BM4040 Mechanobiology

Assignment 2

Deadline: February 23, 2024 **Total marks: 60**

Instructions

1. You have to write down the answers to each question clearly.
2. Submit the assignment in PDF format at this link.

Questions

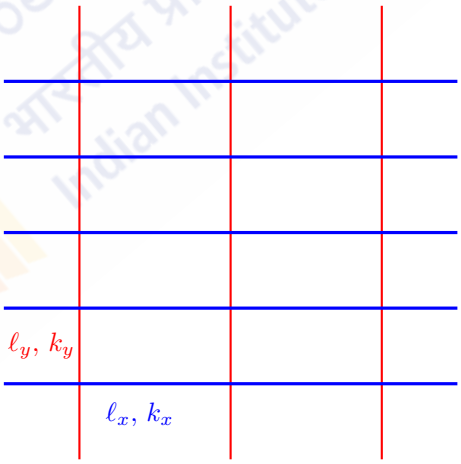
1. **(20)** For networks with four-fold symmetry we have seen that the free energy density is given by

$$U = \frac{K_a}{2}\left(\epsilon_{xx} + \epsilon_{yy}\right)^2 + \frac{\mu_p}{2}\left(\epsilon_{xx} - \epsilon_{yy}\right)^2 + 2\mu_s\epsilon_{xy}^2.$$

By minimizing enthalpy of the network (microscopic perspective) show that for a network under isotropic tension τ the elastic constants are given by

$$K_a = \frac{k_s - \tau}{2}$$
$$\mu_p = \frac{k_s + \tau}{2}$$
$$\mu_a = \tau$$

2. The spring network with four-fold symmetry (as in the last question) can be generalized to a rectangular network of two spring types (two-fold symmetry), as shown in the figure below



(a) **(10)** Derive the expression for macroscopic free energy density.

(b) **(10)**Prove that, once equilibrated at a specific tension τ the strain in the x direction is given by

$$\epsilon_{xx} = \frac{\tau\ell_y k_y + \tau^2 l_x}{k_x k_y - \tau^2}$$

(c) **(10)** Calculate the strain in the y direction under tension τ.

(d) **(10)** To first order in τ, what is the area per vertex as a function of tension?