

# Dealing with CPT violation in mixing for neutral pseudoscalar meson decaying to two vectors

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# CPT Symmetry:

- \* **CPT invariance**  $\rightarrow$  a fundamental symmetry of nature.
- \* **CPT theorem:** Any quantum field theory involving point-particles in flat Minkowski space, delineated by Hermitian, local, Lorentz-invariant Lagrangian (or Hamiltonian), is certainly CPT invariant.

*~ G. Lüders, W. Pauli, R. Jost, J. Schwinger, R. Streater, A. Wightman, J.S. Bell*

- \* **Proof:**

- 1 Hamiltonian formalism of QFT.
- 2 Axiomatic formalism of QFT.

*~ G. Lüders [Ann. Phys. 2, 1–15 (1957)]*

*~ R. Jost, [Helv. Phys. Acta 30, 409 (1957)]*

# But!

- C,P & T were assumed to be conserved separately.

- ▢ **Parity violation was discovered.**

- ~ Lee & Yang [Phys. Rev. 104, 254 (1956)]; Wu et al. [Phys. Rev. 105, 1413 (1957)]*

- CP was assumed to be conserved.

- ▢ **CP violation was observed in neutral kaons.**

- ~ J. Cronin et al. [Phys. Rev. Lett. 13 (1964), pp. 138-140]*

- It's very important to test CPT invariance experimentally.

# CPT Violating Models:

- Non-point-like object     *~ V.A. Kostelecký & R. Potting [Nucl. Phys. B 359, 545–570 (1991)], ...*
- Lorentz violation             *~ D. Colladay & V.A. Kostelecký [Phys. Rev. D 55, 6760 (1997)], ...*
- Non-trivial space-time topology     *~ F.R. Klinkhamer [Nucl. Phys. B 578, 277–289(2000)], ...*
- Non-locality                     *~ M. Chaichian, et al. [Phys. Lett. B 699, 177–180 (2011)], ...*
- Non-point interactions             *~ P. Carruthers [Phys. Rev. Lett. 18, 353 (1967)], ...*
- Modified QM due to gravity             *~ S.W. Hawking, [Phys. Rev. D 14, 2460 (1976)], ...*
- Decoherence in QG             *~ N.E. Mavromatos [J. Phys. Conf. Ser. 873, 012006 (2017)], ...*
- Abelian CS like term     *~ C. Adam & F.R. Klinkhamer [Nucl. Phys. B 607, 247–267 (2001)], ...*
- Fields with infinite components     *~ E. Abers et al. [Phys. Rev. 159, 1222 (1967)], ...*
- More details                     *~ R. Lehnert [Symmetry 2016, 8, 114], ...*

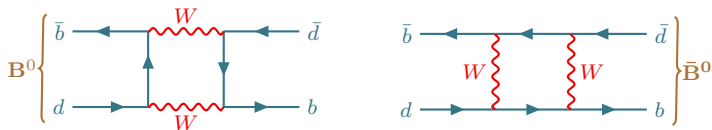
# Where To Look For CPT Violation?

- ⇒ Differences in masses and decay-widths of particle and anti-particle  
~ P.A. Zyla et al. (PDG) [*Prog. Theor. Exp. Phys.* 2020, 083C01 (2020)]
- ⇒ Astrophysical observations ~ J. D. Tasson [*Symmetry* 2016, 8, 111], ...
- ⇒ Neutrino sector ~ J. S. Díaz [*Symmetry* 2016, 8, 105], ...
- ⇒ Neutral meson mixing ~ L. Lavoura & J. P. Silva [*PR D* 60, 056003 (1999)], ...
- ⇒ Others

# Why Neutral Meson Mixing?

~ Lavoura, Silva [Phys. Rev. D 60, 056003 (1999)]

- ❖ CPT violating effects (if exists) must be tiny.
- ❖ Mixing of neutral pseudoscalar mesons ( $K^0, D^0, B_d^0, B_s^0$ ) with their own antiparticles.
- ❖ It is mediated by weak interaction.
- ❖ It is second order in  $\alpha_W$ .



- ❖ Generally semi-leptonic, two pseudoscalars and one pseudoscalar plus one vector decay-modes of neutral meson are considered to study CP and CPT violation.
- ❖ We have studied two vectors decay modes.

# Data On CPT Violation From Meson Mixing:

- ✳ **KTeV:** Semi-leptonic decays of  $K^0$  and  $\bar{K}^0$ ,  
 $Re(\delta) = (2.51 \pm 2.25) \times 10^{-4}$ ,  $Im(\delta) = (-1.5 \pm 1.6) \times 10^{-5}$ .  
~ E. Abouzaid et al., (KTeV Collaboration) [Phys. Rev. D 83, 092001 (2011)]
- ✳ **Focus:** Time dependent rates for  $D^0 \rightarrow K^- \pi^+$  and  $\bar{D}^0 \rightarrow K^+ \pi^-$ ,  
 $Re(\xi)y - Im(\xi)x = 0.0083 \pm 0.0065 \pm 0.0041$ ,  
 with  $x = \frac{\Delta M}{\Gamma}$  and  $y = \frac{\Delta \Gamma}{2\Gamma}$ .  
~ J.M. Link et al., (FOCUS Collaborations) [Phys. Lett. B 556, 7–13 (2003)]
- ✳ **Belle:** Rates for various hadronic and semileptonic modes of  $B_d^0$ ,  
 $Re(z) = (1.9 \pm 3.7 \pm 3.3) \times 10^{-2}$ ,  $Im(z) = (-5.7 \pm 3.3 \pm 3.3) \times 10^{-3}$ .  
~ T. Higuchi et al., (Belle Collaboration), [Phys. Rev. D 85, 071105 (2012)]
- ✳ **LHCb:** Time dependent decay rate for  $B_s^0 \rightarrow J/\psi K^+ K^-$ ,  
 $Re(z) = -0.022 \pm 0.033 \pm 0.005$ ,  $Im(z) = 0.004 \pm 0.011 \pm 0.002$ .  
~ R. Aaij et al., (LHCb Collaboration), [Phys. Rev. Lett. 116, 241601 (2016)]

# Why Two Vectors Decay Modes?

- \* A large number of observables can be constructed from time-dependent decay rates of these modes and they can be measured independently.
- \* Independent theoretical parameters are very less in number.
- \* Many relations among observables will emerge and their violation would indicate the type of new Physics scenario (if any).
- \* Due to these relations the CPT violating parameters can be expressed in several ways involving different observables. So, these parameters can be estimated multiple times from a single experiment. This will help to keep the errors low.
- \* Kaons cannot decay to two vector meson due to lack of phase space.
- \* Different modes like:  $D^0 \rightarrow K^{*+}K^{*-}$ ,  $K^{*\pm}\rho^\mp$ , etc;  $B_d^0 \rightarrow \phi\phi$ ,  $\omega\omega$ ,  $J/\psi K^*$ , etc;  $B_s^0 \rightarrow J/\psi\phi$ ,  $D_s^{*+}D_s^{*-}$ ,  $D_s^{*\pm}\rho^\mp$ , etc can be used.



## $P^0 - \bar{P}^0$ Mixing:

- ▶  $P^0$  and  $\bar{P}^0$  are *orthonormal* flavour eigenstates.

$$|P^0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\bar{P}^0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- ▶  $|\psi(t)\rangle = a(t)|P^0\rangle + b(t)|\bar{P}^0\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$

- ▶ Time evolution:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \mathcal{H} |\psi(t)\rangle$$

- ▶  $\mathcal{H}$  is *non-hermitian*. Since both  $P^0$  and  $\bar{P}^0$  are decaying, probability of finding them must not conserve.
- ▶ Find the time evolved states  $|P^0(t)\rangle$  and  $|\bar{P}^0(t)\rangle$ .

# $P^0 - \bar{P}^0$ Mixing:

- Any matrix can be written as sum of a hermitian and an anti-hermitian matrix.
- $\mathcal{H} = M - \frac{i}{2}\Gamma$   $M$  and  $\Gamma$  are  $2 \times 2$  hermitian matrices.



## Definition

Eigenvectors of  $\mathcal{H}$  :  $|B_L\rangle$  and  $|B_H\rangle \Rightarrow$  **Mass eigenstates**

Eigenvalues of  $\mathcal{H}$  :  $H_L$  and  $H_H$

$$H_L = M_L - \frac{i}{2}\Gamma_L, \quad H_H = M_H - \frac{i}{2}\Gamma_H$$

$$M = \frac{1}{2}(M_L + M_H), \quad \Gamma = \frac{1}{2}(\Gamma_L + \Gamma_H)$$

$$\Delta M = (M_H - M_L), \quad \Delta\Gamma = (\Gamma_H - \Gamma_L)$$

- As  $\mathcal{H}$  is non-hermitian,  $\langle B_H|B_L\rangle \neq 0$  in general.

# Mixing Parameters:

~ CP Violation by Bigi & Sanda

✱  $\mathcal{H} = M - \frac{i}{2}\Gamma = E \sin \theta \cos \phi \sigma_1 + E \sin \theta \sin \phi \sigma_2 + E \cos \theta \sigma_3 - iD I_{2 \times 2}$   
 where,  $E$ ,  $\theta$ ,  $\phi$  and  $D$  are complex parameters.

✱  $E \sin \theta \cos \phi = \text{Re}(M_{12}) - \frac{i}{2} \text{Re}(\Gamma_{12})$   
 $E \sin \theta \sin \phi = -\text{Im}(M_{12}) + \frac{i}{2} \text{Im}(\Gamma_{12})$   
 $E \cos \theta = \frac{1}{2}(M_{11} - M_{22}) - \frac{i}{4}(\Gamma_{11} - \Gamma_{22})$   
 $D = \frac{i}{2}(M_{11} + M_{22}) + \frac{1}{4}(\Gamma_{11} + \Gamma_{22})$

✱  $|B_L\rangle = p_1 |P^0\rangle + q_1 |\bar{P}^0\rangle$ ,  $|B_H\rangle = p_2 |P^0\rangle - q_2 |\bar{P}^0\rangle$

$p_1 = N_1 \cos \frac{\theta}{2}$ ,  $q_1 = N_1 e^{i\phi} \sin \frac{\theta}{2}$ ;  $p_2 = N_2 \sin \frac{\theta}{2}$ ,  $q_2 = N_2 e^{i\phi} \cos \frac{\theta}{2}$

$H_L = E - iD$ ,  $H_H = -E - iD$

✱ So,  $\theta$  and  $\phi$  are mixing parameters.

# T And CPT Theorems:

~ Particle Physics And Introduction To Field Theory by T. D. Lee

- \* *CPT* invariance (independent of *T* invariance) implies

$$M_{11} = M_{22} \ \& \ \Gamma_{11} = \Gamma_{22} \implies \theta = \pi/2$$

$\implies \text{Re}(\theta) - \pi/2$  and  $\text{Im}(\theta)$  are *CPT* violating parameters.

- \* *T* invariance (independent of *CPT* invariance) implies

$$\frac{\Gamma_{12}^*}{M_{12}^*} = \frac{\Gamma_{12}}{M_{12}} \implies \text{Im}(\phi) = 0$$

$\implies \text{Im}(\phi)$  is *T* violating parameter.

- \* *CPT* and *T* violation  $\implies$ 

$$\theta = \pi/2 + \epsilon_1 + i\epsilon_2$$

$$\phi = -2\beta + i\epsilon_3$$

where  $\beta$  is CP violating weak phase involving  $P^0 - \bar{P}^0$  mixing.

~ A. Karan et al. [Phys. Lett. B 781, 459–463 (2018)]

## $(P^0, \bar{P}^0) \rightarrow V_1 V_2$ Transition:

$$\otimes (P^0, \bar{P}^0) \xrightarrow[\text{be violated}]{\mathcal{H} : \text{CPT might}} (P^0(t), \bar{P}^0(t)) \xrightarrow[\text{conserved}]{H_D : \text{CPT}} f$$

$$\otimes \text{Transition amplitudes: } \mathcal{A}_f = \langle f | H_D | P^0 \rangle, \quad \bar{\mathcal{A}}_f = \langle f | H_D | \bar{P}^0 \rangle$$

- $\otimes$  If  $f = V_1 V_2$ , then there exist three polarizations for final state  $\{0, \perp, \parallel\}$  depending on the orbital angular momentum quantum number  $\{0, 1, 2\}$ .

$$\otimes \mathcal{A}_f(P^0 \rightarrow V_1 V_2) = A_0 g_0 + A_{\parallel} g_{\parallel} + i A_{\perp} g_{\perp},$$

$$\bar{\mathcal{A}}_f(\bar{P}^0 \rightarrow V_1 V_2) = \bar{A}_0 g_0 + \bar{A}_{\parallel} g_{\parallel} - i \bar{A}_{\perp} g_{\perp},$$

where,  $A_{\lambda}$  and  $\bar{A}_{\lambda}$  with  $\lambda \in \{0, \perp, \parallel\}$  are transversity amplitudes in linear polarization basis and  $g_{\lambda}$  depend on kinematic angles only.

*~ D. London, N. Sinha, R. Sinha [Phys. Rev. Lett. 85, 1807–1810 (2000)]*

$$\otimes \text{Find } |\langle f | H_D | P^0(t) \rangle|^2 \text{ and } |\langle f | H_D | \bar{P}^0(t) \rangle|^2$$

# $P^0 \rightarrow V_1 V_2$ Observables:

Mode

$$\begin{aligned} \frac{d\Gamma}{dt}(P^0(t) \rightarrow f) = & \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma t}{2}\right) \left\{ (1 + |\cos\theta|^2) |\mathcal{A}_f|^2 + |e^{i\phi} \sin\theta|^2 |\bar{\mathcal{A}}_f|^2 \right. \right. \\ & \left. \left. + 2\text{Re}\left(e^{i\phi} \cos\theta^* \sin\theta \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} + \sinh\left(\frac{\Delta\Gamma t}{2}\right) \left\{ 2\text{Re}\left(\cos\theta |\mathcal{A}_f|^2 + e^{i\phi} \sin\theta \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} \right. \\ & \left. + \cos(\Delta Mt) \left\{ (1 - |\cos\theta|^2) |\mathcal{A}_f|^2 - |e^{i\phi} \sin\theta|^2 |\bar{\mathcal{A}}_f|^2 - 2\text{Re}\left(e^{i\phi} \cos\theta^* \sin\theta \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} \right. \\ & \left. - \sin(\Delta Mt) \left\{ 2\text{Im}\left(\cos\theta |\mathcal{A}_f|^2 + e^{i\phi} \sin\theta \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} \right] \end{aligned}$$

$$\frac{d\Gamma}{dt}(P^0(t) \rightarrow V_1 V_2) =$$

$$e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left[ \Lambda_{\lambda\sigma} \cosh\left(\frac{\Delta\Gamma t}{2}\right) + \eta_{\lambda\sigma} \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \Sigma_{\lambda\sigma} \cos(\Delta Mt) - \rho_{\lambda\sigma} \sin(\Delta Mt) \right] g_{\lambda} g_{\sigma}$$

- ◆ Here,  $\Lambda$ ,  $\eta$ ,  $\rho$ ,  $\Sigma$  are observables for the mode.
- ◆ Let's denote them as  $\mathcal{O}(A_\lambda, \bar{A}_\sigma, \epsilon_j)$ .
- ◆ Number of observables =  $4 \times 6 = 24$ .
- ◆ If  $\Delta\Gamma \rightarrow 0$ , then  $\eta_{\lambda\sigma}$  cannot be measured and one has to work with rest of the 18 observables.

# $\bar{P}^0 \rightarrow V_1 V_2$ Observables:

## Conjugate Mode

$$\begin{aligned} \frac{d\Gamma}{dt}(\bar{P}^0(t) \rightarrow f) &= \frac{e^{-\Gamma t}}{2} \left[ \cosh\left(\frac{\Delta\Gamma t}{2}\right) \left\{ (1 + |\cos\theta|^2) |\bar{\mathcal{A}}_f|^2 + |e^{-i\phi} \sin\theta|^2 |\mathcal{A}_f|^2 \right. \right. \\ &\quad \left. \left. - 2\text{Re}\left(e^{i\phi^*} \cos\theta \sin\theta^* \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} + \sinh\left(\frac{\Delta\Gamma t}{2}\right) \left\{ 2\text{Re}\left(-\cos\theta^* |\bar{\mathcal{A}}_f|^2 + e^{i\phi^*} \sin\theta^* \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} \right. \\ &\quad \left. + \cos(\Delta Mt) \left\{ (1 - |\cos\theta|^2) |\bar{\mathcal{A}}_f|^2 - |e^{-i\phi} \sin\theta|^2 |\mathcal{A}_f|^2 + 2\text{Re}\left(e^{i\phi^*} \cos\theta \sin\theta^* \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} \right. \\ &\quad \left. + \sin(\Delta Mt) \left\{ 2\text{Im}\left(-\cos\theta^* |\bar{\mathcal{A}}_f|^2 + e^{i\phi^*} \sin\theta^* \mathcal{A}_f^* \bar{\mathcal{A}}_f\right) \right\} \right] \end{aligned}$$

$$\frac{d\Gamma}{dt}(\bar{P}^0(t) \rightarrow V_1 V_2) =$$

$$e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left[ \bar{\Lambda}_{\lambda\sigma} \cosh\left(\frac{\Delta\Gamma t}{2}\right) + \bar{\eta}_{\lambda\sigma} \sinh\left(\frac{\Delta\Gamma t}{2}\right) + \bar{\Sigma}_{\lambda\sigma} \cos(\Delta Mt) + \bar{\rho}_{\lambda\sigma} \sin(\Delta Mt) \right] g_{\lambda} g_{\sigma}$$

- ◆ Here,  $\bar{\Lambda}$ ,  $\bar{\eta}$ ,  $\bar{\rho}$ ,  $\bar{\Sigma}$  are observables for the mode.
- ◆ Let's denote them as  $\bar{\mathcal{O}}(A_{\lambda}, \bar{A}_{\sigma}, \epsilon_j)$ .
- ◆ Number of observables =  $4 \times 6 = 24$ .
- ◆ For  $\Delta\Gamma \rightarrow 0$ ,  $\bar{\eta}_{\lambda\sigma}$  also cannot be measured and one should work with remaining 18 observables only.

## Unknown Parameters To Solve:

- ✓  $\mathcal{O} = \mathcal{O}(A_\lambda, \bar{A}_\sigma, \epsilon_j)$  and  $\bar{\mathcal{O}} = \bar{\mathcal{O}}(A_\lambda, \bar{A}_\sigma, \epsilon_j)$   
where  $(\lambda, \sigma) \in \{0, \perp, \parallel\}$  and  $j \in \{1, 2, 3\}$ .
- ✓ Fourteen theoretical parameters to solve for: 3 of  $|A_\lambda|$ , 3 of  $|\bar{A}_\lambda|$ , 3 of  $\epsilon_j$  and 5 phases of transversity amplitudes relative to  $A_\perp$ .
- ✓ Instead of 5 relative phases we use 5 other angular quantities for convenience: 3 of  $\varphi_\lambda^{meas}$  and 2 of  $\omega_{\perp i}$  ( $i \in \{0, \parallel\}$ ), defined as:  

$$\varphi_\lambda^{meas} = -2\beta + \text{Arg}[\bar{A}_\lambda] - \text{Arg}[A_\lambda], \quad \omega_{\perp i} = \text{Arg}[A_\perp] - \text{Arg}[A_i].$$
- ✓ The parameter  $\beta$  cannot be measured in general. It can only be measured if there exists no direct CP violation.



## Solutions:

$$\Rightarrow |A_\lambda| = \sqrt{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}} \quad \text{and} \quad |\bar{A}_\lambda| = \sqrt{\bar{\Lambda}_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda}},$$

$$\Rightarrow \sin^3 \Phi_\lambda^{meas} - \left( \frac{\rho_{\lambda\lambda}^r + \bar{\rho}_{\lambda\lambda}^r}{\sqrt{1-C_\lambda^2}} \right) \sin^2 \Phi_\lambda^{meas} \\ + \left( \frac{C_\lambda^2 - \Sigma_{\lambda\lambda}^r - \bar{\Sigma}_{\lambda\lambda}^r}{1-C_\lambda^2} \right) \sin \Phi_\lambda^{meas} + \frac{C_\lambda}{\sqrt{1-C_\lambda^2}} \left[ \frac{\rho_{\lambda\lambda}^r}{1+C_\lambda} - \frac{\bar{\rho}_{\lambda\lambda}^r}{1-C_\lambda} \right] = 0,$$

$$\Rightarrow \epsilon_1 = - \left( \frac{2}{\sin 2\varphi_\lambda^{meas}} \right) \left[ \frac{\bar{\rho}_{\lambda\lambda}^r}{1-C_\lambda} - \frac{\rho_{\lambda\lambda}^r}{1+C_\lambda} - \frac{\Sigma_{\lambda\lambda}^r - \bar{\Sigma}_{\lambda\lambda}^r}{\sqrt{1-C_\lambda^2}} \sin \Phi_\lambda^{meas} \right],$$

$$\Rightarrow \epsilon_2 = -(\rho_{\lambda\lambda}^r + \bar{\rho}_{\lambda\lambda}^r) + \sqrt{1-C_\lambda^2} \sin \Phi_\lambda^{meas},$$

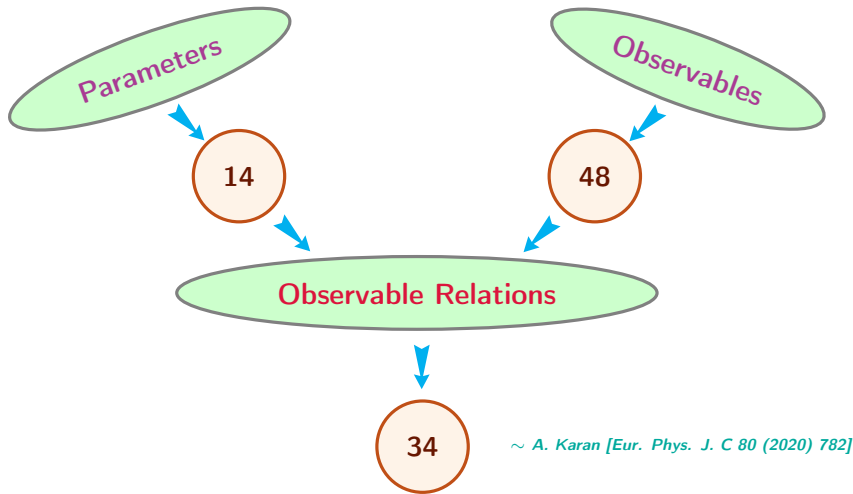
$$\Rightarrow \epsilon_3 = -C_\lambda - \csc \Phi_\lambda^{meas} \left[ \rho_{\lambda\lambda}^r \sqrt{\frac{1-C_\lambda}{1+C_\lambda}} - \bar{\rho}_{\lambda\lambda}^r \sqrt{\frac{1+C_\lambda}{1-C_\lambda}} \right],$$

$$\Rightarrow \omega_{\perp i} = \sin^{-1} \left( - \frac{\Lambda_{\perp i} + \Sigma_{\perp i}}{2\sqrt{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{ii} + \Sigma_{ii})}} \right),$$

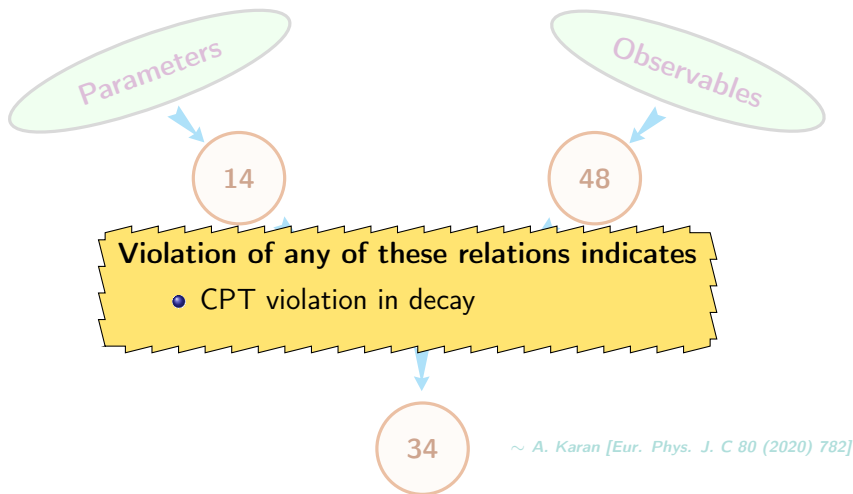
$$\text{Here,} \quad \Phi_i^{meas} = \varphi_i^{meas} \quad \text{and} \quad \Phi_{\perp}^{meas} = -\varphi_{\perp}^{meas},$$

$$C_\lambda = \frac{(\Lambda_{\lambda\lambda} - \bar{\Lambda}_{\lambda\lambda} + \Sigma_{\lambda\lambda} - \bar{\Sigma}_{\lambda\lambda})}{(\Lambda_{\lambda\lambda} + \bar{\Lambda}_{\lambda\lambda} + \Sigma_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda})} \quad \text{and} \quad Y_{\lambda\lambda}^r = \frac{Y_{\lambda\lambda}}{(\Lambda_{\lambda\lambda} + \bar{\Lambda}_{\lambda\lambda} + \Sigma_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda})}.$$

# Independent Relations Among Observables:



# Independent Relations Among Observables:



## Special Case I :

## Standard Model Scenario

- ~ *D. London, N. Sinha, and R. Sinha [Phys. Rev. D 69, 114013 (2004)],*
- ~ *A. Karan and A. K. Nayak [Phys. Rev. D 101 (2020) 015027],*
- ~ *A. Karan [Eur. Phys. J. C 80 (2020) 782].*

# Solutions To The Unknown Parameters:

## → Constrains:

- ①  $A_\lambda = \bar{A}_\lambda \forall \lambda \in \{0, \perp, \parallel\}$ ,
- ②  $\epsilon_j = 0 \forall j \in \{1, 2, 3\}$ .

## → Unknown parameters: 6;

three of  $|A_\lambda|$ , two of  $\omega_{\perp i}$  ( $i \in \{0, \parallel\}$ ) and  $\beta (\equiv \varphi_\lambda^{meas})$ .

## → Solutions:

- ①  $|A_\lambda| = \sqrt{\Lambda_{\lambda\lambda}}$ ,
- ②  $\sin \omega_{\perp i} = - \left( \frac{\Sigma_{\perp i}}{2\sqrt{\Lambda_{\perp\perp} \Lambda_{ii}}} \right)$ ,
- ③  $\sin 2\beta = - \left( \frac{\rho_{00}}{\Lambda_{00}} \right)$ .

## → Independent relations: $48 - 6 = 42$ .

# Independent Relations Among Observables:

$$\textcircled{1} \quad \Lambda_{\lambda\sigma} = \bar{\Lambda}_{\lambda\sigma}, \quad \eta_{\lambda\sigma} = \bar{\eta}_{\lambda\sigma}, \quad \Sigma_{\lambda\sigma} = -\bar{\Sigma}_{\lambda\sigma}, \quad \rho_{\lambda\sigma} = \bar{\rho}_{\lambda\sigma}, \quad \textcircled{24}$$

$$\textcircled{2} \quad \Sigma_{\lambda\lambda} = 0, \quad \Sigma_{\parallel 0} = 0, \quad \Lambda_{\perp i} = 0, \quad \textcircled{06}$$

$$\textcircled{3} \quad \frac{\rho_{ii}}{\Lambda_{ii}} = -\frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}} = \frac{\rho_{\parallel 0}}{\Lambda_{\parallel 0}}, \quad \frac{\eta_{ii}}{\Lambda_{ii}} = -\frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp}} = \frac{\eta_{\parallel 0}}{\Lambda_{\parallel 0}}, \quad \textcircled{06}$$

$$\textcircled{4} \quad \frac{\eta_{\perp i}}{\rho_{\perp i}} + \frac{\eta_{\parallel 0}}{\rho_{\parallel 0}} = 0, \quad \eta_{\parallel 0}^2 + \rho_{\parallel 0}^2 = \Lambda_{\parallel 0}^2, \quad \textcircled{03}$$

$$\textcircled{5} \quad \frac{\rho_{\perp i}^2}{4\Lambda_{\perp\perp}\Lambda_{ii} - \Sigma_{\perp i}^2} = \frac{\Lambda_{00}^2 - \rho_{00}^2}{\Lambda_{00}^2}, \quad \textcircled{02}$$

$$\textcircled{6} \quad \Lambda_{\parallel 0} = \frac{1}{2\Lambda_{\perp\perp}} \left[ \Sigma_{\perp 0}\Sigma_{\perp\parallel} + \rho_{\perp 0}\rho_{\perp\parallel} \left( \frac{\Lambda_{00}^2}{\Lambda_{00}^2 - \rho_{00}^2} \right) \right]. \quad \textcircled{01}$$

Take  $(\lambda, \sigma) \in \{0, \perp, \parallel\}$  and  $i \in \{0, \parallel\}$ .

# Independent Relations Among Observables:

$$\textcircled{1} \quad \Lambda_{\lambda\sigma} = \bar{\Lambda}_{\lambda\sigma}, \quad \eta_{\lambda\sigma} = \bar{\eta}_{\lambda\sigma}, \quad \Sigma_{\lambda\sigma} = -\bar{\Sigma}_{\lambda\sigma}, \quad \rho_{\lambda\sigma} = \bar{\rho}_{\lambda\sigma},$$

24

$$\textcircled{2} \quad \Sigma_{\lambda\lambda} = 0, \quad \Sigma_{\parallel 0} = 0, \quad \Lambda_{\perp i} = 0,$$

06

$$\textcircled{3} \quad \frac{\rho_{ii}}{\Lambda_{ii}} = -$$

**Violation of any of these relations indicates**

06

$$\textcircled{4} \quad \frac{\eta_{\perp i}}{\rho_{\perp i}} + \frac{\eta_{\parallel i}}{\rho_{\parallel i}}$$

- Direct CP violation
- CPT violation in decay
- T & CPT violation in mixing

03

$$\textcircled{5} \quad \frac{\rho_{\perp i}^2}{4\Lambda_{\perp\perp}\Lambda_{ii} - \Sigma_{\perp i}^2} = \frac{\Lambda_{00}^2}{\Lambda_{00}^2 - \rho_{00}^2},$$

02

$$\textcircled{6} \quad \Lambda_{\parallel 0} = \frac{1}{2\Lambda_{\perp\perp}} \left[ \Sigma_{\perp 0}\Sigma_{\perp\parallel} + \rho_{\perp 0}\rho_{\perp\parallel} \left( \frac{\Lambda_{00}^2}{\Lambda_{00}^2 - \rho_{00}^2} \right) \right].$$

01

Take  $(\lambda, \sigma) \in \{0, \perp, \parallel\}$  and  $i \in \{0, \parallel\}$ .

# Special Case II :

# SM Plus Direct CP Violation

~ A. Karan [*Eur. Phys. J. C* 80 (2020) 782].



# Solutions To The Unknown Parameters:

## → Constrains:

$$\textcircled{1} \quad \epsilon_j = 0 \quad \forall j \in \{1, 2, 3\}.$$

## → Unknown parameters: 11;

three of  $|A_\lambda|$ , three of  $|\bar{A}_\lambda|$ , three of  $\varphi_\lambda^{meas}$ , two of  $\omega_{\perp i}$  ( $i \in \{0, ||\}$ ).

## → Solutions:

$$\textcircled{1} \quad |A_\lambda| = \sqrt{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}},$$

$$\textcircled{2} \quad |\bar{A}_\lambda| = \sqrt{\Lambda_{\lambda\lambda} - \Sigma_{\lambda\lambda}},$$

$$\textcircled{3} \quad \sin \Phi_\lambda^{meas} = \frac{\rho_{\lambda\lambda}}{\sqrt{\Lambda_{\lambda\lambda}^2 - \Sigma_{\lambda\lambda}^2}},$$

$$\textcircled{4} \quad \sin \omega_{\perp i} = - \left( \frac{\Lambda_{\perp i} + \Sigma_{\perp i}}{2\sqrt{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{ii} + \Sigma_{ii})}} \right).$$

## → Independent relations: $48 - 11 = 37$ .

# Independent Relations Among Observables:

$$\textcircled{1} \quad \Lambda_{\lambda\sigma} = \bar{\Lambda}_{\lambda\sigma}, \quad \eta_{\lambda\sigma} = \bar{\eta}_{\lambda\sigma}, \quad \Sigma_{\lambda\sigma} = -\bar{\Sigma}_{\lambda\sigma}, \quad \rho_{\lambda\sigma} = \bar{\rho}_{\lambda\sigma},$$

24

$$\textcircled{2} \quad \Lambda_{\lambda\lambda} = \sqrt{\Sigma_{\lambda\lambda}^2 + \rho_{\lambda\lambda}^2 + \eta_{\lambda\lambda}^2},$$

03

$$\textcircled{3} \quad \left[ \frac{\left(\frac{\rho_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right) + \left(\frac{\rho_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) - 2\left(\frac{\rho_{\lambda\sigma}}{\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}}\right)}{\left(\frac{\eta_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right) + \left(\frac{\eta_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) - 2\left(\frac{\eta_{\lambda\sigma}}{\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}}\right)} \right] = - \left[ \frac{\left(\frac{\eta_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) - \left(\frac{\eta_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right)}{\left(\frac{\rho_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) - \left(\frac{\rho_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right)} \right],$$

03

$$\textcircled{4} \quad 4 \left[ \frac{(\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda})(\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma})}{(\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma})^2} \right] - \left[ \frac{\left(\frac{\rho_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) + \left(\frac{\rho_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right) - 2\left(\frac{\rho_{\lambda\sigma}}{\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}}\right)}{\left(\frac{\eta_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) - \left(\frac{\eta_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right)} \right]^2 = 1,$$

03

$$\textcircled{5} \quad \left[ 4 \left( \frac{\eta_{\lambda\lambda} \rho_{\sigma\sigma} - \eta_{\sigma\sigma} \rho_{\lambda\lambda}}{\Lambda_{\lambda\sigma}^2 - \Sigma_{\lambda\sigma}^2} \right) + \left\{ \frac{\left(\frac{\rho_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) + \left(\frac{\rho_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right) - 2\left(\frac{\rho_{\lambda\sigma}}{\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}}\right)}{\left(\frac{\eta_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}\right) - \left(\frac{\eta_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}}\right)} \right\}^2 \right] \\ = 4 \left[ \frac{(\Lambda_{\lambda\lambda} - \Sigma_{\lambda\lambda})(\Lambda_{\sigma\sigma} - \Sigma_{\sigma\sigma})}{(\Lambda_{\lambda\sigma} - \Sigma_{\lambda\sigma})^2} \right] - 1,$$

03

$$\textcircled{6} \quad \left[ \frac{(\Lambda_{\parallel 0} + \Sigma_{\parallel 0})^2}{(\Lambda_{00} + \Sigma_{00})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})} \right] - \left[ \frac{(\Lambda_{\perp 0} + \Sigma_{\perp 0})(\Lambda_{\perp\parallel} + \Sigma_{\perp\parallel})(\Lambda_{\parallel 0} + \Sigma_{\parallel 0})}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{00} + \Sigma_{00})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})} \right] \\ = 4 - \left[ \frac{(\Lambda_{\perp 0} + \Sigma_{\perp 0})^2}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{00} + \Sigma_{00})} \right] - \left[ \frac{(\Lambda_{\perp\parallel} + \Sigma_{\perp\parallel})^2}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})} \right].$$

01

Here,  $(\lambda, \sigma) \in \{0, \perp, \parallel\}$ . In third, fourth and fifth equations  $\lambda \neq \sigma$ .

# Independent Relations Among Observables:

$$\textcircled{1} \quad \Lambda_{\lambda\sigma} = \bar{\Lambda}_{\lambda\sigma}, \quad \eta_{\lambda\sigma} = \bar{\eta}_{\lambda\sigma}, \quad \Sigma_{\lambda\sigma} = -\bar{\Sigma}_{\lambda\sigma}, \quad \rho_{\lambda\sigma} = \bar{\rho}_{\lambda\sigma},$$

24

$$\textcircled{2} \quad \Lambda_{\lambda\lambda} = \sqrt{\Sigma_{\lambda\lambda}^2 + \rho_{\lambda\lambda}^2 + \eta_{\lambda\lambda}^2},$$

03

$$\textcircled{3} \quad \left[ \frac{\left( \frac{\rho_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}} \right) + \left( \frac{\rho_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}} \right) - 2 \left( \frac{\rho_{\lambda\sigma}}{\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma}} \right)}{\left( \frac{\eta_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}} \right)} \right] \left[ \left( \frac{\eta_{\lambda\lambda}}{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}} \right) - \left( \frac{\eta_{\sigma\sigma}}{\Lambda_{\sigma\sigma} + \Sigma_{\sigma\sigma}} \right) \right]$$

03

$$\textcircled{4} \quad 4 \left[ \frac{(\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda})}{(\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma})} \right]$$

**Violation of any of these relations indicates**

- CPT violation in decay
- T & CPT violation in mixing

03

$$\textcircled{5} \quad 4 \left( \frac{\eta_{\lambda\lambda} \rho_{\sigma\sigma}}{\Lambda_{\lambda\lambda}^2} \right)$$

03

$$\textcircled{6} \quad \left[ \frac{(\Lambda_{\parallel 0} + \Sigma_{\parallel 0})^2}{(\Lambda_{00} + \Sigma_{00})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})} \right] - \left[ \frac{(\Lambda_{\perp 0} + \Sigma_{\perp 0})(\Lambda_{\perp\parallel} + \Sigma_{\perp\parallel})(\Lambda_{\parallel 0} + \Sigma_{\parallel 0})}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{00} + \Sigma_{00})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})} \right]$$

$$= 4 - \left[ \frac{(\Lambda_{\perp 0} + \Sigma_{\perp 0})^2}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{00} + \Sigma_{00})} \right] - \left[ \frac{(\Lambda_{\perp\parallel} + \Sigma_{\perp\parallel})^2}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})} \right].$$

01

Here,  $(\lambda, \sigma) \in \{0, \perp, \parallel\}$ . In third, fourth and fifth equations  $\lambda \neq \sigma$ .

Special Case III :

## SM Plus T and CPT Violations In Mixing

~ A. Karan and A. K. Nayak [*Phys. Rev. D* 101 (2020) 015027],

~ A. Karan [*Eur. Phys. J. C* 80 (2020) 782].

# Solutions To The Unknown Parameters:

## ➔ Constrains:

$$\textcircled{1} A_\lambda = \bar{A}_\lambda \forall \lambda \in \{0, \perp, \parallel\}.$$

## ➔ Unknown parameters: 9;

three of  $|A_\lambda|$ , two of  $\omega_{\perp i}$  ( $i \in \{0, \parallel\}$ ), three of  $\epsilon_j$  ( $j \in \{1, 2, 3\}$ ) and  $\beta (\equiv \varphi_\lambda^{meas})$ .

## ➔ Solutions:

$$\textcircled{1} |A_\lambda| = \sqrt{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}},$$

$$\textcircled{4} \epsilon_1 = -\frac{1}{2} \left( \frac{\eta_{00}}{\Lambda_{00} + \Sigma_{00}} + \frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}} \right),$$

$$\textcircled{2} \sin 2\beta = -\frac{1}{2} \left( \frac{\rho_{00}}{\Lambda_{00}} - \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}} \right),$$

$$\textcircled{5} \epsilon_2 = -\frac{1}{2} \left( \frac{\rho_{00}}{\Lambda_{00} + \Sigma_{00}} + \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}} \right),$$

$$\textcircled{3} \epsilon_3 = \frac{1}{2} \left( \frac{\Sigma_{00}}{\Lambda_{00} + \Sigma_{00}} + \frac{\Sigma_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}} \right),$$

$$\textcircled{6} \sin \omega_{\perp i} = - \left( \frac{\Lambda_{\perp i} + \Sigma_{\perp i}}{2\sqrt{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})(\Lambda_{ii} + \Sigma_{ii})}} \right).$$

## ➔ Independent relations: $48 - 9 = 39$ .

# Independent Relations Among Observables:

$$\textcircled{1} \quad \frac{\Lambda_{0\parallel}}{\Lambda_{ii}} = \frac{\Sigma_{0\parallel}}{\Sigma_{ii}} = \frac{\rho_{0\parallel}}{\rho_{ii}} = \frac{\eta_{0\parallel}}{\eta_{ii}}, \quad \textcircled{06}$$

$$\textcircled{2} \quad \frac{\rho_{0\parallel}^2 + \eta_{0\parallel}^2}{\Lambda_{0\parallel}^2} = \frac{\rho_{\perp\perp}^2 + \eta_{\perp\perp}^2}{\Lambda_{\perp\perp}^2}, \quad \textcircled{01}$$

$$\textcircled{3} \quad \eta_{\perp i} = \frac{1}{2} \left[ \frac{\Sigma_{\perp i}}{\Lambda_{ii} \Lambda_{\perp\perp}} \left\{ \eta_{\perp\perp} (\Lambda_{ii} + \Sigma_{ii}) + \eta_{ii} (\Lambda_{\perp\perp} + \Sigma_{\perp\perp}) \right\} + X_i \left\{ \Lambda_{\perp\perp} \rho_{ii} - \Lambda_{ii} \rho_{\perp\perp} \right\} \right], \quad \textcircled{02}$$

$$\textcircled{4} \quad \rho_{\perp i} = \frac{1}{2} \left[ \frac{\Sigma_{\perp i}}{\Lambda_{ii} \Lambda_{\perp\perp}} \left\{ \rho_{\perp\perp} (\Lambda_{ii} + \Sigma_{ii}) + \rho_{ii} (\Lambda_{\perp\perp} + \Sigma_{\perp\perp}) \right\} - X_i \left\{ \Lambda_{\perp\perp} \eta_{ii} - \Lambda_{ii} \eta_{\perp\perp} \right\} \right], \quad \textcircled{02}$$

$$\textcircled{5} \quad \left[ \frac{(\Lambda_{\perp i} + \Sigma_{\perp i})^2}{(\Lambda_{ii} + \Sigma_{ii})(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})} \right] + 4X_i^2 \Lambda_{ii}^2 \Lambda_{\perp\perp}^2 \left[ \frac{(\Lambda_{ii} + \Sigma_{ii})(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})}{(\Lambda_{\perp\perp} \Sigma_{ii} + \Lambda_{ii} \Sigma_{\perp\perp} + 2\Lambda_{\perp\perp} \Lambda_{ii})^2} \right] = 4, \quad \textcircled{02}$$

$$\textcircled{6} \quad \text{Replace every observable } \vartheta \text{ by } \bar{\vartheta} \text{ in all the equations above.} \quad \textcircled{13}$$

$$\text{Here, } i \in \{0, \parallel\} \text{ and } X_i = \left[ \frac{(\Lambda_{\perp i} - \Sigma_{\perp i})(\Lambda_{\perp\perp} \Sigma_{ii} + \Lambda_{ii} \Sigma_{\perp\perp}) + 2(\Lambda_{ii} \Lambda_{\perp\perp} \Lambda_{\perp i} - \Sigma_{ii} \Sigma_{\perp\perp} \Sigma_{\perp i})}{(\eta_{\perp\perp} \rho_{ii} - \eta_{ii} \rho_{\perp\perp})(\Lambda_{ii} + \Sigma_{ii})(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})} \right].$$

# Independent Relations Among Observables:

$$\textcircled{7} \quad \Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda} = \bar{\Lambda}_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda}, \quad \Lambda_{\perp i} + \Sigma_{\perp i} = -(\bar{\Lambda}_{\perp i} + \bar{\Sigma}_{\perp i}), \quad \Lambda_{\parallel 0} + \Sigma_{\parallel 0} = \bar{\Lambda}_{\parallel 0} + \bar{\Sigma}_{\parallel 0}, \quad \textcircled{06}$$

$$\textcircled{8} \quad \left( \frac{\rho_{00}}{\Lambda_{00}} - \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}} \right) = \left( \frac{\bar{\rho}_{00}}{\Lambda_{00}} - \frac{\bar{\rho}_{\perp\perp}}{\Lambda_{\perp\perp}} \right), \quad \left( \frac{\eta_{00}}{\Lambda_{00}} - \frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp}} \right) = \left( \frac{\bar{\eta}_{00}}{\Lambda_{00}} - \frac{\bar{\eta}_{\perp\perp}}{\Lambda_{\perp\perp}} \right), \quad \textcircled{02}$$

$$\textcircled{9} \quad \left( \frac{\rho_{00} - \bar{\rho}_{00}}{\Lambda_{00} + \Sigma_{00}} \right) + \left( \frac{\rho_{\perp\perp} - \bar{\rho}_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}} \right) = 0, \quad \left( \frac{\eta_{00} - \bar{\eta}_{00}}{\Lambda_{00} + \Sigma_{00}} \right) + \left( \frac{\eta_{\perp\perp} + \bar{\eta}_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}} \right) = 0 \quad \textcircled{02}$$

$$\textcircled{10} \quad \left( \frac{\Sigma_{00} + \bar{\Sigma}_{00}}{\Lambda_{00} + \Sigma_{00}} \right) + \left( \frac{\Sigma_{\perp\perp} + \bar{\Sigma}_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}} \right) = 0, \quad \left( \frac{\rho_{00}}{\Lambda_{00}} - \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}} \right)^2 + \left( \frac{\eta_{00}}{\Lambda_{00}} - \frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp}} \right)^2 = 4, \quad \textcircled{02}$$

$$\textcircled{11} \quad \left( \Lambda_{0\parallel} + \Sigma_{0\parallel} \right) - \frac{1}{2} \left[ \frac{(\Lambda_{\perp 0} + \Sigma_{\perp 0})(\Lambda_{\perp\parallel} + \Sigma_{\perp\parallel})}{(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})} \right] \\ = \left[ \frac{2X_0 X_{\parallel} \Lambda_{00} \Lambda_{\parallel\parallel} \Lambda_{\perp\perp}^2 (\Lambda_{00} + \Sigma_{00})(\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel})(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})}{(\Lambda_{\perp\perp} \Sigma_{00} + \Lambda_{00} \Sigma_{\perp\perp} + 2\Lambda_{\perp\perp} \Lambda_{00})(\Lambda_{\perp\perp} \Sigma_{\parallel\parallel} + \Lambda_{\parallel\parallel} \Sigma_{\perp\perp} + 2\Lambda_{\perp\perp} \Lambda_{\parallel\parallel})} \right], \quad \textcircled{01}$$

Here  $\lambda \in \{0, \perp, \parallel\}$  and  $i \in \{0, \parallel\}$ .

# Independent Relations Among Observables:

$$\textcircled{7} \quad \Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda} = \bar{\Lambda}_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda}, \quad \Lambda_{\perp i} + \Sigma_{\perp i} = -(\bar{\Lambda}_{\perp i} + \bar{\Sigma}_{\perp i}), \quad \Lambda_{\parallel 0} + \Sigma_{\parallel 0} = \bar{\Lambda}_{\parallel 0} + \bar{\Sigma}_{\parallel 0}, \quad \textcircled{06}$$

$$\textcircled{8} \quad \left( \frac{\rho_{00}}{\Lambda_{00}} - \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}} \right) - \left( \bar{\rho}_{00} - \bar{\rho}_{\perp\perp} \right) - \left( \eta_{00} - \eta_{\perp\perp} \right) - \left( \bar{\eta}_{00} - \bar{\eta}_{\perp\perp} \right) \quad \textcircled{02}$$

$$\textcircled{9} \quad \left( \frac{\rho_{00} - \bar{\rho}_{00}}{\Lambda_{00} + \Sigma_{00}} \right) \quad \text{Violation of any of these relations indicates} \quad \textcircled{02}$$

$$\textcircled{10} \quad \left( \frac{\Sigma_{00} + \bar{\Sigma}_{00}}{\Lambda_{00} + \Sigma_{00}} \right) \quad \bullet \text{ CP violation in decay} \quad \bullet \text{ CPT violation in decay} \quad \textcircled{02}$$

$$\textcircled{11} \quad \left( \Lambda_{0\parallel} + \Sigma_{0\parallel} \right) \quad \bullet \text{ CP violation in decay} \quad \bullet \text{ CPT violation in decay} \quad \textcircled{02}$$

$$= \left[ \frac{2X_0 X_{\parallel} \Lambda_{00} \Lambda_{\parallel\parallel} \Lambda_{\perp\perp}^2 (\Lambda_{00} + \Sigma_{00}) (\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel}) (\Lambda_{\perp\perp} + \Sigma_{\perp\perp})}{(\Lambda_{\perp\perp} \Sigma_{00} + \Lambda_{00} \Sigma_{\perp\perp} + 2\Lambda_{\perp\perp} \Lambda_{00}) (\Lambda_{\perp\perp} \Sigma_{\parallel\parallel} + \Lambda_{\parallel\parallel} \Sigma_{\perp\perp} + 2\Lambda_{\perp\perp} \Lambda_{\parallel\parallel})} \right], \quad \textcircled{01}$$

Here  $\lambda \in \{0, \perp, \parallel\}$  and  $i \in \{0, \parallel\}$ .



# Summary

- ✍ A new formalism has been developed to deal with CPT violation in mixing for two vector decay modes of neutral pseudoscalar mesons.
- ✍ Three special cases have also been studied using this formalism.
- ✍ A large number of observables can be constructed from time-dependent decay rates of these modes.
- ✍ Measuring the observables, all the unknown parameters including CPT violation in mixing can be estimated.
- ✍ Due to the relations among observables, CPT violating parameters can be expressed in various ways involving different observables which in turn would allow us to measure those parameters multiple times from a single experiment. It can help to reduce errors in their measurement.
- ✍ The relations among observables can also help in predicting the nature of new Physics contributions (if present).

