ 0.035±0.016	–	–	–	–
20	251.52/256	56.7	1.8	0.928	$\Delta C_9 \to$ -1.15±0.14 $C_{10}' \to$ -0.1±0.104	–	–	–	–
10	251.97/256	55.9	1.4	0.932	$C_7' \to$ 0.01±0.015 $\Delta C_9 \to$ -1.15±0.14	0.31	−0.87	−1.06	1.22
46	250.14/255	57.4	1.3	0.922	$C_7' \to$ 0.0058±0.0155 $\Delta C_9 \to$ -1.15±0.14 $C_9' \to$ 0.24±0.18	0.3	−0.87	−1.06	1.22
74	250.16/255	57.4	1.3	0.925	$\Delta C_9 \to$ -1.16±0.15 $C_9' \to$ 0.26±0.17 $\Delta C_{10} \to$ -0.041±0.118	–	–	–	–
75	250.21/255	57.3	1.2	0.923	$\Delta C_9 \to$ -1.12±0.14 $C_9' \to$ 0.3±0.26 $C_{10}' \to$ 0.04±0.157	–	–	–	–

TABLE V: Same as the table IV, but with the 'Likelihood' estimate of the angular observables. Selected models are obtained from fig. 2b.

Model Index	$\chi^2_{\rm Min}$/ DOF	p-val (%)	$\omega^{\Delta{\rm AIC}_c}$ (%)	MSE$_{X-{\rm val}}$	Parameter Values	Deviation in σ			
						B→$X_s\gamma$	$B^+ \to K^*\gamma$	ΔB$^0 \to K^*\gamma$	ΔB$_s \to \phi\gamma$
132	217.02/210	35.5	3.7	0.985	$C_7' \to$ 0.04±0.015 $\Delta C_9 \to$ -1.39±0.13 $C_9' \to$ 0.45±0.2 $C_S \to$ -0.042±0.013	0.44	−0.82	−1.02	1.27
133	217.07/210	35.4	3.6	0.986	$C_7' \to$ 0.04±0.015 $\Delta C_9 \to$ -1.39±0.13 $C_9' \to$ 0.45±0.2 $C_S' \to$ 0.042±0.013	0.44	−0.82	−1.02	1.27
130	217.58/210	34.5	2.8	0.976	$C_7' \to$ 0.044±0.015 $\Delta C_9 \to$ -1.42±0.14 $C_9' \to$ 0.32±0.19 $\Delta C_{10} \to$ -0.16±0.11	0.47	−0.81	−1.01	1.28
46	219.66/211	32.7	2.8	0.988	$C_7' \to$ 0.04±0.015 $\Delta C_9 \to$ -1.34±0.13 $C_9' \to$ 0.33±0.2	0.44	−0.82	−1.02	1.27
47	220.36/211	31.5	2.	0.995	$C_7' \to$ 0.048±0.015 $\Delta C_9 \to$ -1.43±0.15 $\Delta C_{10} \to$ -0.16±0.11	0.5	−0.8	−1.	1.29
10	222.46/212	29.7	1.9	1.001	$C_7' \to$ 0.043±0.015 $\Delta C_9 \to$ -1.33±0.13	0.46	−0.81	−1.01	1.28
257	216.47/209	34.7	1.7	0.99	$C_7' \to$ 0.042±0.015 $\Delta C_9 \to$ -1.42±0.14 $C_9' \to$ 0.41±0.21 $\Delta C_{10} \to$ -0.091±0.123 $C_S \to$ -0.036±0.017	0.46	−0.82	−1.01	1.28
258	216.51/209	34.6	1.7	0.99	$C_7' \to$ 0.042±0.015 $\Delta C_9 \to$ -1.42±0.14 $C_9' \to$ 0.41±0.21 $\Delta C_{10} \to$ -0.092±0.123 $C_S' \to$ 0.036±0.017	0.46	−0.82	−1.01	1.28